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ISOCHRONOUS SYSTEMS

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Abstract. Recently – via a simple trick, amounting essentially to a change of independent, and possibly as well of dependent, variables – the possibility has been noted to modify a quite general evolution system so that the modified system possess a lot of *completely periodic*, indeed *isochronous*, solutions. Generally these *isochronous* solutions emerge out of an *open* domain of initial data having full dimensionality in the space of initial data. And many of the isochronous systems obtained in this manner seem rather interesting. In this paper these developments are reviewed, mainly in the context of dynamical systems (systems of ODEs – in particular, systems interpretable as many-body problems), and some specific examples are discussed in detail, including an analysis of the transition (to motions with higher periods, or *aperiodic*, or perhaps *chaotic*) occurring when the initial data get outside of the region producing *isochronous* motions. The applicability of this approach in the context of nonlinear evolution PDEs is also outlined.

Introduction

This review paper covers the material presented at the International Conference on Geometry, Integrability and Quantization held in Varna (Bulgaria) in June 2004, via four lectures organized as follows: 1. Overview: "isochronous systems are not rare"; 2. The "goldfish": theory and simulations; 3. Novel technique to identify solvable dynamical systems and a solvable extension of the goldfish many-body problem. 4. Isochronous PDEs.

Lecture 1

1. Overview

An **isochronous system** is characterized by the property to possess an *open* domain having full dimensionality in phase space such that *all* the motions evolving from