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SYMMETRY APPROACH AND EXACT SOLUTIONS IN HYDRODYNAMICS

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Abstract. The application of symmetry analysis in hydrodynamics is illustrated by two examples. First is a description of all irrotational barochronous motions of ideal gas. The second is an exact solution of magnetohydrodynamics equations for infinitely conducting media, which describes the flow of so called "special vortex" type.

1. Introduction

The group-theoretical method is proved to be one of the most powerful tool for the construction of exact solutions for various nonlinear differential equations [4, 5]. The method is based on the continuous symmetries of the investigated equations. The complete set of the continuous transformations, which preserve the equations, generates its Lie group of symmetries. Each subgroup of the symmetry group gives the source of an exact solution or a symmetry reduction for the equations. The systematic use of group analysis method to study concrete models of mathematical physics consists of the following three steps. These are: calculation of symmetry group, construction of its optimal system of subgroups and obtaining of classes of both invariant and partially invariant solutions. Realization of all these steps is algorithmic and approved for the wide set of mathematical models by many authors.

In the present work we observe two particular examples of exact solutions for Euler equations of ideal compressible fluid and for ideal magnetohydrodynamics equations (MHD). First we describe all irrotational ideal gas motions, which are simultaneously **barochronous**, i.e., $\mathrm{rot}\,\mathbf{u} = \mathbf{0}$ and $p = p(t)$ (pressure depends only on time). This class of solutions is a partially invariant from group-theoretical point of view. The Chupakhin's results on investigation of barochronous gas motions allow to reduce the stated problem to the following: how to describe all