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QUANTUM GROUPS AND STOCHASTIC MODELS

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Abstract. The aim of this paper is to show that stochastic models provide a very good playground to enhance the utility of quantum groups. Quantum groups arise naturally and the deformation parameter has a direct physical meaning for diffusion systems where it is just the ratio of left/right probability rate. In the matrix product state approach to diffusion processes the stationary probability distribution is expressed as a matrix product state with respect to a quadratic algebra which defines a noncommutative space with a quantum group action as its symmetry. Boundary processes amount for the appearance of parameter-dependent linear terms in the algebra which leads to a reduction of the bulk symmetry.

1. Introduction

Stochastic reaction-diffusion processes are of both theoretical and experimental interest not only because they describe various mechanisms in physics and chemistry but they also provide a way of modelling phenomena like traffic flow, kinetics of biopolymerization, interface growth [11, 8, 12].

A stochastic process is described in terms of a master equation for the probability distribution $P(s_i, t)$ of a stochastic variable $s_i = 0, 1, 2, \ldots, n-1$ at a site $i = 1, 2, \ldots, L$ of a linear chain. A configuration on the lattice at a time t is determined by the set of occupation numbers $s_1, s_2, \ldots, s_L$ and a transition to another configuration s' during an infinitesimal time step $\mathrm{d}t$ is given by the probability $\Gamma(s, s')\,\mathrm{d}t$. The time evolution of the stochastic system is governed by the master equation

$$\frac{\mathrm{d}P(s,t)}{\mathrm{d}t} = \sum_{s'} \Gamma(s, s') P(s', t)$$

for the probability $P(s, t)$ of finding the configuration s at a time t. With the restriction of dynamics to changes of configuration only at two adjacent sites the