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ONE DIMENSIONAL QUASI-EXACTLY SOLVABLE DIFFERENTIAL EQUATIONS

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Abstract. In this paper by means of similarity transformation we find some one-dimensional quasi-exactly solvable differential equations and their related Hamiltonians which appear in physical problems. We have provided also two examples with application of these differential equations.

1. Introduction

During the last decade a remarkable new class of quasi-exactly solvable spectral problems was introduced in [5]. These occupy an intermediate position between exactly solvable and unsolvable models in the sense that exact solution in an algebraic form exists only for a part of the spectrum.

In this paper we suggest a generalization of Bender-Dunne [1] approach to possible one-dimensional elliptic quasi-exactly solvable second order differential equations.

For this purpose, and with an attention to applications of elliptic potential we are motivated to obtain generalized master functions $A(x)$ that lead to elliptic quasi-exactly solvable potentials. By appropriate choice of the generalized master function $A(x)$ we obtain some one dimensional quasi-exactly solvable potentials that in all cases are functions of **Jacobi elliptic function**. These functions are periodic functions.

The paper is organized as follows: In Section 2 we show that we can generalize the usual quadratic master function to a master function of at most four order polynomials, then the most general elliptic quasi-exactly solvable differential operators related to generalized master function of degree $k = 3$ and $k = 4$ are given. Also by expanding their solutions in powers of x, we get three-term and four-term recursion relations among their coefficients, where Bender–Dunne factorization follows