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ON THE INVERSE PROBLEM OF THE SCATTERING THEORY FOR A BOUNDARY-VALUE PROBLEM

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Abstract. In the present work the inverse problem of the scattering theory for Sturm-Liouville differential equation with a spectral parameter in the boundary condition is investigated. The Gelfand–Marchenko–Levitan fundamental equation is obtained, the uniqueness of the solution of the inverse problem is proved and some properties of the scattering data are given.

1. Introduction

We consider the boundary problem generated by the differential equation

$$-y'' + q(x)y = \lambda^2 y \quad (0 < x < \infty) \tag{1}$$

and the boundary condition

$$(\alpha_2 + \mathrm{i}\beta_2\lambda)y'(0) - (\alpha_1 + \mathrm{i}\beta_1\lambda)y(0) = 0 \tag{2}$$

where $q(x)$ is a real-valued function satisfying the condition

$$\int_0^{+\infty} (1+x)|q(x)|\,\mathrm{d}x < \infty \tag{3}$$

and α_i, β_i $(i = 1, 2)$ are real numbers such that

$$\delta = \begin{vmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{vmatrix} > 0.$$

In the present work the **inverse problem of scattering theory** (IPST) for the boundary problem of (1)–(3) is investigated. For the equation (1) IPST was completely solved in [6], [9], [10] when the boundary condition (2) was not including any spectral parameter. When the boundary condition (2) was including a spectral parameter, the similar problem was discussed in [7], [8] and the inverse problem with respect to the spectral function was investigated in [12], also with respect to