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# BREATHER SOLUTIONS OF $N$-WAVE EQUATIONS

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**Abstract.** We consider $N$-wave type equations related to symplectic and orthogonal algebras. We obtain their soliton solutions in the case when two different $\mathbb{Z}_2$ reductions (or equivalently one $\mathbb{Z}_2 \times \mathbb{Z}_2$-reduction) are imposed. For that purpose we apply a particular case of an auto-Bäcklund transformation – the Zakharov–Shabat dressing method. The corresponding dressing factor is consistent with the $\mathbb{Z}_2 \times \mathbb{Z}_2$-reduction. These soliton solutions represent $N$-wave breather-like solitons. The discrete eigenvalues of the Lax operators connected with these solitons form "quadruplets" of points which are symmetrically situated with respect to the coordinate axes.

## 1. Introduction

The $N$-wave equation related to a semisimple Lie algebra $\mathfrak{g}$ is a matrix system of nonlinear differential equations of the type

$$\mathrm{i}[J, Q_t(x,t)] - \mathrm{i}[I, Q_x(x,t)] + [[I, Q(x,t)], [J, Q(x,t)]] = 0 \qquad (1)$$

where the squared brackets denote the commutator of matrices and the subscript means a partial derivative with the respect to independent variables $t$ and $x$. The constant matrices $I$ and $J$ are regular elements of the Cartan subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$. The matrix-valued function $Q(x,t) \in \mathfrak{g}$ can be expanded as follows

$$Q = \sum_{\alpha\in\Delta} Q_\alpha(x,t) E_\alpha$$

where $\Delta$ denotes the root system of $\mathfrak{g}$ and $E_\alpha$ are elements of Weyl basis of Lie algebra $\mathfrak{g}$ parametrized by roots of $\mathfrak{g}$. It is also assumed that $Q(x,t)$ satisfies a vanishing boundary condition, i.e., $\lim_{x\to\pm\infty} Q(x,t) = 0$.