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THE MYLAR BALLOON: NEW VIEWPOINTS AND GENERALIZATIONS

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Abstract. Elsewhere we gave a parametrization of the Mylar balloon in terms of elliptic functions. Here, we parametrize other linear Weingarten surfaces using the incomplete Beta function *and* the hypergeometric function. We also provide a corresponding variational characterization and Maple procedures which plot the linear Weingarten surfaces.

1. Introduction

Since the publication of [10], various authors (e.g., [2] and [5]) have studied balloon shapes from different perspectives. In this paper, we want to do two things: first, we want to derive the Mylar balloon from physical principles (based on [2] and [5]); secondly, we want to view the Mylar balloon as a specific example of a linear Weingarten surface and show how a specific parametrization may be derived, both from the defining geometric condition and from a variational problem generalizing the one characterizing the balloon.

The principal curvatures, k_1, k_2, at a point on a surface are the maximum and minimum curvatures of curves through the point obtained by slicing the surface with planes spanned by a chosen tangent vector and the unit normal of the surface at the point. There are certain situations where imposing a condition on principal curvatures characterizes a surface M. For instance, if we require that $k_1 = k_2$ at every point of a compact surface M, then M is a sphere (see [11, Theorem 3.5.1]). If we insist that M be a surface of revolution for which $k_1 = -k_2$, then the mean (or average) curvature vanishes: $H = (k_1 + k_2)/2 = 0$. Hence, M is a *minimal* surface of revolution. The only non-planar surfaces of this kind are *catenoids*,