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DIFFERENTIAL SUBORDINATIONS FOR CERTAIN ANALYTIC FUNCTIONS MISSING SOME COEFFICIENTS

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ABSTRACT. For a positive integer n , applying Schwarz's lemma related to analytic functions $w(z) = c_n z^n + \dots$ in the open unit disk \mathbb{U} , some assertion in a certain lemma which is well-known as Jack's lemma proven by Miller and Mocanu [J. Math. Anal. Appl. 65 (1978), 289–305] is given. Further, by using a certain method of the proof of subordination relation which was discussed by Suffridge [Duke Math. J. 37 (1970), 775–777] and MacGregor [J. London Math. Soc. (2) 9 (1975), 530–536], some differential subordination property concerning with the subordination

$$p(z) \prec q(z^n) \quad (z \in \mathbb{U})$$

for functions $p(z) = a + a_n z^n + \dots$ and $q(z) = a + b_1 z + \dots$ which are analytic in \mathbb{U} is deduced, and an extension of some subordination relation is given.

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