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ON QUASI $*$-PARANORMAL OPERATORS

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ABSTRACT. An operator $T \in B(H)$ is called quasi $*$-paranormal if $||T^*Tx||^2 \leq ||T^3x||||Tx||$ for all $x \in H$. If μ is an isolated point of the spectrum of T, then the Riesz idempotent E of T with respect to μ is defined by

$$E := \frac{1}{2\pi i}\int_{\partial D}(\mu I - T)^{-1}d\mu,$$

where D is a closed disk centered at μ which contains no other points of the spectrum of T. Stampfli [Trans. Amer. Math. Soc., 117 (1965), 469–476], showed that if T satisfies the growth condition G_1, then E is self-adjoint and $E(H) = N(T-\mu)$. Recently, Uchiyama and Tanahashi [Integral Equations and Operator Theory, 55 (2006), 145–151] obtained Stampfli's result for paranormal operators. In general even though T is a paranormal operator, the Riesz idempotent E of T with respect to $\mu \in \text{iso}\sigma(T)$ is not necessary self-adjoint. In this paper 2×2 matrix representation of a quasi $*$-paranormal operator is given. Using this representation we show that if E is the Riesz idempotent for a nonzero isolated point λ_0 of the spectrum of a quasi $*$-paranormal operator T, then E is self-adjoint if and only if the null space of $T - \lambda_0$ satisfies $N(T - \lambda_0) \subseteq N(T^* - \overline{\lambda_0})$. Other related results are also given.

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