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ON CERTAIN PROJECTIONS OF C^*-MATRIX ALGEBRAS

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ABSTRACT. In 1955, H. Dye defined certain projections of a C^*-matrix algebra by

$$\begin{aligned} P_{i,j}(a) &= (1+aa^*)^{-1} \otimes E_{i,i} + (1+aa^*)^{-1}a \otimes E_{i,j} \\ &+ a^*(1+aa^*)^{-1} \otimes E_{j,i} + a^*(1+aa^*)^{-1}a \otimes E_{j,j}, \end{aligned}$$

which was used to show that in the case of factors not of type I_{2n}, the unitary group determines the algebraic type of that factor. We study these projections and we show that in $\mathbb{M}_2(\mathbb{C})$, the set of such projections includes all the projections. For infinite C^*-algebra A, having a system of matrix units, we have $A \simeq \mathbb{M}_n(A)$. M. Leen proved that in a simple, purely infinite C^*-algebra A, the $*$-symmetries generate $\mathcal{U}_0(A)$. Assuming $K_1(A)$ is trivial, we revise Leen's proof and we use the same construction to show that any unitary close to the unity can be written as a product of eleven $*$-symmetries, eight of such are of the form $1-2P_{i,j}(\omega)$, $\omega \in \mathcal{U}(A)$. In simple, unital purely infinite C^*-algebras having trivial K_1-group, we prove that all $P_{i,j}(\omega)$ have trivial K_0-class. Consequently, we prove that every unitary of $\mathcal{O}_n$ can be written as a finite product of $*$-symmetries, of which a multiple of eight are conjugate as group elements.

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