



Ann. Funct. Anal. 5 (2014), no. 2, 138–146

ANNALS OF FUNCTIONAL ANALYSIS

ISSN: 2008-8752 (electronic)

URL: www.emis.de/journals/AFA/

A NORMAL VARIATION OF THE HORN PROBLEM: THE RANK 1 CASE

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*Dedicated to Professor Tsuyoshi Ando,
a valued colleague and friend, with admiration and respect*

Communicated by M. S. Moslehian

ABSTRACT. Given three n -tuples $\{\lambda_i\}_{i=1}^n, \{\mu_i\}_{i=1}^n, \{\nu_i\}_{i=1}^n$ of complex numbers, we introduce the problem of when there exists a pair of normal matrices A and B such that $\sigma(A) = \{\lambda_i\}_{i=1}^n, \sigma(B) = \{\mu_i\}_{i=1}^n$, and $\sigma(A + B) = \{\nu_i\}_{i=1}^n$, where $\sigma(\cdot)$ denote the spectrum. In the case when $\lambda_k = 0, k = 2, \dots, n$, we provide necessary and sufficient conditions for the existence of A and B . In addition, we show that the solution pair (A, B) is unique up to unitary similarity. The necessary and sufficient conditions reduce to the classical A. Horn inequalities when the n -tuples are real.

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Date: Received: November 20, 2013; Accepted: December 5, 2013.

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2010 *Mathematics Subject Classification.* Primary 15A18; Secondary 47B15.

Key words and phrases. The problem of A. Horn, normal matrices, upper Hessenberg.