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A CHARACTERIZATION OF THE INNER PRODUCT SPACES INVOLVING TRIGONOMETRY

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ABSTRACT. In this paper we will give a new characterization of the inner product space which use the trigonometry. We conclude that a normed space $(X, \|\cdot\|)$ is an inner product space if and only if there exists $\alpha \in \mathbb{R}\backslash\pi\mathbb{Q}$ so that

$$\|x\cos\alpha + y\sin\alpha\|^2 + \|y\cos\alpha - x\sin\alpha\|^2 = \|x\|^2 + \|y\|^2,$$

for any $x, y \in X$.

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