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## MAJORIZATION OF SINGULAR INTEGRAL OPERATORS WITH CAUCHY KERNEL ON $L^2$

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*This paper is dedicated to Professor Tsuyoshi Ando*

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ABSTRACT. Let  $a, b, c$  and  $d$  be functions in  $L^2 = L^2(\mathbb{T}, d\theta/2\pi)$ , where  $\mathbb{T}$  denotes the unit circle. Let  $\mathcal{P}$  denote the set of all trigonometric polynomials. Suppose the singular integral operators  $A$  and  $B$  are defined by  $A = aP + bQ$  and  $B = cP + dQ$  on  $\mathcal{P}$ , where  $P$  is an analytic projection and  $Q = I - P$  is a co-analytic projection. In this paper, we use the Helson–Szegő type set  $(HS)(r)$  to establish the condition of  $a, b, c$  and  $d$  satisfying  $\|Af\|_2 \geq \|Bf\|_2$  for all  $f$  in  $\mathcal{P}$ . If  $a, b, c$  and  $d$  are bounded measurable functions, then  $A$  and  $B$  are bounded operators, and this is equivalent to that  $B$  is majorized by  $A$  on  $L^2$ , i.e.,  $A^*A \geq B^*B$  on  $L^2$ . Applications are then presented for the majorization of singular integral operators on weighted  $L^2$  spaces, and for the normal singular integral operators  $aP + bQ$  on  $L^2$  when  $a - b$  is a complex constant.

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