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## KWONG MATRICES AND OPERATOR MONOTONE FUNCTIONS ON $(0, 1)$

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*This paper is dedicated to Professor Tsuyoshi Ando*

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ABSTRACT. In this paper we study positive operator monotone functions on  $(0, 1)$  which have some differences from those on  $(0, \infty)$ : we show that for a concave operator monotone function  $f$  on  $(0, 1)$ , the Kwong matrices  $K_f(s_1, \dots, s_n)$  are positive semidefinite for all  $n$  and  $s_i \in (0, 1)$ , and  $f(s^p)^{1/p}$  for  $0 < p \leq 1$  and  $s/f(s)$  are operator monotone. We also give a sufficient condition for the Kwong matrices to be positive semidefinite.

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