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AN OPERATOR INEQUALITY IMPLYING THE USUAL AND CHAOTIC ORDERS

JUN ICHI FUJII*[1], MASATOSHI FUJII[2] AND RITSUO NAKAMOTO[3]

Dedicated to Professor Tsuyoshi Ando for his significant contributions to our areas

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ABSTRACT. We prove that if positive invertible operators A and B satisfy an operator inequality $(B^{s/2}A^{(s-t)/2}B^tA^{(s-t)/2}B^{s/2})^{\frac{1}{2s}} \geq B$ for some $t > s > 0$, then

(1) If $t \geq 3s - 2 \geq 0$, then $\log B \geq \log A$, and if $t \geq s + 2$ is additionally assumed, then $B \geq A$.

(2) If $0 < s < 1/2$, then $\log B \geq \log A$, and if $t \geq s + 2$ is additionally assumed, then $B \geq A$.

It is an interesting application of the Furuta inequality. Furthermore we consider some related results.

[1] DEPARTMENT OF ARTS AND SCIENCES (INFORMATION SCIENCE), OSAKA KYOIKU UNIVERSITY, KASHIWARA, OSAKA 582-8582, JAPAN.
E-mail address: fujii@cc.osaka-kyoiku.ac.jp

[2] DEPARTMENT OF MATHEMATICS, OSAKA KYOIKU UNIVERSITY, KASHIWARA, OSAKA 582-8582, JAPAN.
E-mail address: mfujii@cc.osaka-kyoiku.ac.jp

[3] 1-4-13, DAIHARA-CHO, HITACHI, IBARAKI 316-0021, JAPAN.
E-mail address: r-naka@net1.jway.ne.jp

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* Corresponding author.
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