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A NOTE ON SOME GENERALIZED SUMMABILITY METHODS

E. SAVAS, P. DAS and S. DUTTA

Abstract. In this paper, we continue our investigations in line of our recent papers, Savas and Das [**16**] and Das, Savas and Ghosal [**5**]. We introduce the notion of $A^{\mathcal{I}}$-statistical convergence which includes the new summability methods studied in [**16**] and [**5**] as special cases and make certain observations on this new and more general summability method.

1. Introduction

The idea of convergence of a real sequence was extended to statistical convergence by Fast [**8**] (see also [**18**]) as follows: If $\mathbb{N}$ denotes the set of natural numbers and $K \subset \mathbb{N}$, then $K(m,n)$ denotes the cardinality of $K \cap [m,n]$. The upper and lower natural (or asymptotic) densities of the subset K are defined by

$$\bar{d}(K) = \limsup_{n\to\infty} \frac{K(1,n)}{n} \quad \text{and} \quad \underline{d}(K) = \liminf_{n\to\infty} \frac{K(1,n)}{n}.$$

If $\bar{d}(K) = \underline{d}(K)$, then we say that the natural density of K exists and it is simply denoted by $d(K)$. Clearly $d(K) = \lim_{n\to\infty} \frac{K(1,n)}{n}$.

A sequence $\{x_k\}_{k\in\mathbb{N}}$ of real numbers is said to be statistically convergent to L if for arbitrary $\varepsilon > 0$, the set $K(\varepsilon) = \{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\}$ has natural density zero. Statistical convergence turned out to be one of the most active areas of research in summability theory after the works of Fridy [**9**] and Šalát [**15**] (also see [**2**], [**3**]).

The notion of statistical convergence was further extended to $\mathcal{I}$-convergence [**12**] using the notion of ideals of $\mathbb{N}$. Many interesting investigations using the ideals can be found in [**5, 6**] where more references are mentioned. In particular, in [**5**] and [**16**] ideals were used to introduce new concepts of $\mathcal{I}$-statistical convergence, $\mathcal{I}$-lacunary statistical convergence and $\mathcal{I}$-λ-statistical convergence. Recently these ideas were extended to double sequences in [**1**].

On the other hand, the idea of A-statistical convergence was introduced by Kolk [**10**] using a non-negative regular matrix A (which subsequently included

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the ideas of statistical, lacunary statistical or λ-statistical convergence as special cases). More recent work in this line can be found in [**7**], [**11**], [**14**] where many references are mentioned.

In this paper, we naturally unify the above two approaches and introduce the idea of $A^{\mathcal{I}}$-statistical convergence and make certain observations.

2. Main results

Throughout the paper $\mathbb{N}$ will denote the set of all positive integers. A family $\mathcal{I} \subset 2^Y$ of subsets of a nonempty set Y is said to be an ideal in Y if (i) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$; (ii) $A \in \mathcal{I}$, $B \subset A$ implies $B \in \mathcal{I}$, while an admissible ideal $\mathcal{I}$ of Y further satisfies $\{x\} \in \mathcal{I}$ for each $x \in Y$. If $\mathcal{I}$ is a proper ideal in Y (i.e., $Y \notin \mathcal{I}$, $Y \neq \emptyset$), then the family of sets $F(\mathcal{I}) = \{M \subset Y :$ there exists $A \in \mathcal{I}$ such that $M = Y \setminus A\}$ is a filter in Y. It is called the filter associated with the ideal $\mathcal{I}$. Throughout, $\mathcal{I}$ will stand for a proper non-trivial admissible ideal of $\mathbb{N}$.

A sequence $\{x_k\}_{k\in\mathbb{N}}$ of real numbers is said to be $\mathcal{I}$-convergent to $x \in \mathbb{R}$ if for each $\varepsilon > 0$, the set $A(\varepsilon) = \{n \in \mathbb{N} : |x_n - x| \geq \varepsilon\} \in \mathcal{I}$ [**12**].

If $x = \{x_k\}_{k\in\mathbb{N}}$ is a sequence of real numbers and $A = (a_{nk})_{n,k=1}^{\infty}$ is an infinite matrix, then Ax is the sequence whose n-th term is given by

$$A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k.$$

We say that x is A-summable to L if $\lim_{n\to\infty} A_n(x) = L$.

Let X and Y be two sequence spaces and $A = (a_{nk})$ be an infinite matrix. If for each $x \in X$, the series $A_n(x) = \sum\limits_{k=1}^{\infty} a_{nk} x_k$ converges for each n and the sequence $Ax = \{A_n(x)\} \in Y$, we say that A maps X into Y. By (X, Y) we denote the set of all matrices which maps X into Y, and in addition, if the limit is preserved, then we denote the class of such matrices by $(X, Y)_{\text{reg}}$. A matrix A is called regular if $A \in (c, c)$ and $\lim_{k\to\infty} A_k(x) = \lim_{k\to\infty} x_k$ for all $x = \{x_k\}_{k\in\mathbb{N}} \in c$ when c, as usual, stands for the set of all convergent sequences. It is well-known that the necessary and sufficient conditions for A to be regular are

$$(R1) \qquad \|A\| = \sup_n \sum_k |a_{nk}| < \infty;$$

$$(R2) \qquad \lim_n a_{nk} = 0, \qquad \text{for each } k;$$

$$(R3) \qquad \lim_n \sum_k a_{nk} = 1.$$

For a non-negative regular matrix $A = (a_{nk})$ following [**10**], a set K is said to have A-density if $\delta_A(K) = \lim_n \sum_{k\in K} a_{nk}$ exists.

The real number sequence $x = \{x_k\}_{k\in\mathbb{N}}$ is A-statistically convergent to L provided that for every $\varepsilon > 0$, the set $K(\varepsilon) := \{k \in \mathbb{N} : \ |x_k - L| \geq \varepsilon\}$ has A-density zero [**10**].

Now we introduce the main concept of this paper, namely the notion of $A^{\mathcal{I}}$-statistical convergence.

Definition 2.1. Let A be a non-negative regular matrix. A sequence $x=\{x_k\}_{k\in\mathbb{N}}$ is said to be $A^{\mathcal{I}}$-statistically convergent to L if for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in \mathbb{N} \; : \; \sum_{k\in K(\varepsilon)} a_{nk} \geq \delta\right\} \in \mathcal{I}$$

where $K(\varepsilon) = \{k \in \mathbb{N} \; : \; |x_k - L| \geq \varepsilon\}$.

In this case we write $x_k \xrightarrow{A^{\mathcal{I}}-st} L$. We will denote the set of all $A^{\mathcal{I}}$-statistically convergent sequences by $S_A(\mathcal{I})$. It can be easily verified that $S_A(\mathcal{I})$ is a linear subspace of the space of all real sequences. Also note that for $\mathcal{I} = \mathcal{I}_{\text{fin}}$, the ideal of all finite subsets of $\mathbb{N}$, $A^{\mathcal{I}}$-statistical convergence becomes A-statistical convergence [**10**].

(1) If we take $A = (a_{nk})$ as

$$a_{nk} = \begin{cases} \dfrac{1}{n} & \text{if } n \geq k \\ 0 & \text{otherwise,} \end{cases}$$

then $A^{\mathcal{I}}$-statistical convergence becomes $\mathcal{I}$-statistical convergence [**5**].

(2) If we take $A = (a_{nk})$ as

$$a_{nk} = \begin{cases} \dfrac{1}{\lambda_n} & \text{if } k \in \mathcal{I}_n = [n - \lambda_n + 1, n] \\ 0 & \text{otherwise,} \end{cases}$$

where $\{\lambda_n\}_{n\in\mathbb{N}}$ is a non-decreasing sequence of positive numbers tending to ∞ and $\lambda_{n+1} \leq \lambda_n + 1$ then A$^{\mathcal{I}}$-statistical convergence coincides with $\mathcal{I}$-λ-statistical convergence [**16**].

(3) By a lacunary sequence $\theta = (k_r), r = 0, 1, 2, \ldots$ where $\mathrm{k}_0 = 0$ we mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $\mathcal{I}_r = (k_{r-1}, k_r]$ and let $h_r = k_r - k_{r-1}$. If $A = (a_{nk})$ is given by

$$a_{nk} = \begin{cases} \dfrac{1}{h_r} & \text{if } k_{r-1} < k \leq k_r \\ 0 & \text{otherwise,} \end{cases}$$

then $A^{\mathcal{I}}$-statistical convergence coincides with $\mathcal{I}$-lacunary statistical convergence [**5**].

Non-trivial examples of such sequences can be seen in ([**5**], [**16**]). We now give another example of a sequence which is $A^{\mathcal{I}}$-statistically convergent.

Example 1. Let $\mathcal{I}$ be a non-trivial admissible ideal of $\mathbb{N}$. Choose an infinite subset

$$C = \{p_1 < p_2 < p_3 < \ldots\}$$

from $\mathcal{I}$. Let $x = \{x_k\}_{k\in\mathbb{N}}$ be given by

$$x_k = \begin{cases} 1 & k \text{ is odd} \\ 0 & k \text{ is even.} \end{cases}$$

Let $A = (a_{nk})$ be given by

$$a_{nk} = \begin{cases} 1 & \text{if } n = p_i, k = 2p_i \text{ for some } i \in \mathbb{N} \\ 1 & \text{if } n \neq p_i, \text{ for any } i, k = 2n+1 \\ 0 & \text{otherwise.} \end{cases}$$

Now for $0 < \varepsilon < 1$, $K(\varepsilon) = \{k \in \mathbb{N} : |x_k - 1| \geq \varepsilon\}$ is the set of all even integers. Observe that

$$\sum_{k\in K(\varepsilon)} a_{nk} = \begin{cases} 1 & \text{if } n = p_i \text{ for some } i \in \mathbb{N} \\ 0 & \text{if } n \neq p_i, \text{ for any } i \in \mathbb{N}. \end{cases}$$

Thus for any $\delta > 0$, $\left\{n \in \mathbb{N} : \sum_{k\in K(\varepsilon)} a_{nk} \geq \delta\right\} = C \in \mathcal{I}$ showing that x is $A^{\mathcal{I}}$-statistically convergent to 1.

Note that for any $L \in \mathbb{R}$ and $0 < \varepsilon < \frac{1}{2}$, $\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ contains either the set of even integers or the set of all odd integers or both and consequently for $\delta = \frac{1}{100}$, $\left\{n \in \mathbb{N} : \frac{|\{k\leq n: |x_k - L| \geq \varepsilon\}|}{n} \geq \delta\right\} \notin \mathcal{I}$ as it must be equal to $\mathbb{N}$ or $\mathbb{N} \setminus \{1\}$. Hence x is not $\mathcal{I}$-statistically convergent. Further note that if $\mathcal{I} \neq \mathcal{I}_d$ and we choose C from $\mathcal{I} \setminus \mathcal{I}_d$, the ideal of all subset of $\mathbb{N}$ with natural density zero, then x is not A-statistically convergent.

We now prove the following result which establishes the topological character of the space $S_A(\mathcal{I})$.

Theorem 2.1. *$S_A(\mathcal{I}) \cap l_\infty$ is a closed subset of l_∞ where as usual, l_∞ is the space of all bounded real sequences endowed with the superior norm.*

Proof. Suppose that $\{x^n\}_{n\in\mathbb{N}} \subset S_A(\mathcal{I}) \cap l_\infty$ is a convergent sequence and it converges to $x \in l_\infty$. We have to show that $x \in S_A(\mathcal{I}) \cap l_\infty$. Let $x^n \xrightarrow{A^{\mathcal{I}}-st} L_n$ for all $n \in \mathbb{N}$. Take a sequence $\{\varepsilon_n\}_{n\in\mathbb{N}}$ where $\varepsilon_n = \frac{1}{2^{n+1}}$ $\forall n \in \mathbb{N}$. We can find $n \in \mathbb{N}$ such that $||x - x^j||_\infty < \frac{\varepsilon_n}{4}$ $\forall$ $j \geq n$. Choose $0 < \delta < \frac{1}{3}$.
Now

$$A = \left\{m \in \mathbb{N} : \sum_{k\in M_1} a_{mk} < \delta\right\} \in F(\mathcal{I}) \text{ where } M_1 = \left\{k \in \mathbb{N} : |x_k{}^n - L_n| \geq \frac{\varepsilon_n}{4}\right\}$$

and

$$B = \left\{m \in \mathbb{N} : \sum_{k\in M_2} a_{mk} < \delta\right\} \in F(\mathcal{I}) \text{ where } M_2 = \left\{k \in \mathbb{N} : |x_k{}^{n+1} - L_{n+1}| \geq \frac{\varepsilon_n}{4}\right\}.$$

Since $A \cap B \in F(\mathcal{I})$ and $\mathcal{I}$ is admissible, $A \cap B$ must be infinite. So we can choose $m \in A \cap B$ such that $|\sum_k a_{mk} - 1| < \frac{\delta}{2}$. But $\sum_{k\in M_1\cup M_2} a_{mk} \leq 2\delta < 1 - \frac{\delta}{2}$ while $\sum_k a_{mk} > 1 - \frac{\delta}{2}$.

Hence there must exist $k \in \mathbb{N}\backslash(M_1 \cup M_2)$ for which we have both $|x_k{}^n - L_n| < \frac{\varepsilon_n}{4}$ and $|x_k{}^{n+1} - L_{n+1}| < \frac{\varepsilon_n}{4}$. Then it follows that

$$\begin{aligned} |L_n - L_{n+1}| &\leq |L_n - x_k{}^n| + |x_k{}^n - x_k{}^{n+1}| + |x_k{}^{n+1} - L_{n+1}| \\ &\leq |L_n - x_k{}^n| + |x_k{}^{n+1} - L_{n+1}| + \|x - x^n\|_\infty + \|x - x^{n+1}\|_\infty \\ &\leq \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} = \varepsilon_n. \end{aligned}$$

This implies that $\{L_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in $\mathbb{R}$. Let $L_n \to L \in \mathbb{R}$ as $n \to \infty$. We shall prove that $x \xrightarrow{A^{\mathcal{I}}-st} L$. Choose $\varepsilon > 0$ and $n \in \mathbb{N}$ such that $\varepsilon_n < \frac{\varepsilon}{4}$, $\|x - x^n\|_\infty < \frac{\varepsilon}{4}$, $|L_n - L| < \frac{\varepsilon}{4}$. Now since

$$\sum_{k\in\{k\in\mathbb{N}:\ |x_k - L| \geq \varepsilon\}} a_{nk} \leq \sum_{k\in\{k\in\mathbb{N}:\ |x_k - x_k{}^n| + |x_k{}^n - L_n| + |L_n - L| \geq \varepsilon\}} a_{nk},$$

it follows that

$$\left\{n \in \mathbb{N} : \sum_{k\in\{k\in\mathbb{N}\ :\ |x_k - L| \geq \varepsilon\}} a_{nk} \geq \delta\right\} \subset \left\{n \in \mathbb{N} : \sum_{k\in\{k\in\mathbb{N}:\ |x_k{}^n - L_n| \geq \frac{\varepsilon}{2}\}} a_{nk} \geq \delta\right\} \in \mathcal{I}$$

for any given $\delta > 0$. Since the set on the right hand side belongs to $\mathcal{I}$, this shows that $x \xrightarrow{A^{\mathcal{I}}-st} L$. This completes the proof of the result. □

Remark 1. We can say that the set of all bounded $A^{\mathcal{I}}$-statistically convergent sequences of real numbers forms a closed linear subspace of l_∞. Also it is obvious that $S_A(\mathcal{I}) \cap l_\infty$ is complete.

We now define another related summability method and establish its relation with $A^{\mathcal{I}}$-statistical convergence.

Definition 2.2. Let $A = (a_{nk})_{n,k=1}^{\infty}$ be a non-negative regular matrix. Then we say that $x = \{x_k\}_{k\in\mathbb{N}}$ is $A^{\mathcal{I}}$-summable to L if the sequence $\{A_n(x)\}_{n\in\mathbb{N}}$ $\mathcal{I}$-converges to L.

For $\mathcal{I} = \mathcal{I}_d$, $A^{\mathcal{I}}$-summability reduces to statistical A-summability of [**7**].

Theorem 2.2. *If a sequence is bounded and $A^{\mathcal{I}}$-statistically convergent to L, then it is $A^{\mathcal{I}}$-summable to L.*

Proof. Let $x = \{x_k\}_{k\in\mathbb{N}}$ be bounded and $A^{\mathcal{I}}$-statistically convergent to L and for $\varepsilon > 0$, let $K(\frac{\varepsilon}{2}) := \{k \in \mathbb{N} : |x_k - L| \geq \frac{\varepsilon}{2}\}$ as before. Then

$$\begin{aligned} |A_n(x) - L| &\leq \left|\sum_{k\notin K(\frac{\varepsilon}{2})} a_{nk}(x_k - L)\right| + \left|\sum_{k\in K(\frac{\varepsilon}{2})} a_{nk}(x_k - L)\right| \\ &\leq \frac{\varepsilon}{2} \sum_{k\notin K(\frac{\varepsilon}{2})} a_{nk} + \sup_k |(x_k - L)| \left|\sum_{k\in K(\frac{\varepsilon}{2})} a_{nk}\right| \leq \frac{\varepsilon}{2} + B. \sum_{k\in K(\frac{\varepsilon}{2})} a_{nk}, \end{aligned}$$

where $B = \sup_k |x_k - L|$. It now follows that

$$\left\{n \in \mathbb{N} : |A_n(x) - L| \geq \varepsilon\right\} \subset \left\{n \in N : \sum_{k \in K(\frac{\varepsilon}{2})} a_{nk} \geq \frac{\varepsilon}{2B}\right\}.$$

Since x is $A^{\mathcal{I}}$-statistically convergent to L, the set on the right hand side belongs to $\mathcal{I}$ and this consequently implies that x is $A^{\mathcal{I}}$-summable to L. □

The converse of the above result is not generally true.

Example 2. Let $A = (a_{nk})$ be given by

$$a_{nk} = \begin{cases} \frac{1}{n+1} & 0 \leq k \leq n+1 \\ 0 & \text{otherwise} \end{cases}$$

and let

$$x_k = \begin{cases} 1 & \text{if k is odd} \\ 0 & \text{if k is even.} \end{cases}$$

Then $x = \{x_k\}_{k \in \mathbb{N}}$ is A-summable to $1/2$, so is $A^{\mathcal{I}}$-summable to $1/2$ for any admissible ideal $\mathcal{I}$. But note that for any $L \in \mathbb{R}$ and for $0 < \varepsilon < \frac{1}{2}$, $K(\varepsilon) = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ contains either the set of all even integers or the set of all odd integers or both. Consequently, $\sum\limits_{k \in K(\varepsilon)} a_{nk} = \infty$ for any $n \in \mathbb{N}$ and so for any $\delta > 0$, $\left\{n \in \mathbb{N} : \sum_{k \in K(\varepsilon)} a_{nk} \geq \delta\right\} \notin \mathcal{I}$. This shows that $x = \{x_k\}_{k \in \mathbb{N}}$ is not $A^{\mathcal{I}}$-statistically convergent for any non-trivial ideal $\mathcal{I}$.

Example 3. As before, let $\mathcal{I}$ be a non-trivial admissible ideal of $\mathbb{N}$. Choose an infinite subset $C = \{p_1 < p_2 < p_3 < \ldots\}$ from $\mathcal{I}$. Let x be the same sequence defined in Example 1. Let $A = (a_{nk})$ be given by

$$a_{nk} = \begin{cases} \frac{1}{2} & \text{if } n \neq p_i \text{ for any } i \in \mathbb{N} \text{ and } k = n^2, n^2+1 \\ 1 & \text{if } n = p_i, k = p_i^2 \\ 0 & \text{otherwise.} \end{cases}$$

Then

$$y_n = \sum_{k=1}^{\infty} a_{nk} x_k = \begin{cases} \frac{1}{2} & \text{if } n \neq p_i \text{ for any } i \in \mathbb{N} \\ 0 & \text{if } n = p_i, p_i^2 \text{ is even} \\ 1 & \text{if } n = p_i, p_i^2 \text{ is odd .} \end{cases}$$

Now

$$\left\{n \in \mathbb{N} : |y_n - \frac{1}{2}| \geq \varepsilon\right\} = C \in \mathcal{I},$$

so x is $A^{\mathcal{I}}$-summable to $\frac{1}{2}$. Note that if $\mathcal{I} \neq \mathcal{I}_d$ and if $C \in \mathcal{I} \setminus \mathcal{I}_d$, then x is not statistically A-summable also.

Further for any $L \in \mathbb{R}$ and $0 < \varepsilon < \frac{1}{2}$, $\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ contains either the set of all even integers or the set of all odd integers or both and hence $\sum_{k \in K(\varepsilon)} a_{nk} \geq \frac{1}{2}$ for all $n \in \mathbb{N} \setminus C$. It is clear that for $0 < \delta < \frac{1}{2}$,

$\Big\{n \in \mathbb{N} : \sum_{k \in K(\varepsilon)} a_{nk} \geq \delta\Big\} \supset \mathbb{N} \setminus C$, so can not belong to $\mathcal{I}$. This shows that x is not $A^{\mathcal{I}}$-statistically convergent.

We now prove that continuity preserves the $A^{\mathcal{I}}$-statistical convergence.

Theorem 2.3. *If for a sequence $x = \{x_k\}_{k \in \mathbb{N}}$, $x_k \xrightarrow{A^{\mathcal{I}}-st} L$ and g is a real valued function which is continuous, then $g(x_k) \xrightarrow{A^{\mathcal{I}}-st} g(L)$.*

Proof. Since g is continuous at $y = L$, for a given $\varepsilon > 0$, there is $\delta > 0$ such that $|y - L| < \delta$ implies $|g(y) - g(L)| < \varepsilon$. Hence $|g(y) - g(L)| \geq \varepsilon$ implies $|y - L| \geq \delta$. In particular, $|g(x_k) - g(L)| \geq \varepsilon$ implies $|x_k - L| \geq \delta$. Thus

$$K = \{k \in \mathbb{N} : \ |g(x_k) - g(L)| \geq \varepsilon\} \subset \ K' := \{k \in \mathbb{N} : \ |x_k - L| \geq \delta\}.$$

Hence for any $\sigma > 0$,

$$\Big\{n \in \mathbb{N} : \sum_{k \in K} a_{nk} \geq \sigma\Big\} \subset \Big\{n \in \mathbb{N} : \sum_{k \in K'} a_{nk} \geq \sigma\Big\} \in \mathcal{I}.$$

Therefore, $g(x_k) \xrightarrow{A^{\mathcal{I}}-st} g(L)$. □

We now establish an equivalent criteria for $A^{\mathcal{I}}$-statistical convergence. For this we will need the following result.

Lemma 2.1 (Ideal version of Dominated Convergence Theorem)**.** *If $\{f_n\}_{n \in N}$ is a sequence of real valued functions with $\mathcal{I}\text{-}\lim_n f_n = f$ and if $|f_n| \leq g$ for all $n \in \mathbb{N}$ for some function $g > 0$ with $\int g < \infty$, then*

$$\mathcal{I}\text{-}\lim_n \int f_n = \int \mathcal{I}\text{-}\lim_n f_n.$$

The proof is parallel to the proof of Lebesgue Dominated Convergence Theorem with little modifications, so it is omitted.

Theorem 2.4. *A sequence $x = \{x_k\}_{k \in \mathbb{N}}$ is $A^{\mathcal{I}}$-statistically convergent to L iff for each real number t, we have*

$$\mathcal{I} - \lim_n \sum_{k=1}^{\infty} a_{nk} \mathrm{e}^{itx_k} = \mathrm{e}^{itL} \tag{1}$$

Proceeding as in [**4**, Theorem 2] and using the ideal version of Bounded convergence Theorem, we can prove this theorem.

Actually we can show that for a sequence $x = \{x_k\}_{k \in \mathbb{N}}$ belonging to the space,

$$S^* = \Big\{x : \{\sum_{k=1}^{\infty} a_{nk}|x_k|\}_{n=1}^{\infty} \in l_\infty\Big\}$$

(1) holds for every rational number t iff x is $A^{\mathcal{I}}$-statistically convergent.

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E. Savas, Istanbul Ticaret University, Department of Mathematics, Üsküdar-Istanbul, Turkey, *e-mail*: `ekremsavas@yahoo.com`

P. Das, Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India, *e-mail*: `pratulananda@yahoo.co.in`

S. Dutta, Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India, *e-mail*: `dutta.sudipta@ymail.com`