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CANAL SURFACES WITH MODIFIED ORTHOGONAL FRAME IN MINKOSWKI 3-SPACE

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Abstract. In this paper, we study canal surfaces with modified orthogonal frame in Minkowski 3-space. Some characterizations of the canal surface will be given using curvature-modified orthogonal frame in Minkowski 3-space.

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1. Introduction

The theory of surfaces has an important role in differential geometry.Canal surfaces, on the other hands, are special class of surface theory and are quite often used in studies. Canal surfaces may be formed by either by sweeping a particular circular cross-section of the sphere along the same path. It is parameterized with the help of the spheres forming itself. A canal surface M can be parameterized as follows:

$$x(s,\theta) = c(s) + r(s)((-r'(s))t + \sqrt{1 - r'(s)^2}(cos\theta n + sin\theta b)) \tag{1}$$

where $c(s)$ is a unit speed curve parameterized by arc-lenght s. $\{t, n, b\}$ is the frame of $c(s)$. $\{t, n, b\}$ is the unit tangent, principal normal,and binormal vector fields, respectively. Here the curve $c(s)$ called center curve and $r(s)$ is called the radial function of M. If the radius function $r(s) = r$, then surface of the canal given by 1;

$$L(s,\theta) = c(s) + r(cos\theta n + sin\theta b) \tag{2}$$

surface is also called tube(pipe)surface[5].

Canal and tube surfaces also have applications in various science. It is used in computer aided design (CAGD) especially for surface modeling, shape reconstruction,planing of robot move-ments,construction of blending surfaces. It is also convenient for viewing long and thin objects, such as pipe, ropes, poles or living intestines.

The canal surfaces first addressed by Monge have been consider from a different angles many geometricians. Xu Z., Feng R. and Sun J. studied the analytical and geometric properties of the canal surface.Karacan M.K.,Es H.,Yaylı Y.,Yoon D.W.,Tuncer Y.,Yüksel N., Bükcü B. consider the canal surfaces and tubular surfaces in Euclidian 3-space,Minkowski space,Galilean and Pseudo space[12, 13, 15, 16]. Doğan F. and Yaylı Y., invastigated canal surfaces and tube surfaces given by the different frames of their curve and generalized some properties of them,for instance the equation of tubular surfaces given by Bishop frame of its spine curve and the equation of tube surface given by Darbox frame of its spine curve[2, 3]

The Lorentz-Minkowski space is the basic space model of quantum that plays an important role in general relativity. In recent years, with development of the theory of relativity physicians and geometers extended the topics in classical differantial geometry of Rieamanian manifolds. It is clearly demonstrated by the fact that many works in Eucldian space have found their counterparts in Minkowski space[7].At present,the properties of canal surfaces have been researched in E^3[6].Similar to generating process of canal surfaces in E^3,a canal surface in Minkowski 3-space E_1^3 can be obtained as the envelop of a family at pseudo spheres S_1^2,pseudohyperbolic spheres H_0^2 or lightlike cones Q^2 whose centers lie on a space curve (resp.spacelike curve,timelike curve or null curve).The classification of canal surfaces was obtained by Ucum A.and Ilarslan K. in[10].Fu X.,Jung S.,Qian J. and Su M. study classify the canal surfaces foaliated by pseudo spheres S_1^2 along a space curve in E_1^3[4].

In this study we examine and illustrade the canal surfaces that were previously made various characterizations and studied according to modified orthogonal frame by sphere $S_1^2(s)$ in Minkowski 3-space. All the surfaces under consideration are assumed to be smooth, regular and topologically connected unless otherwise stated.

2. Preliminaries

In this section, we review some basic facts for curves and canal surfaces in Minkowski 3-space. The Minkowski 3-space E_1^3 is the Euclidian 3-space E^3 equipped with standart flat metric given by $<,>= -dx_1^2 + dx_2^2 + dx_3^2$ where $x = (x_1, x_2, x_3)$ is a rectangular coordinat system of E_1^3.Recall that a vector $v \epsilon E_1^3 - \{0\}$ can be spacelike if $g(v, v) > 0$,timelike if $g(v, v) < 0$ and null(lightlike) if $g(v, v) = 0$ and $v \neq 0$.The norm of a vector v is given $\|v\| = \sqrt{g < v, v >}$ and two vectors v and w are said to be orthogonalif $g(v, w) = 0$.An orbitrary curve $\alpha(s)$ in E_1^3 can locally be *spacelike,timelike* or *null(lightlike)*, if all of its velocity vectors $\alpha'(t) = t$ are respectively spacelike, timelike or null(lightlike)[8].The Lorentizian vector product U and V defined by

$$U \wedge V = (u_3v_2 - u_2v_3, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$$

The formulas of any curve in Minkowski 3-space are given[11].Now let α be any curve in Minkoowski 3-space and suppose that we can reparameterize α by the arc lenght s.We define modified orthogonal frame $\{T, N, B\}$ as follows

$$T = \frac{d\alpha}{ds},\ N = \frac{dT}{ds},\ B = T \wedge N \tag{3}$$

The relations between those and the Frenet frame $\{t, n, b\}$ at non-zero points of curvature κ

$$\begin{aligned} T &= t \\ N &= \kappa n \\ B &= \kappa b \end{aligned} \tag{4}$$

From equation (4) we obtained next theorem.

Theorem 1. *Let $\alpha(s)$ be a unit speed curve classical Frenet frame is $\{t, n, b\}$ and modified orthogonal frame is $\{T, N, B\}$ in Minkowski 3-space[1].*

Case1:If α is a spacelike curve with a spacelike principal normal n, then the modified orthogonal frame is

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\kappa^2 & \frac{\kappa'}{\kappa} & \tau \\ 0 & \tau & \frac{\kappa'}{\kappa} \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{5}$$

$$\begin{aligned} &< T, T >= 1, < N, N >= \kappa^2, < B, B >= -\kappa^2 \\ &< T, N >=< T, B >=< N, B >= 0 \end{aligned}$$

$$T \wedge N = B,\ N \wedge B = -\kappa^2 T,\ B \wedge T = -N$$

Case2:If α is a spacelike curve with a spacelike binormal b, then the modified orthogonal frame is

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \kappa^2 & \frac{\kappa'}{\kappa} & \tau \\ 0 & \tau & \frac{\kappa'}{\kappa} \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{6}$$

$$< T, T >= 1, < N, N >= -\kappa^2, < B, B >= \kappa^2$$
$$< T, N >=< T, B >=< N, B >= 0$$

$$T \wedge N = -B,\ N \wedge B = -\kappa^2 T,\ B \wedge T = N$$

Case3:If α is a timelike curve, then the modified orthogonal frame is

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \kappa^2 & \frac{\kappa'}{\kappa} & \tau \\ 0 & -\tau & \frac{\kappa'}{\kappa} \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{7}$$

$$< T, T >= -1, < N, N >=< B, B > \kappa^2$$
$$< T, N >=< T, B >=< N, B >= 0$$

$$T \wedge N = -B,\ N \wedge B = \kappa^2 T,\ B \wedge T = -N$$

Case4:If α is a pseudo null then the modified orthogonal frame is

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{\kappa'}{\kappa} + \tau & 0 \\ -\kappa^2 & 0 & \frac{\kappa'}{\kappa} - \tau \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{8}$$

$$< T, T >= 1, < N, B >= \kappa^2,$$
$$< N, N >=< B, B >=< T, N >=< T, B >= 0$$

Case5:If α is a null then the modified orthogonal frame is

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \kappa\tau & \frac{\kappa'}{\kappa} & -\kappa \\ 0 & -\tau & \frac{\kappa'}{\kappa} \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{9}$$

$$< T, B >= \kappa, < N, N >= \kappa^2,$$
$$< T, T >=< N, B >=< T, N >=< B, B >= 0$$

Let p be a fixed point in E_1^3 and $r > 0$ be a costant.The pseudo-Rieamannian sphere is defined by

$$S_1^2(p,r) = \{x \epsilon E_1^3 :< x - p, x - p >= r^2\}$$

The surface M in E_1^3 is called a canal surface which is formed as the envelope of a family pseudo spheres S_1^2 whose centers lie on a space curve $c(s)$ framed by $\{T, N, B\}$,then M can be parameterized by

$$x(s,\theta) = c(s) + m(s,\theta)T(s) + p(s,\theta)N(s) + q(s,\theta)B(s) \tag{10}$$

where $m.p$ and q differantiable functions of s and θ.Additionally if M is foliated by pseudo spheres S_1^2 is said to be of type M_+ and canal surface of type M_+ can be divided into three types.If $c(s)$ is spacelike(resp.timelike or null) it is said to be of type M_+^1(resp.M_+^2 or M_+^3).Also M_+^1 can be divided into M_+^{11},M_+^{12} and M_+^{13}.When $c(s)$ is the first kind spacelike curve, the second spacelike curve and null type spacelike curve,respectively[4].

3. Canal Surfaces with Modified Orthogonal Frame in Minkowski 3-Space

In this section we study some properties of different types of canal surfaces formed by the movement of pseudo spheres S_1^2 along a space curve in Minkowski 3-space according to the modified orthogonal frame.

3.1. Canal Surfaces of Type M_+^{11} and M_+^{12}

We assume M be canal surface formed by the movement of the pseudo spheres S_1^2 along a first kind spacelike curve $c(s)$,i.e,M_+^{11}. Now we parameterized M_+^{11} canal surface using the modified orthogonal frame of the $c(s)$ center curve;

$$x(s,\theta) = c(s) + m(s,\theta)T(s) + p(s,\theta)N(s) + q(s,\theta)B(s)$$

$$\|x(s,\theta)-c(s)\| = r(s)$$
$$< x(s,\theta)-c(s), x_s >= 0$$
$$m^2+\kappa^2p^2-\kappa^2q^2 = r^2$$
$$x_s = (1+m_s-p\kappa^2)T(s)+(m+p_s+p\frac{\kappa^{'}}{\kappa}+\tau q)N(s)+(\tau p+q_s+q\frac{\kappa^{'}}{\kappa})B(s)$$
$$m = -rr^{'}$$
$$p = \pm\frac{r(s)}{\kappa(s)}\sqrt{1-r'(s)^2}cosh\theta$$
$$q = \pm\frac{r(s)}{\kappa(s)}\sqrt{1-r'(s)^2}sinh\theta$$

Then M_+^{11} can be parameterized by

$$M_+^{11} = X(s,\theta) = c(s)-r(s)r'(s)T\pm\frac{r(s)}{\kappa(s)}\sqrt{1-r'(s)^2}(cosh\theta N(s)+sinh\theta B(s) \quad (11)$$

where $c(s)$ parameterized by arc lenght s and $\kappa\neq 0$. If the radius function $r(s)=r$ for $\kappa\neq 0$,then the parameterization of the tube surface can be as following;

$$L_+^{11}(s,\theta) = c(s)+\frac{r}{\kappa(s)}(cosh\theta N(s)+sinh\theta B(s)) \quad (12)$$

From (11) we may assume $-r^{'}(s) = cos\mu(s)$ for some smooth function $\mu=\mu(s)$.Then the canal surface M_+^{11} can be written as;

$$X(s,\theta) = c(s)+r(s)(\cos\mu T+\frac{1}{\kappa}\sin\mu\cosh\theta N+\frac{1}{\kappa}\sin\mu\sinh\theta B) \quad (13)$$

$\mu\epsilon[0,\pi]$. Inıtally we have,

$$X_s = X_s^1T+X_s^2N+X_s^3B \qquad X_\theta = X_\theta^1T+X_\theta^2N+X_\theta^3B \quad (14)$$

$$X_s^1 = \sin^2\mu-rr''-r\kappa\sin\mu\cosh\theta$$
$$X_s^2 = \frac{1}{\kappa}(r'\sin\mu\cosh\theta-\kappa rr'-\mu'rr'\cosh\theta+\tau r\sin\mu\sinh\theta)$$
$$X_s^3 = \frac{1}{\kappa}(r'\sin\mu\sinh\theta+\tau r\sin\mu\cosh\theta-\mu'rr'\sinh\theta) \quad (15)$$

$$X_\theta^1 = 0$$
$$X_\theta^2 = \frac{r}{\kappa}\sin\mu\sinh\theta$$
$$X_\theta^3 = \frac{r}{\kappa}\sin\mu\cosh\theta \tag{16}$$

Then the component functions of the first fundamental form are given by;

$$E =< X_s, X_s >$$
$$= (X_s^1)^2 + \kappa^2(X_s^2)^2 - \kappa^2(X_s^3)^2$$

$$= \sin^2\mu + r^2(\kappa^2\sin^2\mu\cosh^2\theta + r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 +$$
$$2\mu'\kappa\cosh\theta - 2r'\tau\kappa\sin\mu\sinh\theta) - 2(rr^{''} + \kappa r\sin\mu\cosh\theta)$$

$$F =< X_s, X_\theta >$$
$$= \kappa^2 X_s^2 X_\theta^2 - \kappa^2 X_s^3 X_\theta^3$$
$$= -\tau r^2\sin^2\mu - \kappa r^{'} r^2\sin\mu\sinh\theta$$
$$G =< X_\theta, X_\theta >$$
$$= \kappa^2(X_\theta^2)^2 - \kappa^2(X_\theta^3)^2$$
$$= -r^2\sin^2\mu \tag{17}$$

And $EG - F^2 = -r^2(rr'' + \kappa r\sin\mu\cosh\theta - \sin^2\mu)^2$ From (15) and (16) we have,

$$X_s \wedge X_\theta = \begin{vmatrix} . & . & . \\ X_s^1 & X_s^2 & X_s^3 \\ X_\theta^1 & X_\theta^2 & X_\theta^3 \end{vmatrix}$$

$$= -\kappa^2(X_s^2 X_\theta^3 - X_s^3 X_\theta^2)T + (X_\theta^3 X_S^1)N + (X_s^1 X_\theta^2)B$$
$$= -rr'(-rr'' - \kappa r\sin\mu\cosh\theta + \sin^2\mu)T$$
$$+\frac{r}{\kappa}(-rr'' - \kappa r\sin\mu\cosh\theta + \sin^2\mu)(\sin\mu\cosh\theta)N$$
$$+\frac{r}{\kappa}(-rr'' - \kappa r\sin\mu\cosh\theta + \sin^2\mu)(\sin\mu\sinh\theta)B \tag{18}$$

$$\begin{aligned}\|X_s \wedge X_\theta\| &= \sqrt{EG - F^2} \\ &= r(-rr'' - \kappa r \sin\mu\cos\theta + \sin^2\mu)\end{aligned}$$

The unit normal vector field U to M_+^{11} is given by

$$\begin{aligned}U &= \frac{X_s \wedge X_\theta}{\|X_s \wedge X_\theta\|} \\ U(s) &= \cos\mu T + \frac{1}{\kappa}\sin\mu\cosh\theta N + \frac{1}{\kappa}\sin\mu\sinh\theta B\end{aligned} \tag{19}$$

$$\begin{aligned}U_s = (-r'' - \kappa\sin\mu\cosh\theta)T + \frac{1}{\kappa}(-r'\kappa - \mu' r' \cosh\theta + \tau\sin\mu\sinh\theta)N \\ + \frac{1}{\kappa}(-r'\mu' \sinh\theta + \tau\sin\mu\cosh\theta)B\end{aligned}$$

$$U_\theta = \frac{1}{\kappa}(\sin\mu\sinh\theta)N + \frac{1}{\kappa}(\sin\mu\cosh\theta)B \tag{20}$$

Then the component functions of the second fundamental form are given by

$$L = - < X_s, U_s >$$

$$\begin{aligned}= -r(\kappa^2\sin^2\mu\cosh^2\theta + r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 + \\ 2\mu'\kappa\cosh\theta - 2r'\tau\kappa\sin\mu\sinh\theta) + (r'' + \kappa\sin\mu\cosh\theta)\end{aligned}$$

$$\begin{aligned}M &= - < X_\theta, U_s > \\ &= \tau r\sin^2\mu + \kappa r r'\sin\mu\sinh\theta \\ N &= - < X_\theta, U_\theta > \\ &= r\sin^2\mu\end{aligned} \tag{21}$$

And as well component functions of the third fundamental form are given by

$$e = < U_s, U_s >$$

$$\begin{aligned}= \kappa^2\sin^2\mu\cosh^2\theta + r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 \\ +2\mu'\kappa\cosh\theta - 2r'\tau\kappa\sin\mu\sinh\theta\end{aligned} \tag{22}$$

$$\begin{aligned} f &= < U_\theta, U_s > \\ &= -\tau \sin^2\mu - \kappa r' \sin\mu \sinh\theta \\ g &= < U_\theta, U_\theta > \\ &= -\sin^2\mu \end{aligned} \tag{23}$$

Lemma 2. *The first,second and third fundamental forms of canal surface M_+^{11} satisfy;*

$$\begin{array}{lll} L = \dfrac{E + P_1}{-r} & M = \dfrac{F}{-r} & N = \dfrac{G}{-r} \\ e = \dfrac{L - Q_1}{-r} & f = \dfrac{M}{-r} & g = \dfrac{N}{-r} \\ EG - F^2 = -r^2 P_1^2 & LN - M^2 = -rP_1Q_1 & eg - f^2 = -Q_1^2 \end{array} \tag{24}$$

$$\begin{aligned} P_1 &= rr'' + \kappa r \sin\mu \cosh\theta - \sin^2\mu = rQ_1 - \sin^2\mu \\ Q_1 &= r'' + \kappa \sin\mu \cosh\theta \end{aligned} \tag{25}$$

Remark 1. *Due to regularity, we see that $P_1 \neq 0$ everywhere by (24);*

From Lemma.2 the Gaussian curvature K the mean curvature H of M_+^{11} are given by respectively.

$$K = \frac{LN - M^2}{EG - F^2} = \frac{Q_1}{rP_1} \tag{26}$$

$$H = \frac{EN + GL - 2FM}{2(EG - F^2)} = \frac{\sin^2\mu + 2P_1}{-2rP_1} \tag{27}$$

Now we denote M_+^{12} canal surface according to modified orthogonal frame.By the definitaion of M_+^{12} and from (10) we get,

$$\begin{aligned} & m^2 - \kappa^2 p^2 + \kappa^2 q^2 = r^2 \\ & m = -rr' \\ & p = \pm \frac{r(s)}{\kappa(s)} \sqrt{1 - r'(s)^2} sinh\theta \\ & q = \pm \frac{r(s)}{\kappa(s)} \sqrt{1 - r'(s)^2} cosh\theta \end{aligned}$$

$$M_+^{12} = X(s,\theta) = c(s) - r(s)r'(s)T \pm \frac{r(s)}{\kappa(s)} \sqrt{1 - r'(s)^2} (\sinh\theta N(s) + cosh\theta B(s) \tag{28}$$

where $c(s)$ parameterized by arc lenght s and $\kappa \neq 0$. If the radius function $r(s) = r$ for $\kappa \neq 0$,then the parameterization of the tube surface can be as following;

$$L_{+}^{12}(s,\theta) = c(s) + \frac{r}{\kappa(s)}(sinh\theta N(s) + cosh\theta B(s)) \tag{29}$$

From (28) we may assume $-r^{'}(s) = cos\mu(s)$ for some smooth function $\mu = \mu(s)$.Then the canal surface M_{+}^{12} can be written as;

$$X(s,\theta) = c(s) + r(s)(\cos\mu T + \frac{1}{\kappa}\sin\mu\sinh\theta N + \frac{1}{\kappa}\sin\mu\cosh\theta B) \tag{30}$$

$\mu\epsilon[0,\pi]$. From (14) we have,

$$\begin{aligned} X_s^1 &= \sin^2\mu - rr'' + r\kappa\sin\mu\sinh\theta \\ X_s^2 &= \frac{1}{\kappa}(r'\sin\mu\sinh\theta - \kappa rr' - \mu' rr'\sinh\theta + \tau r\sin\mu\cosh\theta) \\ X_s^3 &= \frac{1}{\kappa}(r'\sin\mu\cosh\theta + \tau r\sin\mu\sinh\theta - \mu' rr'\cosh\theta) \end{aligned} \tag{31}$$

$$\begin{aligned} X_\theta^1 &= 0 \\ X_\theta^2 &= \frac{r}{\kappa}\sin\mu\cosh\theta \\ X_\theta^3 &= \frac{r}{\kappa}\sin\mu\sinh\theta \end{aligned} \tag{32}$$

Then the component functions of the first fundamental form are given by;

$$\begin{aligned} E = \sin^2\mu + r^2(\kappa^2\sin^2\mu\sinh^2\theta - r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 - \\ 2\mu'\kappa\sinh\theta + 2r'\tau\kappa\sin\mu\cosh\theta) - 2(rr^{''} - \kappa r\sin\mu\sinh\theta) \end{aligned}$$

$$\begin{aligned} F &= -\tau r^2\sin^2\mu + \kappa r^{'} r^2\sin\mu\cosh\theta \\ G &= -r^2\sin^2\mu \end{aligned} \tag{33}$$

$$EG - F^2 = -r^2(rr'' - \kappa r\sin\mu\sinh\theta - \sin^2\mu)^2$$

$$U(s) = \cos\mu T + \frac{1}{\kappa}\sin\mu\sinh\theta N + \frac{1}{\kappa}\sin\mu\cosh\theta B \tag{34}$$

$$U_s = (-r^{''} + \kappa \sin\mu \sinh\theta)T + \frac{1}{\kappa}(-r^{'}\kappa - \mu^{'}r^{'}\sinh\theta + \tau\sin\mu\cosh\theta)N + \frac{1}{\kappa}(-r^{'}\mu^{'}\cosh\theta + \tau\sin\mu\sinh\theta)B$$

$$U_\theta = \frac{1}{\kappa}(\sin\mu\cosh\theta)N + \frac{1}{\kappa}(\sin\mu\sinh\theta)B \tag{35}$$

Then the component functions of the second fundamental form are given by

$$L = -r(\kappa^2\sin^2\mu\sinh^2\theta - r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 - 2\mu'\kappa\sinh\theta + 2r'\tau\kappa\sin\mu\sinh\theta) + (r^{''} - \kappa\sin\mu\sinh\theta)$$

$$\begin{aligned} M &= \tau r \sin^2\mu - \kappa r r' \sin\mu\cosh\theta \\ N &= r\sin^2\mu \end{aligned} \tag{36}$$

And as well component functions of the third fundamental form are given by

$$e = \kappa^2\sin^2\mu\sinh^2\theta - r'^2\kappa^2 - \tau^2\sin^2\mu + \mu'^2 - 2\mu'\kappa\sinh\theta + 2r'\tau\kappa\sin\mu\cosh\theta$$

$$\begin{aligned} f &= -\tau\sin^2\mu + \kappa r'\sin\mu\cosh\theta \\ g &= -\sin^2\mu \end{aligned} \tag{37}$$

Lemma 3. *The first,second and third fundamental forms of canal surface M_+^{12} satisfy;*

$$\begin{aligned} L &= \frac{E+P_2}{-r} & M &= \frac{F}{-r} & N &= \frac{G}{-r} \\ e &= \frac{L-Q_2}{-r} & f &= \frac{M}{-r} & g &= \frac{N}{-r} \\ EG - F^2 &= -r^2P_2^2 & LN - M^2 &= -rP_2Q_2 & eg - f^2 &= -Q_2^2 \end{aligned} \tag{38}$$

$$\begin{aligned} P_2 &= rr'' - \kappa r\sin\mu\sinh\theta - \sin^2\mu = rQ_2 - \sin^2\mu \\ Q_2 &= r'' - \kappa\sin\mu\sinh\theta \end{aligned} \tag{39}$$

Remark 2. *Due to regularity, we see that* $P_2 \neq 0$ *everywhere by (38);*

From Lemma.3 the Gaussian curvature K *the mean curvature* H *of* M_+^{12} *are given by respectively.*

$$K = \frac{LN - M^2}{EG - F^2} = \frac{Q_2}{rP_2} \tag{40}$$

$$H = \frac{EN + GL - 2FM}{2(EG - F^2)} = \frac{\sin^2 \mu + 2P_2}{-2rP_2} \tag{41}$$

Theorem 4. *The Gaussian curvature* K *and mean curvature* H *of canal surface* M_+^{11} *and* M_+^{12} *satisfy;*

$$H = \frac{-1}{2}(Kr + \frac{1}{r}) \tag{42}$$

Proof. For M_+^{11} from (26) and (27), for M_+^{12} from (40) and (41) we get conclusion.

Theorem 5. M_+^{11} *canal surface is not flat surface.*

Proof. We assume that M_+^{11} is flat.In case Gaussian curvature $K \equiv 0$.By (26) we have $Q_1 \equiv 0$.From (25) we get

$$r^{''}(s) + \kappa(s) \sin \mu(s) \cosh \theta = 0$$

It follows that $r^{''} = 0$ and $\kappa(s) \sin \mu(s) = 0$. Since M_+^{11} is regular, $\sin \mu \neq 0$. In case $r^{''}(s)$ and $\kappa(s) = 0$.If $\kappa(s) = 0$,then M_+^{11} canal surface is non regular. Therefore M_+^{11} is not flat.

Theorem 6. M_+^{11} *canal surface is not minimal surface.*

Proof. We asuume that M_+^{11} is minimal. So that mean curvature $H \equiv 0$,then (27) implies

$$2P_1 + \sin^2 \mu = 0$$

from (25) we get

$$2rr^{''} - \sin^2 \mu + 2\kappa r \sin \mu \cosh \theta = 0$$

Therefore $2rr^{''} - \sin^2 \mu = 0$ and $\kappa r \sin \mu \cosh \theta = 0$. Since $r \neq 0, \sin \mu \neq 0$ then $\kappa = 0$. Hence M_+^{11} canal surface is non regular. So M_+^{11} is not minimal.

For M_+^{12} by doing similar calculations according to M_+^{11} same proofs and results are obtained.

Theorem 7. M_+^{12} *canal surface is not flat surface.*

Theorem 8. M_+^{12} *canal surface is not minimal surface.*

Corollary 9. *Since $\kappa = 0$ is not, M_+^{11} and M_+^{12} is not revolution surface.*

Definition 1. *For a pair $(X,Y), X \neq Y$ of the curvatures K, H of a canal surface M,if M satisfies*

$$\Phi = (X,Y) = 0$$

then it is said to be a (X,Y)-Weingarten canal surface,where Φ is the Jacobi function defined by $\Phi = XY - YX$[6]

Definition 2. *For a pair $(X,Y), X \neq Y$ of the curvatures K, H of a canal surface M,if M satisfies*

$$aX + bY = c$$

then it is said to be a (X,Y)-linear Weingarten canal surface,where $(a,b,c)\epsilon R$ and$(a,b,c) \neq (0,0,0)$[9]

Lemma 10. *Partial derivates of the K Gaussian curvature and H mean curvature of the surface of the canal surface M_+^{11} are as follows;*

$$K_s = \frac{1}{r^2 P_1^2}(-2rr^{'}\kappa^2 \sin^2\mu \cosh^2\theta + (r^{'}\kappa - r\kappa^{'})\sin^3\mu\cosh\theta - 5rr^{'}r^{''}\kappa\sin\mu\cosh\theta + r^{'}r^{''}\sin^2\mu - rr^{'''}\sin^2\mu - 4rr^{'}r^{''2}) \quad (43)$$

$$K_\theta = -\frac{\kappa\sin^3\mu\sinh\theta}{rP_1^2} \quad (44)$$

$$H_s = \frac{1}{2r^2 P_1^2}(2r^2r^{'}\kappa^2\sin^2\mu\cosh^2\theta - (2rr^{'}\kappa - r^2\kappa^{'})\sin^3\mu\cosh\theta + 5r^2r^{'}r^{''}\kappa\sin\mu\cosh\theta - 2rr^{'}r^{''}\sin^2\mu + r^2r^{'''}\sin^2\mu + 4r^2r^{'}r^{''2} + r^{'}\sin^4\mu) \quad (45)$$

$$H_\theta = \frac{\kappa\sin^3\mu\sinh\theta}{2P_1^2} \quad (46)$$

Theorem 11. *M_+^{11} canal surface is a (K,H) -Weingarten canal surface if and only if it is a tube surface.*

Proof. $\Rightarrow$: (K,H) -Weingarten canal surface M_+^{11} satisfies Jacobi equation

$$H_s K_\theta = H_\theta K_s \quad (47)$$

from (43), (44), (45) and (46)

$$(Kr^{'} - \frac{r^{'}}{r^2})K_\theta = 0 \tag{48}$$

by (48) we consider $K_\theta = 0$ of M_+^{11} from (44) $\sin\mu \neq 0$ (or else $P_1 = 0$,M_+^{11} is not regular) we have $\kappa = 0$ and M_+^{11} is non regular. Thus $K_\theta \neq 0$. Then we have

$$r^{'}(K - \frac{1}{r^2}) = 0$$

from (48) $K \neq \frac{1}{r^2}$ otherwise M_+^{11} is not regular.

$$K = \frac{Q_1}{rP_1} = \frac{1}{r^2}$$

from (25) $\sin\mu \neq 0$. Hence $r^{'} = 0$ on $M_+^{11}, r = constant$ and M_+^{11} is tube.

⇐:If M_+^{11} a tube (i.e $r = constant$),then from (25)-(27) we have,

$$\begin{aligned} P_1 &= \kappa r \cosh\theta - 1 \\ Q_1 &= \kappa \cosh\theta \\ K &= \frac{\kappa\cosh\theta}{r(r\kappa\cosh\theta - 1)} \\ H &= \frac{-2r\kappa\cosh\theta + 1}{2r(r\kappa\cosh\theta - 1)} \end{aligned}$$

Therefore their partial derivate are given by

$$K_s = -\frac{\kappa^{'}\cosh\theta}{rP_1^2} \tag{49}$$

$$K_\theta = -\frac{\kappa\sinh\theta}{rP_1^2} \tag{50}$$

$$H_s = \frac{\kappa^{'}\cosh\theta}{2P_1^2} \tag{51}$$

$$H_\theta = \frac{\kappa\sinh\theta}{2P_1^2} \tag{52}$$

By (49)-(52) the Jacobi equation (47) is satisfied everywhere.

Theorem 12. *Let M_+^{11} be a linear Weingarten canal surface . Then a tube with radius $r = -\dfrac{b}{a}$*

Proof. A (K, H)-linear Weingarten canal surfaces satisfies

$$aK + bH = 1$$

where $a, b \epsilon R$ and $(a, b) \neq 0$ from (3.32) we obtained

$$K(2ar - br^2) = b + 2r$$

By (26) we get

$$\frac{(2ar - br^2)(r'' + \kappa \sin\mu \cosh\theta)}{r(rr'' + \kappa r \sin\mu \cosh\theta - \sin^2\mu)} = b + 2r$$

$$2\kappa(r^2 + br - a)\sin\mu\cosh\theta + 2(r^2 + br - a)r'' - (b + 2r)(1 - r'^2) = 0$$

Therefore we get

$$\kappa(r^2 + br - a)\sin\mu = 0,$$

$$2(r^2 + br - a)r'' - (b + 2r)(1 - r'^2) = 0$$

Case1: If $r^2 + br - a \neq 0$ then $\kappa = 0$. Thus M_+^{11} is non regular.

Case2:If $\kappa \neq 0$ then $r^2 + br - a = 0$. Hence $r = -\dfrac{b}{2}$ is a nonzero constant, M_+^{11} is a tube and a, b satisfy $b^2 + 4a = 0$

Lemma 13. *Partial derivates of the K Gaussian curvature and H mean curvature of the surface of the canal surface M_+^{12} are as follows;*

$$K_s = \frac{1}{r^2 P_2^2}(-2rr'\kappa^2 \sin^2\mu \sinh^2\theta + (-r'\kappa + r\kappa')\sin^3\mu \sinh\theta + 5rr'r''\kappa \sin\mu \sinh\theta + r'r'' \sin^2\mu - rr''' \sin^2\mu - 4rr'r''^2) \quad (53)$$

$$K_\theta = -\frac{\kappa \sin^3\mu \cosh\theta}{rP_2^2} \quad (54)$$

$$H_s = \frac{1}{2r^2 P_2^2}(2r^2 r'\kappa^2 \sin^2\mu \sinh^2\theta - (-2rr'\kappa + r^2\kappa')\sin^3\mu \sinh\theta - 5r^2 r'r''\kappa \sin\mu \sinh\theta - 2rr'r'' \sin^2\mu + r^2 r''' \sin^2\mu + 4r^2 r'r''^2 + r' \sin^4\mu) \quad (55)$$

$$H_\theta = \frac{\kappa \sin^3\mu \cosh\theta}{2P_2^2} \quad (56)$$

Theorem 14. *M_+^{12} canal surface is a (K, H) -Weingarten canal surface if and only if it is a tube surface.*

Theorem 15. *Let M_+^{12} be a linear Weingarten canal surface . Then a tube with radius $r = -\dfrac{b}{a}$*

3.2. Canal Surfaces of Type M_+^2

In this part we denote M_+^2 canal surface according to modified orthogonal frame.By the definitaion of M_+^2 and from (10) we get,

$$\begin{aligned}
&-m^2 + \kappa^2 p^2 + \kappa^2 q^2 = r^2 \\
&m = rr^{'} \\
&p = \pm \frac{r(s)}{\kappa(s)} \sqrt{1 + r'(s)^2} cos\theta \\
&q = \pm \frac{r(s)}{\kappa(s)} \sqrt{1 + r'(s)^2} sin\theta
\end{aligned}$$

$$M_+^2 = X(s, \theta) = c(s) + r(s)r'(s)T \pm \frac{r(s)}{\kappa(s)} \sqrt{1 + r'(s)^2} (\cos\theta N(s) + sin\theta B(s) \quad (57)$$

where $c(s)$ parameterized by arc lenght s and $\kappa \neq 0$. If the radius function $r(s) = r$ for $\kappa \neq 0$,then the parameterization of the tube surface can be as following;

$$L_+^2(s, \theta) = c(s) + \frac{r}{\kappa(s)} (cos\theta N(s) + sin\theta B(s)) \quad (58)$$

From (57) we may assume $r^{'}(s) = \tan\mu(s)$ for some smooth function $\mu = \mu(s)$.Then the canal surface M_+^2 can be written as;

$$X(s, \theta) = c(s) + r(s)(\tan\mu T + \frac{1}{\kappa} \sec\mu \cos\theta N + \frac{1}{\kappa} \sec\mu \sin\theta B) \quad (59)$$

$\mu \epsilon [-\frac{\pi}{2}, \frac{\pi}{2}]$. From (14) we have,

$$\begin{aligned}
X_s^1 &= \sec^2\mu + rr'' + r\kappa \sec\mu \cos\theta \\
X_s^2 &= \frac{1}{\kappa}(r' \sec\mu \cos\theta + \kappa rr' + \mu' rr' \sec\mu \cos\theta - \tau r \sec\mu \sin\theta) \\
X_s^3 &= \frac{1}{\kappa}(r' \sec\mu \sin\theta + \tau r \sec\mu \cos\theta + \mu' rr' \sec\mu \sin\theta)
\end{aligned} \quad (60)$$

$$
\begin{aligned}
X_\theta^1 &= 0 \\
X_\theta^2 &= -\frac{r}{\kappa} \sec\mu \sin\theta \\
X_\theta^3 &= \frac{r}{\kappa} \sec\mu \cos\theta
\end{aligned}
\tag{61}
$$

Then the component functions of the first fundamental form are given by;

$$
\begin{aligned}
E = \sec^2\mu + r^2(-\kappa^2 \sec^2\mu \cos^2\theta + r'^2\kappa^2 + \tau^2 \sec^2\mu - \mu'^2 \sec^2 - \\
2\mu'\kappa \sec\mu \cos\theta - 2r'\tau\kappa \sec\mu \sin\theta) - 2(rr'' + \kappa r \sec\mu \cos\theta)
\end{aligned}
$$

$$
\begin{aligned}
F &= \tau r^2 \sec^2\mu - \kappa r' r^2 \sec\mu \sin\theta \\
G &= r^2 \sec^2\mu
\end{aligned}
\tag{62}
$$

$$EG - F^2 = -r^2(rr'' + \kappa r \sec\mu \cos\theta + \sec^2\mu)^2$$

$$
U(s) = \tan\mu T + \frac{1}{\kappa} \sec\mu \cos\theta N + \frac{1}{\kappa} \sec\mu \sin\theta B
\tag{63}
$$

$$
\begin{aligned}
U_s = (r'' + \kappa \sec\mu \cos\theta)T + \frac{1}{\kappa}(r'\kappa + \mu' r' \sec\mu \cos\theta - \tau \sec\mu \sin\theta)N \\
+ \frac{1}{\kappa}(-r'\mu' \cos\theta + \tau \sin\mu \sin\theta)B
\end{aligned}
$$

$$
U_\theta = -\frac{1}{\kappa}(\sec\mu \sin\theta)N + \frac{1}{\kappa}(\sec\mu \cos\theta)B
\tag{64}
$$

Then the component functions of the second fundamental form are given by

$$
\begin{aligned}
L = r(\kappa^2 \sec^2\mu \cos^2\theta - r'^2\kappa^2 - \tau^2 \sec^2\mu + \mu'^2 \sec^2\mu + \\
2\mu'\kappa \sec\mu \cos\theta + 2r'\tau\kappa \sec\mu \sin\theta) + (r'' + \kappa \sec\mu \cos\theta)
\end{aligned}
$$

$$
\begin{aligned}
M &= -\tau r \sec^2\mu + \kappa r r' \sec\mu \sin\theta \\
N &= -r^2 \sec^2\mu
\end{aligned}
\tag{65}
$$

$$\begin{aligned} e = & -\kappa^2 \sec^2 \mu \cos^2 \theta + r'^2 \kappa^2 + \tau^2 \sec^2 \mu - \mu'^2 \sec^2 \mu \\ & -2\mu' \kappa \sec \mu \cos \theta - 2r' \tau \kappa \sec \mu \sin \theta \end{aligned}$$

$$\begin{aligned} f &= \tau \sec^2 \mu - \kappa r' \sec \mu \sin \theta \\ g &= \sec^2 \mu \end{aligned} \tag{66}$$

Lemma 16. *The first,second and third fundamental forms of canal surface M_+^2 satisfy;*

$$\begin{array}{lll} L = \dfrac{E + P_3}{-r} & M = \dfrac{F}{-r} & N = \dfrac{G}{-r} \\ e = \dfrac{L - Q_3}{-r} & f = \dfrac{M}{-r} & g = \dfrac{N}{-r} \\ EG - F^2 = -\, r^2 P_3^2 & LN - M^2 = -\, r P_3 Q_3 & eg - f^2 = -Q_3^2 \end{array} \tag{67}$$

$$\begin{aligned} P_3 &= r r'' + \kappa r \sec \mu \cos \theta + \sec^2 \mu = r Q_3 + \sec^2 \mu \\ Q_3 &= r'' + \kappa \sec \mu \cos \theta \end{aligned} \tag{68}$$

Remark 3. *Due to regularity, we see that $P_3 \neq 0$ everywhere by (67);*

*From Lemma.*16 *the Gaussian curvature K the mean curvature H of M_+^2 are given by respectively.*

$$K = \frac{LN - M^2}{EG - F^2} = \frac{Q_3}{r P_3} \tag{69}$$

$$H = \frac{EN + GL - 2FM}{2(EG - F^2)} = \frac{\sec^2 \mu - 2P_3}{2r P_3} \tag{70}$$

Theorem 17. *The Gaussian curvature K and mean curvature H of canal surface M_+^2 satisfy;*

$$H = \frac{-1}{2}(Kr + \frac{1}{r}) \tag{71}$$

For M_+^2 by doing similar calculations according to M_+^{11} same proofs and results are obtained.

Theorem 18. *M_+^2 canal surface is not flat surface.*

Theorem 19. *M_+^2 canal surface is not minimal surface.*

Corollary 20. *Since $\kappa = 0$ is not, M_+^2 is not revolution surface.*

Lemma 21. *Partial derivates of the K Gaussian curvature and H mean curvature of the surface of the canal surface M_+^2 are as follows;*

$$K_s = \frac{1}{r^2 P_3^2}(-2rr'\kappa^2 \sec^2\mu\cos^2\theta + (-r'\kappa + r\kappa')\sec^3\mu\cos\theta - 5rr'r''\kappa\sec\mu\cos\theta - r'r''\sec^2\mu + rr'''\sec^2\mu - 4rr'r''^2) \quad (72)$$

$$K_\theta = -\frac{\kappa\sec^3\mu\sin\theta}{rP_3^2} \quad (73)$$

$$H_s = \frac{1}{2r^2P_3^2}(2r^2r'\kappa^2\sec^2\mu\cos^2\theta - (-2rr'\kappa + r^2\kappa')\sec^3\mu\cos\theta + 5r^2r'r''\kappa\sec\mu\cos\theta - 2rr'r''\sec^2\mu - r^2r'''\sec^2\mu + 4r^2r'r''^2 + r'\sec^4\mu) \quad (74)$$

$$H_\theta = \frac{\kappa\sec^3\mu\sin\theta}{2P_3^2} \quad (75)$$

Theorem 22. *M_+^2 canal surface is a (K, H) -Weingarten canal surface if and only if it is a tube surface.*

Theorem 23. *Let M_+^2 be a linear Weingarten canal surface . Then a tube with radius $r = -\dfrac{b}{a}$*

Corollary 24. *When the canal surface center curve is first,second type of spacelike and timelike, canal surface is not flat, minimal, revolution and developable surface according to modified orthogonal frame. Also surface is not circular cylinders, circular cones and catenoids.*

3.3. Canal Surfaces of Type M_+^{13} and M_+^3

In this part we only give M_+^{13} and M_+^3 canal surfaces parameterizations according to modified orthogonal frame.From (10) we get,

$$m^2 + 2\kappa^2 pq = r^2$$
$$m = -rr'$$
$$2pq = \frac{r^2(s)}{\kappa^2(s)}(1 - r'(s)^2)$$

$$M_+^{13} = X(s,\theta) = c(s) - r(s)r'(s)T + p(s,\theta)N(s) + q(s,\theta)B(s)$$

where

$$2p(s,\theta)q(s,\theta) = \frac{r^2(s)}{\kappa^2(s)}(1 - r'^2(s)) \tag{76}$$

and $\kappa \neq 0$

For M_+^3 from (10) we get,

$$\kappa^2 p^2 + 2\kappa mq = r^2$$
$$\kappa q = -rr'$$

$$M_+^3 = X(s,\theta) = c(s) + m(s,\theta)T + p(s,\theta)N(s) - \frac{r(s)r'(s)}{\kappa(s)}B(s)$$

where

$$\kappa^2(s)p^2(s,\theta) + 2\kappa(s)m(s,\theta)q(s,\theta) = r^2(s) \tag{77}$$

Example 1. *Let us consider c(s) center curve spacelike curve with spacelike binormal α as;*

$$\alpha(s) = (\cosh\frac{\sqrt{5}}{3}s, \sinh\frac{\sqrt{5}}{3}s, \frac{2s}{3})$$

Then its Modified frame is;

$$T = (\frac{\sqrt{5}}{3}\sinh\frac{\sqrt{5}}{3}s, \frac{\sqrt{5}}{3}cosh\frac{\sqrt{5}}{3}s, \frac{2}{3})$$
$$N = (\frac{5}{9}\cosh\frac{\sqrt{5}}{3}s, \frac{5}{9}sinh\frac{\sqrt{5}}{3}s, 0)$$
$$B = (\frac{10}{27}\sinh\frac{\sqrt{5}}{3}s, \frac{10}{27}cosh\frac{\sqrt{5}}{3}s, -\frac{5\sqrt{5}}{27})$$

when the radius function $r(s) = \cos s$ the canal surface as;

$$X(s,\theta) = (\cosh\frac{\sqrt{5}}{3}s + \frac{\sqrt{5}}{3}\cos s\sin s\sinh\frac{\sqrt{5}}{3}s + \cos^2 s\sinh\theta\cosh\frac{\sqrt{5}}{3}s + \frac{2}{3}\cos^2 s\cosh\theta\sinh\frac{\sqrt{5}}{3}s,$$
$$\sin\frac{\sqrt{5}}{3}s + \frac{\sqrt{5}}{3}\cos s\sin s\cosh\frac{\sqrt{5}}{3}s + \cos^2 s\sinh\theta\sinh\frac{\sqrt{5}}{3}s + \frac{2}{3}\cos^2 s\cosh\theta\cosh\frac{\sqrt{5}}{3}s,$$
$$\frac{2s}{3} + \frac{2}{3}\cos s\sin s - \frac{\sqrt{5}}{3}\cos^2 s\cosh\theta)$$

We draw graphics in Figure.1

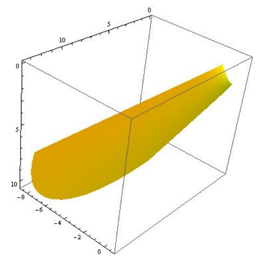

Figure.1 Canal surface M_{+}^{12} with $r(s) = \cos s$.

$$L_{+}^{12}(s,\theta) = (\cosh\frac{\sqrt{5}}{3}s + \sinh\theta\cosh\frac{\sqrt{5}}{3}s + \frac{2}{3}\cosh\theta\sinh\frac{\sqrt{5}}{3}s, \\ \sinh\frac{\sqrt{5}}{3}s + \sinh\theta\sinh\frac{\sqrt{5}}{3}s + \frac{2}{3}\cosh\theta\cosh\frac{\sqrt{5}}{3}s, \frac{2s}{3} - \frac{\sqrt{5}}{3}\cosh\theta)$$

We draw graphics in Figure.2

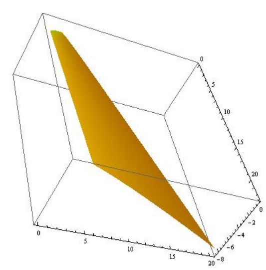

Figure.2 Tube surface L_{+}^{12} with $r(s) = 1$.

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