UNSTEADY FLOW OF A CONDUCTING DUSTY FLUID NEAR AN ACCELERATED PLATE

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Abstract. The Unsteady laminar flow of an electrically conducting dusty viscous incompressible fluid near an accelerated flat, nonconducting plate, under the influence of a uniform magnetic field and pulsatile pressure gradient has been studied using differential geometry techniques i.e., in Frenet frame field system. The particular cases as (i) impulsively moving and (ii) uniformly accelerated motion of plate, have been discussed in detail. Further The velocity profiles for conducting fluid and non-conducting dust particles are determined. The expressions for skin friction at the boundaries are obtained. Finally the conclusions are given on basis the graphs.

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1.Introduction

In fluid mechanics, multi-phase flow is a generalization of the modelling used in two-phase flow i.e., to cases where the two phases are not chemically related (e.g. dusty gases) or where more than two phases are present (e.g. in modelling of propagating steam explosions). Each of the phases is considered to have a separately defined volume fraction and velocity field. Conservation equations for the flow of each species, can then be written down straightforwardly. The momentum equation for each phase is less straightforward. It can be shown that a common pressure field can be defined, and that each phase is subject to the gradient of this field, weighted by its volume fraction. Transfer of momentum between the phases is sometimes less straightforward to determine, and in addition, a very light phase in bubble form has a virtual mass associated with its acceleration.

Considerable work has already been done on such models of dusty fluid flow. Rossow [12] has discussed the flow of a viscous, incompressible and electrically conducting fluid in presence of an external magnetic field due to the impulsive motion of an infinite flat plate. Ong and Nicholls [10] have extended the study to cover the case of flow near an infinite wall which executes simple harmonic motion parallel to itself. P.G.Saffman [13] formulated the basic equations for the flow of dusty fluid. Regarding the plate problems, Liu [7], Michael and Miller [9] have studied the flow produced by the motion of an infinite plane in a steady fluid occupying the semi-infinite space above it. Later, M.C.Baral [4] has discussed the plane parallel flow of conducting dusty gas.

The tools of differential geometry and, in particular, Riemannian geometry have been proved very useful in handling problems in areas of Fluid dynamics. Differential geometry of curves and surfaces has appeared in several areas of physics, ranging from liquid crystals to plasma physics, and from solutions to general relativity, or even in high energy strings and thermodynamics. In all these applications one important common feature arises, which is the application of the Serret-Frenet frame to the motion of curves.

Frenet frames are a central construction in modern differential geometry, in which structure is described with respect to an object of interest rather than with respect to external coordinate systems.

To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [6], Truesdell [15], Indrasena [5], Purushotham [11], Bagewadi, Shantharajappa and Gireesha [14], [1], [2] have applied differential geometry techniques. Further, recently the authors [1], have studied two-dimensional dusty fluid flow in Frenet frame field system, which is one of the moving frame. In this investigation, the differential geometry techniques are used to study the flow of an electrically conducting viscous incompressible fluid with suspended non-conducting small spherical dust particles. The flow of the fluid is due to the influence of uniform magnetic field, accelerated flat plate and pulsatile pressure gradient. Initially both the conducting fluid and the dust particles are assumed to be at rest. Applying Laplace transform technique, the velocity fields for fluid and dust particles have been obtained. Also the particular cases when the flow is due to (i) impulsively moving plate, (ii) uniformly accelerated motion of plate. Further the expressions for skin friction at the boundaries are determined. The graphical representation of velocity profiles are given.

2. Equations of Motion

The modified Saffman's [13] equations for the conducting dusty gas and non-conducting dust particle are:
For fluid phase

$$\nabla.\overrightarrow{u} = 0 \tag{1}$$
$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}.\nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \upsilon\nabla^2\overrightarrow{u} + \frac{KN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) + \frac{1}{\rho}(\overrightarrow{J} \times \overrightarrow{B}) \tag{2}$$

For dust phase

$$\nabla.\overrightarrow{v} = 0 \tag{3}$$
$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \tag{4}$$

We have following nomenclature:
$\overrightarrow{u}-$ velocity of the fluid phase, $\overrightarrow{v}-$ velocity density of dust phase, $\rho-$ Density of the gas, $p-$ Pressure of the fluid, $N-$ Number of density of dust particles, $\upsilon-$ Kinematic viscosity, $K = 6\pi a\mu-$ Stoke's resistance (drag coefficient), $a-$ Spherical radius of dust particle, $m-$ Mass of the dust particle, $\mu-$ the co-efficient of viscosity of fluid particles, $t-$ time and $\overrightarrow{J}-$ and $\overrightarrow{B}-$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla \times \overrightarrow{H} = 4\pi\overrightarrow{J},\ \nabla \times \overrightarrow{B} = 0,\ \nabla \times \overrightarrow{E} = 0,\ \overrightarrow{J} = \sigma[\overrightarrow{E} + \overrightarrow{u} \times \overrightarrow{B}]$$

Here $\overrightarrow{H}-$Magnetic field, $\overrightarrow{J}-$ Current density, $\overrightarrow{B}-$ Magnetic Flux, $\overrightarrow{E}$-Electric field.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\overrightarrow{J} \times \overrightarrow{B}$ of the body force in (2) reduces simply to $-\sigma B_0^2\overrightarrow{u}$.

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively, Geometrical relations are given by Frenet formulae [3]

$$
\begin{aligned}
&i) \quad \frac{\partial \overrightarrow{s}}{\partial s} = k_s \overrightarrow{n}, \ \frac{\partial \overrightarrow{n}}{\partial s} = \tau_s \overrightarrow{b} - k_s \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial s} = -\tau_s \overrightarrow{n} \\
&ii) \quad \frac{\partial \overrightarrow{n}}{\partial n} = k'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial n} = -\sigma'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial n} = \sigma'_n \overrightarrow{b} - k'_n \overrightarrow{n} \\
&iii) \quad \frac{\partial \overrightarrow{b}}{\partial b} = k''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{n}}{\partial b} = -\sigma''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial b} = \sigma''_b \overrightarrow{n} - k''_b \overrightarrow{b} \\
&iv) \quad \nabla . \overrightarrow{s} = \theta_{ns} + \theta_{bs}; \ \nabla . \overrightarrow{n} = \theta_{bn} - k_s; \ \nabla . \overrightarrow{b} = \theta_{nb}
\end{aligned}
\tag{5}
$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. Formulation and Solution of the Problem

Consider a viscous incompressible, dusty fluid bounded by two nonconducting infinite flat plates separated by a distance h. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically nonconducting and neutral. The motion of the fluid is due to magnetic field of uniform strength B_0, accelerated flat plate and pulsatile pressure gradient. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction as shown in the figure 1.

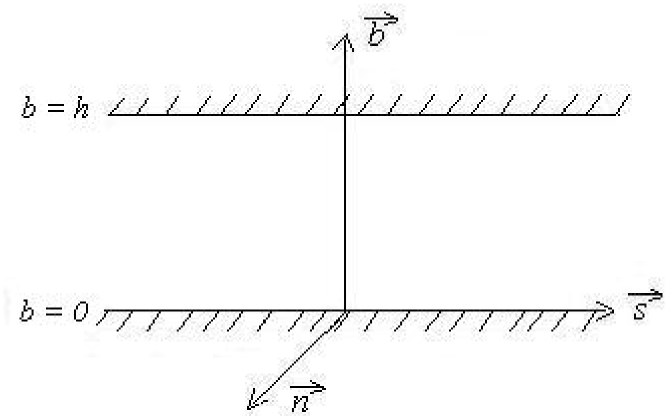

Figure-1: Geometry of the flow

For the above described flow the velocities of fluid and dust are of the form

$$\vec{u} = u_s \vec{s}, \qquad \vec{v} = v_s \vec{s}$$

i.e., $u_n = u_b = 0$ and $v_n = v_b = 0$, where (u_s, u_n, u_b) and (v_s, v_n, v_b) denote the velocity components of fluid and dust respectively.

Since the flow is in between two parallel plates, we can assume the velocity of both fluid and dust particles do not vary along tangential direction. Suppose the fluid extends to infinity in the principal normal direction, then the velocities of both may be neglected in this direction.

By virtue of system of equations (5) the intrinsic decomposition of equations (2) and (4) give the following forms;

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial s} + \nu\left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) - Du_s \tag{6}$$

$$2u_s^2 k_s = -\frac{1}{\rho}\frac{\partial P}{\partial n} + \nu\left[2\sigma_b''\frac{\partial u_s}{\partial b} - u_s k_s^2\right] \tag{7}$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial b} + \nu\left[u_s k_s \tau_s - 2k_b''\frac{\partial u_s}{\partial b}\right] \tag{8}$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s) \tag{9}$$

$$2v_s^2 k_s = 0 \tag{10}$$

where $D = \frac{\sigma B_0^2}{\rho}$ and $C_r = (\sigma_b'^2 + k_n'^2 + k_b'^2 + \sigma_b''^2)$ is called curvature number [2].

From equation (10) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

Equation (6) and (9) are to be solved subject to the initial and boundary conditions;

$$\begin{array}{rl} \text{Initial condition;at } t = 0; & u_s = 0, v_s = 0 \\ \text{Boundary condition;for } t > 0; & u_s = A\ t^n, \text{ at } b = h \\ & u_s = 0, \qquad \text{at } b = 0. \end{array}$$

where A & n are positive constants.

We define Laplace transformations of u_s and v_s as

$$U = \int_0^\infty e^{-xt} u_s dt \quad \text{and} \quad V = \int_0^\infty e^{-xt} v_s dt$$

Suppose $-\frac{1}{\rho}\frac{\partial P}{\partial s} = C + \alpha \cos \beta t$, where C and α are constants and β is the frequency of oscillation.

Applying the Laplace transform to equations (6), (9) and to boundary conditions, then one can obtains

$$xU = \left[\frac{C}{x} + \frac{\alpha x}{x^2 + \beta^2}\right] + \nu \left[\frac{\partial^2 U}{\partial b^2} - C_r U\right] + \frac{L}{\tau}(V - U) - DU \quad (11)$$

$$xV = \frac{1}{\tau}(U - V) \quad (12)$$

$$U = \frac{A\ n!}{x^{n+1}}, \text{ at } b = h \text{ and } U = 0 \text{ at } b = 0 \quad (13)$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (12) implies

$$V = \frac{U}{1 + s\tau} \quad (14)$$

Eliminating V from (11) and (14) we obtain the following equation

$$\frac{d^2U}{db^2} - Q^2 U = -\frac{1}{\nu}\left[\frac{C}{x} + \frac{\alpha x}{x^2 + \beta^2}\right] \quad (15)$$

where $Q^2 = \left(C_r + \frac{s}{\nu} + \frac{D}{\nu} + \frac{sl}{\nu(1+s\tau)}\right)$.

The velocities of fluid and dust particle are obtained by solving the equation (15) subjected to the boundary conditions (13) as follows

$$U = \frac{A\; n!}{x^{n+1}}\frac{\sinh Qb}{\sinh Qh} + \left[\frac{C}{x} + \frac{\alpha x}{x^2+\beta^2}\right]\left[\frac{sinh(Q(b-h)) - sinh(Qb) + sinh(Qh)}{\nu Q^2 sinh(Qh)}\right].$$

Using U in (14) we obtain V as

$$V = \frac{A\; n!}{x^{n+1}(1+x\tau)}\frac{\sinh Qb}{\sinh Qh} + \left[\frac{C}{x} + \frac{\alpha x}{x^2+\beta^2}\right].$$

$$\left[\frac{sinh(Q(b-h)) - sinh(Qb) + sinh(Qh)}{\nu Q^2 (1+x\tau) sinh(Qh)}\right].$$

Case-1: (**Impulsively moving plate**) Suppose $n = 0$, then the flow is due to Impulsively moving plate, then one can obtain the velocity profiles for fluid phase and dust phase as

$$\begin{aligned}
u_s &= \frac{C}{\nu\lambda^2}\left[\frac{sinh(\lambda(b-h)) - sinh(\lambda b)}{sinh(\lambda h)} + 1\right] + \frac{4C}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)} sin\left(\frac{(2r+1)\pi}{h}b\right) \\
&\times \left[\frac{e^{x_1 t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]} + \frac{e^{x_2 t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right] \\
&+ \frac{\alpha}{\nu}\left[\frac{\theta_1\cos\beta t - \theta_2\sin\beta t}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}\right] + \frac{4\alpha}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)} sin\left(\frac{(2r+1)\pi}{h}b\right) \\
&\times \left[\frac{x_1 e^{x_1 t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]} + \frac{x_2 e^{x_2 t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right] + A\frac{\sinh\lambda b}{\sinh\lambda h} \\
&- \frac{2A\pi\nu}{h^2}\sum_{r=0}^{\infty}(-1)^r r\sin\frac{r\pi b}{h}\left[\frac{e^{x_1 t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]} + \frac{e^{x_2 t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}$$

$$\begin{aligned}
v_s &= \frac{C}{\nu\lambda^2}\left[\frac{sinh(\lambda(b-h)) - sinh(\lambda b)}{sinh(\lambda h)} + 1\right] + \frac{4C}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)} sin\left(\frac{(2r+1)\pi}{h}b\right) \\
&\times \left[\frac{e^{x_1 t}(1+x_1\tau)}{x_1\left[(1+x_1\tau)^2+l\right]} + \frac{e^{x_2 t}(1+x_2\tau)}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{\alpha}{\nu}\left[\frac{(\theta_1\cos\beta t-\theta_2\sin\beta t)+\beta\tau(\theta_1\sin\beta t+\theta_2\cos\beta t)}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)(1+\beta^2\tau^2)}\right]\\
&+ \quad \frac{4\alpha}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)}sin\left(\frac{(2r+1)\pi}{h}b\right)\\
&\times \quad \left[\frac{x_1e^{x_1t}(1+x_1\tau)}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]+A\frac{\sinh\lambda b}{\sinh\lambda h}\\
&- \quad \frac{2A\pi\nu}{h^2}\sum_{r=0}^{\infty}(-1)^r r\sin\frac{r\pi b}{h}\left[\frac{e^{x_1t}(1+x_1\tau)}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

Shearing Stress (Skin Friction): The Shear stress at the boundaries $b=0,\quad b=h$ are given by;

$$
\begin{aligned}
D_0 \quad = \quad & \frac{C}{\nu\lambda}\left[\frac{\cosh(\lambda h)-1}{sinh(\lambda h)}\right]+\frac{4C}{h}\sum_{r=0}^{\infty}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
+ \quad & \frac{\alpha[(k_1\alpha_1+k_2\alpha_2)\cos\beta t-(k_2\alpha_1-k_1\alpha_2)\sin\beta t]}{\nu(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}+\frac{A}{\cosh\lambda h}\\
+ \quad & \frac{4\alpha}{h}\sum_{r=0}^{\infty}\left[\frac{x_1e^{x_1t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]\\
- \quad & \frac{2A\pi^2\nu}{h^3}\sum_{r=0}^{\infty}(-1)^r r^2\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
D_h \quad = \quad & \frac{C}{\nu\lambda}\left[\frac{1-\cosh(\lambda h)}{sinh(\lambda h)}\right]-\frac{4C}{h}\sum_{r=0}^{\infty}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

$$
\begin{aligned}
- \quad & \frac{\alpha[(k_1\alpha_1+k_2\alpha_2)\cos\beta t+(k_1\alpha_2-k_2\alpha_1)\sin\beta t]}{\nu(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}+A\\
- \quad & \frac{4\alpha}{h}\sum_{r=0}^{\infty}\left[\frac{x_1e^{x_1t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]\\
- \quad & \frac{2A\pi^2\nu}{h^3}\sum_{r=0}^{\infty}r^2\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

Case-2: If $n=1$, then the flow is due to uniformly accelerated motion of plate, then by taking inverse Laplace transform to U and V, one can obtain the velocity profiles for fluid phase and dust phase as

$$
\begin{aligned}
u_s &= \frac{C}{\nu\lambda^2}\left[\frac{sinh(\lambda(b-h))-sinh(\lambda b)}{sinh(\lambda h)}+1\right]+\frac{4C}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)}sin\left(\frac{(2r+1)\pi}{h}b\right)\\
&\times \left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]+\frac{\alpha}{\nu}\left[\frac{\theta_1\cos\beta t-\theta_2\sin\beta t}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}\right]\\
&+ \frac{4\alpha}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)}sin\left(\frac{(2r+1)\pi}{h}b\right)\\
&\times \left[\frac{x_1e^{x_1t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]\\
&+ \frac{A}{2\lambda\nu}\left[\frac{(2t\nu\lambda\sinh\lambda b+lb\cosh\lambda b)\sinh\lambda h-lh\sinh\lambda b\cosh\lambda h}{\sinh^2\lambda h}\right]\\
&- \frac{2A\pi\nu}{h^2}\sum_{r=0}^{\infty}(-1)^r r\sin\frac{r\pi b}{h}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1^2\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2^2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

$$
\begin{aligned}
v_s &= \frac{C}{\nu\lambda^2}\left[\frac{sinh(\lambda(b-h))-sinh(\lambda b)}{sinh(\lambda h)}+1\right]+\frac{4C}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)}sin\left(\frac{(2r+1)\pi}{h}b\right)\\
&\times \left[\frac{e^{x_1t}(1+x_1\tau)}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
&+ \frac{\alpha}{\nu}\left[\frac{(\theta_1\cos\beta t-\theta_2\sin\beta t)+\beta\tau(\theta_1\sin\beta t+\theta_2\cos\beta t)}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)(1+\beta^2\tau^2)}\right]\\
&+ \frac{4\alpha}{\pi}\sum_{r=0}^{\infty}\frac{1}{(2r+1)}sin\left(\frac{(2r+1)\pi}{h}b\right)\\
&\times \left[\frac{x_1e^{x_1t}(1+x_1\tau)}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]+\frac{A}{2\lambda\nu}\\
&\cdot \left[\frac{(2t\nu\lambda\sinh\lambda b+lb\cosh\lambda b)\sinh\lambda h-(2\tau\lambda\nu\sinh\lambda h+lh\cosh\lambda h)\sinh\lambda b}{\sinh^2\lambda h}\right]\\
&- \frac{2A\pi\nu}{h^2}\sum_{r=0}^{\infty}(-1)^r r\sin\frac{r\pi b}{h}\left[\frac{e^{x_1t}(1+x_1\tau)}{x_1^2\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)}{x_2^2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

Shearing Stress (Skin Friction): The Shear stress at the boundaries

$b=0,\quad b=h$ are given by;

$$
\begin{aligned}
D_0 &= \frac{C}{\nu\lambda}\left[\frac{\cosh(\lambda h)-1}{sinh(\lambda h)}\right]+\frac{4C}{h}\sum_{r=0}^{\infty}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
&+ \frac{\alpha}{\nu}\left[\frac{(k_1\alpha_1+k_2\alpha_2)\cos\beta t-(k_2\alpha_1-k_1\alpha_2)\sin\beta t}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}\right]\\
&+ A\left[\frac{(2t\nu\lambda^2+l)\sinh\lambda h-\lambda lhcosh\lambda h}{2\lambda\nu\sinh^2\lambda h}\right]\\
&+ \frac{4\alpha}{h}\sum_{r=0}^{\infty}\left[\frac{x_1e^{x_1t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]\\
&- \frac{2A\pi^2\nu}{h^3}\sum_{r=0}^{\infty}(-1)^r r^2\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
D_h &= \frac{C}{\nu\lambda}\left[\frac{1-\cosh(\lambda h)}{sinh(\lambda h)}\right]-\frac{4C}{h}\sum_{r=0}^{\infty}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]\\
&- \frac{\alpha}{\nu}\left[\frac{(k_1\alpha_1+k_2\alpha_2)\cos\beta t-(k_2\alpha_1-k_1\alpha_2)\sin\beta t}{(\alpha_1^2+\alpha_2^2)(F_1^2+F_2^2)}\right]\\
&+ A\left[\frac{(2t\nu\lambda^2+l)\sinh\lambda h-\lambda lhcosh\lambda h}{2\lambda\nu\sinh^2\lambda h}\right]\\
&- \frac{4\alpha}{h}\sum_{r=0}^{\infty}\left[\frac{x_1e^{x_1t}(1+x_1\tau)^2}{(x_1^2+\beta^2)\left[(1+x_1\tau)^2+l\right]}+\frac{x_2e^{x_2t}(1+x_2\tau)^2}{(x_2^2+\beta^2)\left[(1+x_2\tau)^2+l\right]}\right]\\
&- \frac{2A\pi^2\nu}{h^3}\sum_{r=0}^{\infty}r^2\left[\frac{e^{x_1t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{e^{x_2t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
x_1 &= -\frac{1}{2\tau}\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{r^2\pi^2}{h^2}\right)\\
&\quad +\frac{1}{2\tau}\sqrt{\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{r^2\pi^2}{h^2}\right)^2-4\tau\left(C_r\nu+D+\nu\frac{r^2\pi^2}{h^2}\right)}\\
x_2 &= -\frac{1}{2\tau}\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{r^2\pi^2}{h^2}\right)\\
&\quad -\frac{1}{2\tau}\sqrt{\left(1+l+D\tau+\nu C_r\tau+\nu\tau\frac{r^2\pi^2}{h^2}\right)^2-4\tau\left(C_r\nu+D+\nu\frac{r^2\pi^2}{h^2}\right)}
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= \sinh(\beta_1(b-h))\cos(\beta_2(b-h)) - \sinh(\beta_1 b)\cos(\beta_2 b) + \sinh\beta_1 h\cos\beta_2 h,\\
E_2 &= \cosh(\beta_1(b-h))\sin(\beta_2(b-h)) - \cosh(\beta_1 b)\sin(\beta_2 b) + \cosh\beta_1 h\sin\beta_2 h,\\
F_1 &= \sinh\beta_1 h\cos\beta_2 h, \quad F_2 = \cosh\beta_1 h\sin\beta_2 h\\
\theta_1 &= \alpha_1(E_1F_1 + E_2F_2) + \alpha_2(E_2F_1 - F_2E_1),\\
\theta_2 &= \alpha_1(E_2F_1 - F_2E_1) - \alpha_2(E_1F_1 + E_2F_2),\\
\alpha_1 &= \left[C_r + \frac{D}{\nu} + \frac{\beta^2\tau l}{\nu(1+\beta^2\tau^2)}\right], \quad \alpha_2 = \left[\frac{\beta}{\nu} + \frac{\beta l}{\nu(1+\beta^2\tau^2)}\right]\\
\beta_1 &= \sqrt{\frac{\alpha_1 + \sqrt{\alpha_1^2+\alpha_2^2}}{2}}, \quad \beta_2 = \sqrt{\frac{\alpha_1 + \sqrt{\alpha_1^2+\alpha_2^2}}{2}}\\
R_1 &= -\beta_2\sinh\beta_1 h\sin\beta_2 h + \beta_1\cosh\beta_1 h\cos\beta_2 h - \beta_1\\
R_2 &= \beta_2\cosh\beta_1 h\cos\beta_2 h + \beta_1\sinh\beta_1 h\sin\beta_2 h - \beta_2\\
k_1 &= F_1R_1 + F_2R_2, \quad k_2 = F_1R_2 - F_2R_1,\ \lambda = \sqrt{C_r + D/\nu}.
\end{aligned}
$$

4. Conclusion

Figures 2 to 5 show the velocity profiles for the fluid and dust particles respectively, which are parabolic. It is concluded that velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field it reduces the velocities of fluid and dust particles. It means that it has an appreciable effect on the velocities of the both the phases. One can observe that if the magnetic field is zero then results are in agreement with the Couette flow. The velocity is symmetrical with the centre of the channel. Further one can observe that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same. Also we see that the fluid particles will reach the steady state earlier than the dust particles.

Graphs are drawn for the following particular values
$h = 1, \quad r = 1, \quad \tau = 0.5, \quad C_r = 1, \quad l = 1, \quad A = 100, \quad \nu = 0.5, \quad D = 0.2, 0.4, 0.6, \quad t = 0.2, \quad \beta = 1.$

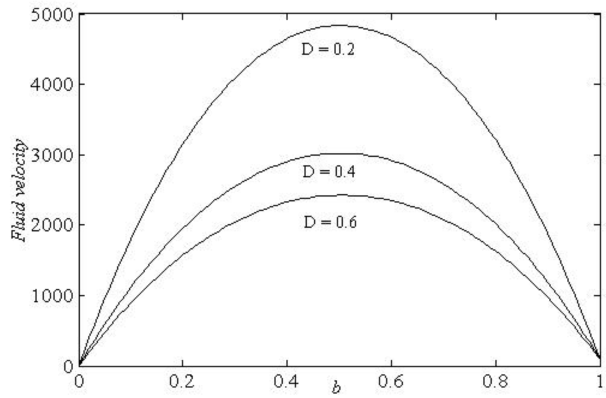

Figure 2: Variation of fluid velocity in case-1 versus b

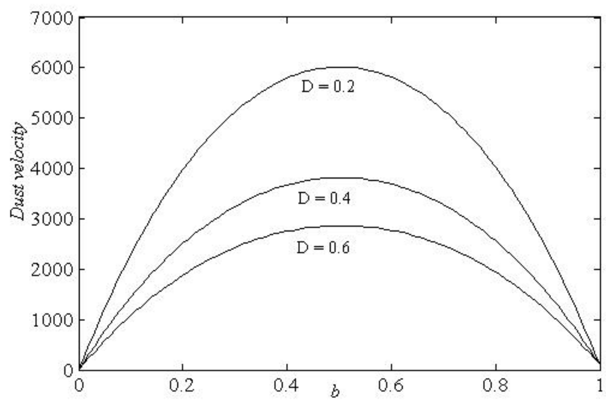

Figure 3: Variation of dust phase velocity in case-1 versus b

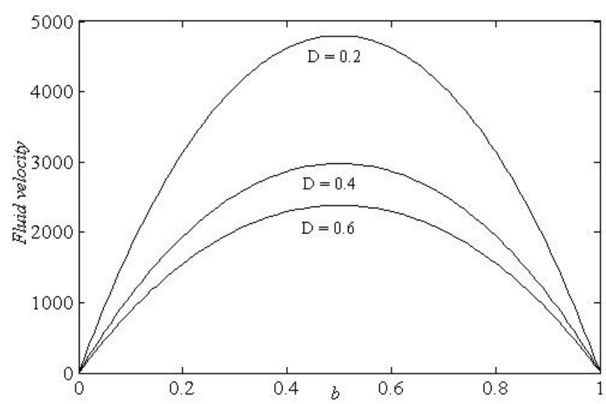

Figure 4: Variation of fluid velocity in case-2 versus b

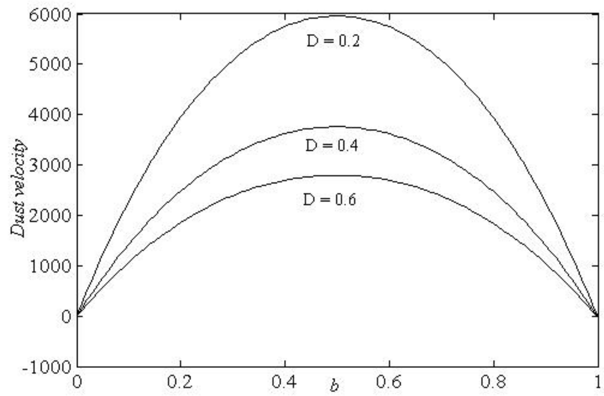

Figure 5: Variation of dust phase velocity in case-2 versus b

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