ASYMMETRY LEVEL AS A FUZZY MEASURE

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Abstract. Some difficulties with the Counterfactual Theory of David K.Lewis (1941-2001) and its applications to the Causation Analysis have appeared. Among them, the *Temporal Asymmetry.* Many criticism appeared against the explanation given by Lewis, as in Horwich´87; Price´92 & ´96; Hausman´98. Here, we will analyze the possibility of obtain a new function: the Asymmetry Level, by the intervention of Fuzzy Measures, which will collaborate in the solution of such problem.

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1.Introduction

Lewis admits that the asymmetry is possibly a contingent characteristic of the actual world, not present in other worlds. So, in a world populated by only one atom such asymmetry on the overdetermination does not hold. For this reason, there exists a possible discontinuity problem in the boundary.

Because if we consider a contractive sequence of subworlds, each of them asymmetric, converging to the monoatomic world, denoted W, where asymmetry does hold, we would have a weakness in the theory.

Geometrically, the situation (relative to such symmetric character) should be: a contractive set, or decreasing collection, of subworlds, each one totally included in the precedent world, and where each one, but the last, shows asymmetries, whereas in the limit, finally, the symmetry appears.

To solve this problem, either we can admit the symmetry as discontinuous function, and so we see that:

$$ASYM \rightarrow ASYM \rightarrow ASYM \rightarrow ... \rightarrow ASYM \rightarrow SYMMETRY$$

Or we may assign a certain value, , as a level of symmetry or of asymmetry (complementarity), with a definition suggested by the belonging degree of elements to fuzzy sets; or as a level of satisfaction of some condition or property, defined so in the limit it is possible to obtain the state of complete symmetry.

$$A_1 \supseteq A_2 \supseteq A_3 \supseteq \; ... \; \supseteq A_n \supseteq \; ... \; \supseteq A = \{a\}$$

So, for instance: with the contractivity condition taken from the concept of cardinality:

$$c(A_1) \geq c(A_2) \geq c(A_3) \geq \; ... \; \geq c(A_n) \geq \; ... \; \geq c(A) = 1$$

Also we can suppose, simplifying, that each world has a cardinal number one less than the precedent world's.

Once classified in decreasing order, reaching some degree of homogeneity among its elements, it is possible to introduce the function "symmetry level", (or asymmetry level, by complementarity). Respectively, denoted L_s and L_a.

2.Towards the asymmetry level function

It would be interesting to make intervene *Specificity* and *Entropy* measures, *Sp* and *H*, which may reflects the progressive contraction of the fields.

Remember that the values of such fuzzy measure, *Sp*, decrease as the size of the considered sets increases.

As we have:

$$\lim \{A_n\} = \{a\}$$

Then:

$$Sp(A_1) \leq Sp(A_2) \leq Sp(A_3) \leq \; ... \; \leq Sp(A_n) \leq \; ... \; \leq Sp(A) = Sp(\{a\})$$

$$Sp(\delta) = 1 \Leftrightarrow \delta = A = \{a\} \; (singleton)$$

Also:

$$0 \leq H\left(A_n\right) \leq 1,\ \forall n \in N \Leftrightarrow 1 \leq \frac{1}{H(A_n)} \leq +\infty \Rightarrow \frac{1}{2} \leq \frac{1}{H(A_n)+1} \leq 1,\ \forall n \in N$$

Whereas:

$$Sp\left(A_n\right) \rightarrow Sp\left(A\right) \equiv Sp\left(\{a\}\right)$$

But:

$$H\left(A_n\right) \rightarrow 0^+,\ if\ n \rightarrow +\infty$$

And so:

$$\frac{1}{H(A_n)} \rightarrow +\infty,\ if\ n \rightarrow +\infty \Rightarrow \frac{1}{H(A_n)+1} \rightarrow 1^+,\ if\ n \rightarrow +\infty$$

From here, basing on:

$$0 \leq Sp\left(A_n\right) \leq 1 \Rightarrow \lim\left\{L_s\left(A_n\right)\right\} = L_s\left(A\right) = L_s\left(\{a\}\right)$$

About the intervention of Entropy Measure, H, in our formula, we must consider that:

$$A_i \subseteq A_j \Rightarrow H\left(A_i\right) \leq H\left(A_j\right) \Rightarrow 1 + H\left(A_i\right) \leq 1 + H\left(A_j\right) \Rightarrow$$
$$\Rightarrow \frac{1}{1+H(A_j)} \leq \frac{1}{1+H(A_i)},\ being\ i \neq j$$

The Entropy degree increases when the cardinal (or number of elements, for finite sets) increases, and reciprocally. In our construction, we have: $j \geq i$.

So, we can obtain a more complete expression of the Level Symmetry Function, through the intervention of Entropy Measure, *H*, depending on the increment or decrement of the cardinal of the set:

$$L_s = \left\{ Sp\left(\tfrac{1-c}{1+c}\right) + (1+H)^{-1} \right\} \Leftrightarrow L_a = 1 - \left\{ Sp\left(\tfrac{1-c}{1+c}\right) + (1+H)^{-1} \right\}$$

But actually, in these cases, we return to cardinal two:

$$R_{INT} \in \{0,\ 1\}$$

Reappearing in such case the discontinuous situation for the described function.

Therefore:

$$[L_s(A_i)] = 1,\ \textit{in the monoatomic world};\ \ [L_s(A_i)] = 0,\ \textit{in other worlds}$$

$$[L_a(A_i)] = 0,\ \textit{in the monoatomic world};\ \ [L_a(A_i)] = 1,\ \textit{in other worlds}$$

3.A new Fuzzy Measure

For this, we need to demonstrate three conditions.
In our case, will be expressed by:

I) $L_a(\varnothing) = 0$

II) $L_a(U) = 1$

III) $If\ A_n \subseteq A_m \Rightarrow L_a(A_n) \leq L_a(A_m)$

First and second conditions will be easily proved.

In our model, we can establish that:

$$\lim_{n\to\infty} A_n = U\ (universe)$$

$$L_a(A) = 1 - \left\{ Sp(A)\left(\frac{1-c(A)}{1+c(A)}\right) + \frac{1}{1+H(A)} \right\} = 1 - \left\{ 1\left(\tfrac{1-1}{1+1}\right) + \tfrac{1}{1+0} \right\} = 0$$

Because here we have:

$$\delta = A\ (\text{singleton}) \Rightarrow Sp(A) = Sp(\delta) = 1\ and\ H(A) = 0$$

(according to the properties of Specificity and Entropy Measures). Also:

$$L_a\left(\varnothing\right)=1-\left\{Sp\left(\varnothing\right)\left(\tfrac{1-c(\varnothing)}{1+c(\varnothing)}\right)+\tfrac{1}{1+H(\varnothing)}\right\}=1-\left\{0\left(\tfrac{1-0}{1+0}\right)+\tfrac{1}{1+0}\right\}=0$$

Given that according to the properties of Specificity Measure:

$$Sp\left(\varnothing\right)=0$$

As:

$$\lim_{n\to\infty}A_n=U$$

$$L_a\left(A_n\right)=1-\left\{Sp\left(A_n\right)\left(\tfrac{1-n}{1+n}\right)+\tfrac{1}{1+H(A_n)}\right\}$$

But:

$$\left(\tfrac{1-n}{1+n}\right)\to-1,\ and\ \left(\tfrac{1-m}{1+m}\right)\to-1,\ if\ m\to+\infty$$

Being:

$$\forall n,m\in N: n\leq m\Rightarrow\tfrac{1-n}{1+n}\geq\tfrac{1-m}{1+m}$$

Then:

$$L_a\left(U\right)=\lim_{n\to\infty}\left(1-\left\{Sp\left(A_n\right)\left(\tfrac{1-n}{1+n}\right)+\tfrac{1}{1+H(A_n)}\right\}\right)=1$$

Equivalently:

$$L_s\left(U\right)=\lim_{n\to\infty}\left\{Sp\left(A_n\right)\left(\tfrac{1-n}{1+n}\right)+\tfrac{1}{1+H(A_n)}\right\}=0$$

Because:

$$Sp\left(A_n\right) \to 0,\ and\ H\left(A_n\right) \to +\infty,\ if\ n \to +\infty$$

With:

$$\lim_{n\to\infty}\left(\frac{1-n}{1+n}\right) = -1$$

Therefore:

$$L_a\left(A_n\right) \to 1,\ when\ n \to +\infty \Rightarrow L_a\left(U\right) = 1$$

In general:

$$if\ n \geq m \Rightarrow A_n \subseteq A_m \Rightarrow L_a\left(A_n\right) \leq L_a\left(A_m\right)$$

The reason is that:

$$n \geq m \Rightarrow \frac{1}{1+H(A_m)} \leq \frac{1}{1+H(A_n)}$$

Furthermore, as:

$$Sp\left(A_n\right) \geq Sp\left(A_m\right)\ and\ H\left(A_n\right) \leq H\left(A_m\right)$$

We deduce:

$$Sp\left(A_m\right)\left(\frac{1-m}{1+m}\right) + \frac{1}{1+H(A_m)} \leq Sp\left(A_n\right)\left(\frac{1-n}{1+n}\right) + \frac{1}{1+H(A_n)}$$

So, concluding which:

$$1 - \left\{Sp\left(A_m\right)\left(\frac{1-m}{1+m}\right) + \frac{1}{1+H(A_m)}\right\} \geq 1 - \left\{Sp\left(A_n\right)\left(\frac{1-n}{1+n}\right) + \frac{1}{1+H(A_n)}\right\}$$

We have, finally:

$$L_a\left(A_n\right) \leq L_a\left(A_m\right) \Leftrightarrow L_s\left(A_n\right) \geq L_s\left(A_m\right)$$

Verifying in this manner the third condition necessary to be L_a a fuzzy measure.□

4.Conclusion

So, we conclude that it is possible, in this way, to introduce a new measure [10] quantifying the asymmetry level of shapes, being in general, applicable to fuzzy sets.

This new normal fuzzy measure may improve the precedent results, when it is applied on new situations. In particular, when we works with Bayesian Networks.

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