FLOW OF AN UNSTEADY CONDUCTING DUSTY FLUID THROUGH CIRCULAR CYLINDER

B.J.Gireesha, P.Venkatesh, C.S.Bagewadi

Abstract. The paper deals with the study of laminar flow of a conducting dusty fluid with uniform distribution of dust particles through a circular cylinder under the influence of time dependent pressure gradient. Initially the fluid and dust particles are at rest. The analytical expressions for velocities of fluid and dust particles are obtained by solving the partial differential equations using variable separable method. The skin friction at the wall of the cylinder also calculated. Finally the changes in the velocity profiles with R is shown graphically.

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Keywords and Phrases: Circular cylinder, laminar flow, conducting dusty fluid; velocity of dust phase and fluid phase, skin friction.

1. Introduction

The fluid flow embedded with dust particles is encountered in a wide variety of engineering problems concerned with atmospheric fallout, dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, batch settling, rainerosion, guided missiles and paint spraying etc.

Based on fundamental equations of motion of P.G.Saffman [10] for the laminar flow of a dusty gas the author Liu [8] has studied the Flow induced by an oscillating infinite plat plate in a dusty gas. Michael and Miller [9] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in a cylinder and between two rotating cylinders. Ghosh [5] have obtained the analytical solutions for the dusty visco-elastic fluid between two infinite parallel plates under the influence of time dependent pressure gradient, using appropriate boundary conditions. Datta and Dalal [4] have studied the Pulsatile flow and heat transfer of a dusty fluid through an infinitely long annular pipe. Amos [1] studied the megnetic effect on pulsatile flow in a axisymmetric tube.

The authors Bagewadi and Gireesha [2], [3] have studied two-dimensional dusty fluid flow in Frenet frame field system. Recently the authors [6], [7] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential in Frenet frame field system. In the present paper laminar flow of an unsteady conducting dusty fluid through a circular channel under the influence of time dependent pressure gradient is considered. Further by considering the fluid and dust particles to be at rest initially, the analytical expressions are obtained for velocities of fluid and dust particles also the skin friction at the boundary is calculated. The graphical representation of the velocity profiles versus r are given.

2.Equations of Motion

The equations of motion of conducting unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [10]:

For fluid phase

$$\begin{aligned} \nabla.\overrightarrow{u} &= 0, \qquad \text{(Continuity)} & (1)\\ \frac{\partial \overrightarrow{u}}{\partial t} &+ (\overrightarrow{u}.\nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \upsilon\nabla^2\overrightarrow{u} & (2)\\ &+ \frac{KN}{\rho}(\overrightarrow{v}-\overrightarrow{u}) + \frac{1}{\rho}(\overrightarrow{J}\times\overrightarrow{B}) \ \text{(Linear Momentum)} \end{aligned}$$

For dust phase

$$\nabla.\overrightarrow{v} = 0, \qquad \text{(Continuity)} \tag{3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u}-\overrightarrow{v}) \qquad \text{(Linear Momentum)} \tag{4}$$

We have following nomenclature:

$\overrightarrow{u}-$velocity of the fluid phase, $\overrightarrow{v}-$velocity density of dust phase, $\rho-$density of the gas, $p-$pressure of the fluid, $N-$number of density of dust particles, $\nu-$kinematic viscosity, $k = 6\pi a\mu-$Stoke's resistance (drag coefficient), $a-$spherical radius of dust particle, $m-$mass of the dust particle, $\mu-$the co-efficient of viscosity of fluid particles, $t-$time and $\overrightarrow{J}$ and $\overrightarrow{B}$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla \times \overrightarrow{H} = 4\pi \overrightarrow{J},\ \nabla \times \overrightarrow{B} = 0,\ \nabla \times \overrightarrow{E} = 0,\ \overrightarrow{J} = \sigma[\overrightarrow{E} + \overrightarrow{u} \times \overrightarrow{B}] \tag{5}$$

Here $\overrightarrow{H}-$magnetic field, $\overrightarrow{J}-$current density, $\overrightarrow{B}-$magnetic Flux, $\overrightarrow{E}$-electric field and $\sigma-$ the electrical conductivity of the fluid.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\overrightarrow{J} \times \overrightarrow{B}$ of the body force in (2.2) reduces simply to $-\sigma B_0^2 \overrightarrow{u}$ where B_0 - the intensity of the imposed transverse magnetic field.

3.Formulation of the Problem

Consider a flow of viscous incompressible, conducting dusty fluid through a circular cylinder of radius a. The flow is due to the influence of time dependent pressure gradient. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. As Figure-1 shows, the axis of the channel is along $z-$axis and the velocity components of both fluid and dust particles are respectively given by:

$$\left.\begin{array}{l} u_r = 0;\ \ u_\theta = 0;\ \ u_z = u_z(r,t); \\ v_r = 0;\ \ v_\theta = 0;\ \ v_z = v_z(r,t) \end{array}\right\} \tag{6}$$

where (u_r, u_θ, u_z) and (v_r, v_θ, v_z) are velocity components of fluid and dust particles respectively.

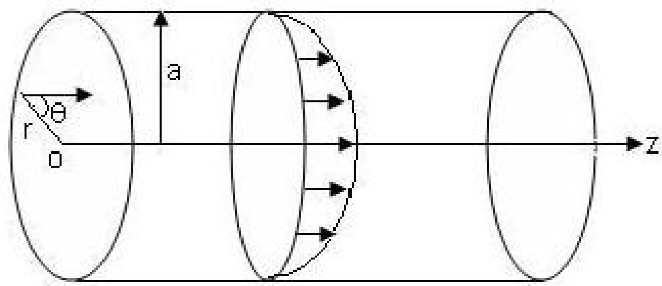

Figure 1: Schematic diagram of dusty fluid flow in a Circular Cylinder.

By virtue of equation (10) the intrinsic decomposition of equations (1) and (4) in cylindrical polar coordinates give the following forms:

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{7}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right] + \frac{kN}{\rho}(v_z - u_z) - \frac{\sigma B_0^2}{\rho}u_z, \tag{8}$$

$$\frac{\partial v_z}{\partial t} = -\frac{k}{m}(u_z - v_z), \tag{9}$$

Let us introduce the following non-dimensional quantities;

$$R = \frac{r}{a}, \quad \bar{z} = \frac{z}{a}, \quad \bar{p} = \frac{pa^2}{\rho\nu^2}, \quad T = \frac{t\nu}{a^2}, \quad u = \frac{u_z a}{\nu}, \quad v = \frac{v_z a}{\nu} \tag{10}$$

$$\beta = \frac{l}{\gamma} = \frac{Nka^2}{\rho\nu}, \quad l = \frac{Nm}{\rho}, \quad \gamma = \frac{\nu m}{ka^2}.$$

Transform the equations (7) to (9) to the non-dimensional forms as

$$-\frac{\nu^2}{a^3}\frac{\partial p}{\partial R} = 0, \tag{11}$$

$$\frac{\partial u}{\partial T} = -\frac{\partial p}{\partial \bar{z}} + \nu\left[\frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u}{\partial R}\right] + \beta(v - u) - M^2 u, \tag{12}$$

$$\gamma\frac{\partial v}{\partial T} = (u - v). \tag{13}$$

where $M = B_0 a\sqrt{(\sigma/\mu)}$ = the Hartmann number.

Since we have assumed that the time dependent pressure gradient to be impressed on the system for $t > 0$, so we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = c + de^{\alpha t}$$

where $c,\ d$ and α are reals.

Eliminating v from (12) and (13) and then substituting the expression for pressure gradient, one can get

$$\begin{aligned}\gamma\frac{\partial^2 u}{\partial T^2} &+ (l+1)\frac{\partial u}{\partial T} - \gamma\frac{\partial}{\partial T}\left[\frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u}{\partial R}\right] \\ &= c + de^{\alpha t} - \left[\frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u}{\partial R}\right] - M^2 u, \qquad (14)\end{aligned}$$

4.Solution Part

Let the solution of the equation (14) in the form [11]

$$u = U(R) + V(R,T) \qquad (15)$$

where U is the steady part and V is the unsteady part of the fluid velocity

Separating the equation (14) in to a steady part and unsteady part as

$$\begin{aligned}\frac{\partial^2 U}{\partial R^2} &+ \frac{1}{R}\frac{\partial U}{\partial R} + M^2 U = c, \qquad (16)\\ \gamma\frac{\partial^2 V}{\partial T^2} &+ (l+1)\frac{\partial V}{\partial T} - \gamma\frac{\partial}{\partial T}\left[\frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R}\right] \\ &= de^{\alpha t} + \left[\frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R}\right] + M^2 V, \qquad (17)\end{aligned}$$

Consider the equation (16) with the following boundary conditions

$$\begin{aligned} U &= 0, \quad \text{at} \quad R = 1,\\ U &= finite, \ \text{at} \ R = 0.\end{aligned}$$

Now, by solving equation (16) with above boundary conditions, one can get

$$U = \frac{c}{M^2}\left(\frac{J_0(MR)}{J_0(M)} - 1\right) \qquad (18)$$

where J_0 is Bessel's function of zeroth order.

Now consider the equation (17) with the following boundary conditions

$$\begin{aligned} \text{Initial condition :} \quad & U = V \quad \text{at} \quad T = 0, \\ \text{Boundary condition :} \quad & V = 0 \quad \text{at} \quad R = 1, \\ & V = 0, \quad \text{at} \quad R = 0. \end{aligned} \tag{19}$$

Assume the solution of the equation (17) is in the form

$$V = g(R)e^{\alpha T} \tag{20}$$

where $g(R)$ is an unknown function to be determine.

Using equation (20) in (17) one can obtain

$$\frac{\partial^2 g}{\partial R^2} + \frac{1}{R}\frac{\partial g}{\partial R}\lambda_1^2 = \lambda_2 \tag{21}$$

where $\lambda_1 = \frac{(M^2-\gamma\alpha^2)-\alpha(1+l)}{(1+\alpha\gamma)}$ and $\lambda_2 = \frac{d}{(1+\alpha\gamma)}$.

One can obtain the solution of (21)

$$g(R) = AJ_0(\lambda_1 R) + BK_0(\lambda_1 R) - \frac{\lambda_2}{\lambda_1^2} \tag{22}$$

where A, B are constants. J_0 and K_0 are Bessel's function of first and second kind order zero respectively.

Since the fluid velocity is finite at the center of the circular tube, we have $B = 0$. Hence using the boundary conditions (19) we get

$$g(R) = \frac{\lambda_2}{\lambda_1^2}\left[\frac{J_0(\lambda_1 R)}{J_0(\lambda_1)} - 1\right] \tag{23}$$

Using this in (20) we get V as

$$V = e^{\alpha T}\frac{\lambda_2}{\lambda_1^2}\left[\frac{J_0(\lambda_1 R)}{J_0(\lambda_1)} - 1\right] \tag{24}$$

Now, using equations (24) and (18) in (4.1) one can obtain the fluid velocity u as

$$u = \frac{c}{M^2}\left[\frac{J_0(MR)}{J_0(M)} - 1\right] + e^{\alpha T}\frac{\lambda_2}{\lambda_1^2}\left[\frac{J_0(\lambda_1 R)}{J_0(\lambda_1)} - 1\right] \tag{25}$$

Also, the dust phase velocity is obtained from equation (13) as

$$\begin{aligned} v &= \frac{c}{M^2}\left[\frac{J_0(MR)}{J_0(M)} - 1\right]\left[1 - e^{-\frac{1}{\gamma}T}\right] \\ &+ e^{\alpha T}\frac{\lambda_2}{\lambda_1^2}\left[\frac{J_0(\lambda_1 R)}{J_0(\lambda_1)} - 1\right]\left[e^{\alpha T} - e^{-\frac{1}{\gamma}T}\right] \end{aligned} \tag{26}$$

Shearing Stress (Skin Friction):
The Shear stress at the boundary $R = 1$ is given by

$$D_1 = \frac{\mu c}{M}\left[\frac{J_0'(M)}{J_0(M)}\right] + e^{\alpha T}\frac{\mu\lambda_2}{\lambda_1}\left[\frac{J_0'(\lambda_1)}{J_0(\lambda_1)}\right] \tag{27}$$

From property of bessel's function we know that $J_0' = -J_1$. Hence The above equation becomes

$$D_1 = -\mu\left[\frac{c}{M}\frac{J_1(M)}{J_0(M)} + e^{\alpha T}\frac{\lambda_2}{\lambda_1}\frac{J_1(\lambda_1)}{J_0(\lambda_1)}\right] \tag{28}$$

4.Conclusion

The flow of the fluid and dust particles are drawn as in figures 2 and 3, which are parabolic in nature. From these it is observed the flow of fluid particles is parallel to that of dust. Also the velocity of both fluid and dust particles, which are nearer to the axis of flow, move with the greater velocity. One can observe that the appreciable effect of Hartmann number on the flow of both fluid and dust phase. This shows that the magnetic field has retarding influence. Further observation shows that if $\gamma \to 0$ the velocities of fluid and dust particles will be the same.

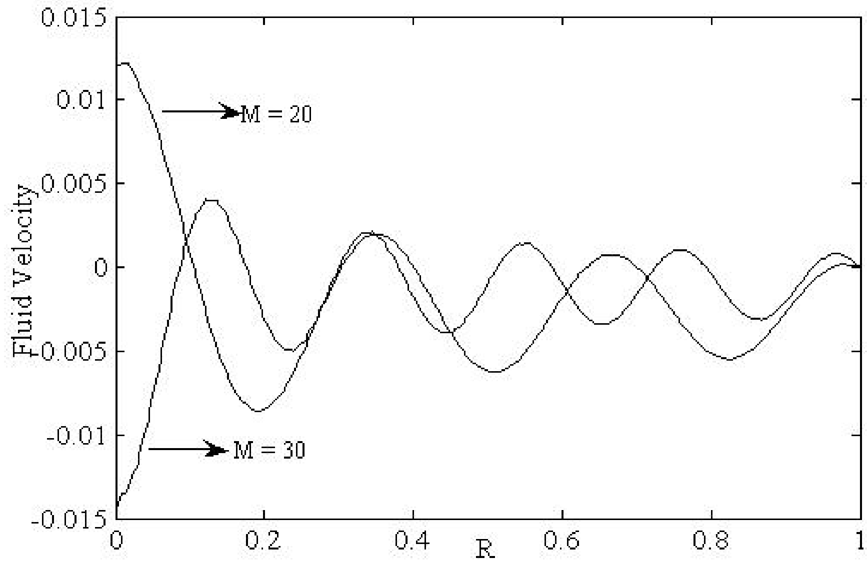

Figure 2: Variation of fluid velocity with R

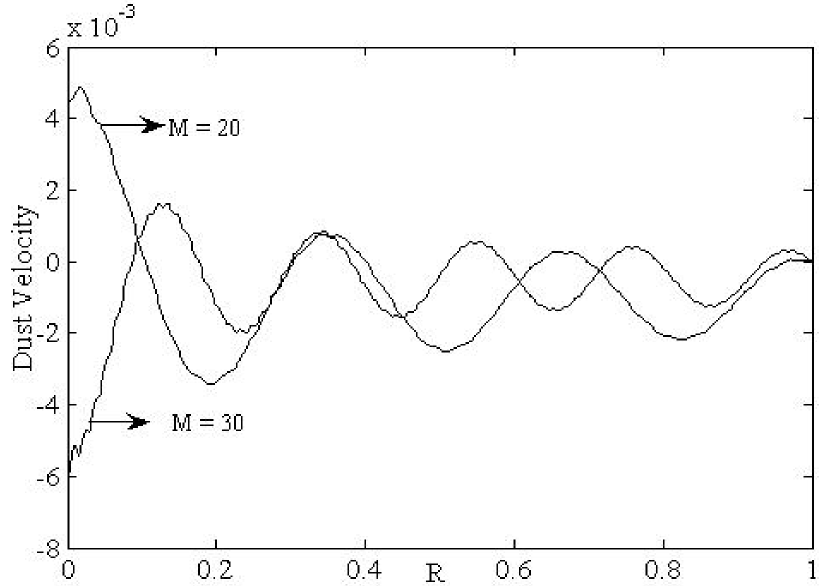

Figure 3: Variation of dust phase velocity with R

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