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ON ORLICZ FUNCTIONS OF GENERALIZED DIFFERENCE SEQUENCE SPACES

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Abstract. In this paper, we define the sequence spaces : $[V, M, p, \Delta_u^n, s]$, $[V, M, p, \Delta_u^n, s]_0$ and $[V, M, p, \Delta_u^n, s]_\infty$, where for any sequence $x = (x_n)$, the difference sequence Δx is given by $\Delta x = (\Delta x_n)_{n=1}^{\infty} = (x_n - x_{n-1})_{n=1}^{\infty}$. We also examine some inclusion relations between these spaces and discuss some properties and results related to them.

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1.Introduction and definitions

Let X be a linear space. A function $p : X \to \mathbb{R}$ is called paranorm if the following are satisfied :

(i) $p(0) \geq 0$

(ii) $p(x) \geq 0$ for all $x \in X$

(iii) $p(x) = p(-x)$ for all $x \in X$

(iv) $p(x + y) \leq p(x) + p(y)$ for all $x \in X$ (triangle inequality)

(v) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda$ $(n \to \infty)$ and (x_n) is a sequence of vectors with $p(x_n - x) \to 0$ $(n \to \infty)$, then $p(\lambda_n x_n - \lambda x) \to 0$ $(n \to \infty)$ (continuity of multiplication by scalars).

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total. It is well known that the metric of any linear metric space is given by some total paranorm (cf.[11]).

Let $\Lambda = (\lambda_n)$ a nondecreasing sequence of positive reals tending to infinity and $\lambda_1 = 1$ and $\lambda_{n+1} \leq \lambda_n + 1$.

The generalized de la Vallėe-Poussin means is defined by :

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,$$

where $I_n = [n - \lambda_n + 1, n]$. A sequence $x = (x_k)$ is said to be $(V, \lambda)-$summable to a number l (see [2]) if $t_n(x) \to l$, as $n \to \infty$.

We write

$$[V,\lambda]_0 = \{x=(x_k): \lim_n \frac{1}{\lambda_n}\sum_{k\in I_n} | x_k |= 0\}$$
$$[V,\lambda] = \{x=(x_k): \lim_n \frac{1}{\lambda_n}\sum_{k\in I_n} | x_k - le |= 0, \text{ for some } l\in\mathbb{C}\}$$

and

$$[V,\lambda]_\infty = \{x=(x_k): \sup_n \frac{1}{\lambda_n}\sum_{k\in I_n} | x_k |< \infty\}.$$

For the set of sequences that are strongly summable to zero, strongly summable and strongly bounded by the de la Vallėe-Poussin method. If $\lambda_n = n$ for $n = 1,2,3,\cdots$, then these sets reduce to ω_0, ω and ω_∞ introduced and studied by Maddox [5].

Following Lidenstrauss and Tzafriri [4], we recall that an Orlicz function M is continuous, convex, nondecreasing function defined for $x \geq 0$ such that $M(0) = 0$ and $M(x) \geq 0$ for $x > 0$.

If convexity of M is replaced by $M(x+y) \leq M(x) + M(y)$, then it is called a modulus function, defined and studied by Nakano [8], Ruckle [10], Maddox [6] and others.

An Orlicz function M is said to satisfy the Δ_2-condition for all values of u, if there exist a constant $K > 0$ such that

$$M(2u) \leq KM(u) \ (u \geq 0).$$

It is easy to see that always $K > 2$. The Δ_2-condition is equivalent to the satisfaction of the inequality

$$M(lu) \leq KlM(u),$$

for all values of u and for $l > 1$.

Lidenstrauss and Tzafriri [4] used the idea of Orlicz function to construct the Orlicz sequence space :

$$l_M := \{x=(x_k): \sum_{k=1}^{\infty} M(\frac{| x_k |}{\rho}) < \infty, \text{ for some } \rho\ > 0\},$$

which is a Banach space with the norm :

$$\| x \|_M = \inf\{\rho\ > 0 : \sum_{k=1}^{\infty} M(\frac{| x_k |}{\rho}) \leq 1\}.$$

If $M(x) = x^p, 1 \leq p < \infty$, the space l_M coincide with the classical sequence space l_p.

Parashar and Choudhary [9] have introduced and examined some properties of four sequence spaces defined by using an Orlicz function M, which generalized the well-known Orlicz sequence space l_M and strongly summable sequence spaces $[C, 1, p], [C, 1, p]_0$ and $[C, 1, p]_\infty$.

Let M be an Orlicz function, $p = (p_k)$ be any sequence of strictly positive real numbers and $u = (u_k)$ be any sequence such that $u_k \neq 0 (k = 1, 2, \cdots)$. Then Alsaedi and Bataineh [1] defined the following sequence spaces :

$[V, M, p, u, \Delta] = \{x = (x_k) : \lim_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} [M(\frac{|u_k \Delta x_k - le|}{\rho})]^{p_k} = 0,$ for some l and $\rho\ > 0\}$,

$[V, M, p, u, \Delta]_0 = \{x = (x_k) : \lim_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} [M(\frac{|u_k \Delta x_k|}{\rho})]^{p_k} = 0,$ for some $\rho\ > 0\}$,
and

$$[V, M, p, u, \Delta]_\infty = \{x = (x_k) : \sup_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} [M(\frac{| u_k \Delta x_k |}{\rho})]^{p_k} < \infty, \text{ for some } \rho\ > 0\}.$$

Now, if n is a nonnegative integer and s is any real number such that $s \geq 0$, then we define the following sequence spaces :

$[V, M, p, \Delta_u^n, s] = \{x = (x_k) : \lim_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} k^{-s} [M(\frac{|\Delta_u^n x_k - le|}{\rho})]^{p_k} = 0,$ for some $l, \rho\ > 0$ and $s \geq 0\}$,

$[V, M, p\Delta_u^n, s]_0 = \{x = (x_k) : \lim_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} k^{-s} [M(\frac{|\Delta_u^n x_k|}{\rho})]^{p_k} = 0,$ for some $\rho\ > 0$ and $s \geq 0\}$,
and

$[V, M, p, \Delta_u^n, s]_\infty = \{x = (x_k) : \sup_n \frac{1}{\lambda_n} \sum_{k\in I_n}^{\infty} k^{-s} [M(\frac{|\Delta_u^n x_k|}{\rho})]^{p_k} < \infty,$ for some $\rho\ > 0$ and $s \geq 0\}$,

where $u = (u_k)$ is any sequence such that $u_k \neq 0$ for each k,and

$$\Delta_u^0 x = u_k x_k,$$

$$\Delta_u^1 x = u_k x_k - u_{k+1} x_{k+1},$$

$$\Delta_u^2 x = \Delta(\Delta_u^1 x),$$

$$\vdots$$

$$\Delta_u^n x = \Delta(\Delta_u^{n-1} x),$$

so that

$$\Delta_u^n x = \Delta_{u_k}^n x_k = \sum_{r=0}^{n} (-1)^r \binom{n}{r} u_{k+r} x_{k+r}.$$

If $n = 0$ and $s = 0$, then these gives the spaces of Alsaedi and Bataineh [1].

2.Main results

We prove the following theorems :

Theorem 1. *For any Orlicz function M and any sequence $p = (p_k)$ of strictly positive real numbers, $[V, M, p, \Delta_u^n, s], [V, M, p, \Delta_u^n, s]_0$ and $[V, M, p, \Delta_u^n, s]_\infty$ are linear spaces over the set of complex numbers.*

Proof. We shall prove only for $[V, M, p, \Delta_u^n, s]_0$. The others can be treated similarly. Let $x, y \in [V, M, p, \Delta_u^n, s]_0$ and $\alpha, \beta \in \mathbb{C}$. In order to prove the result, we need to find some $\rho > 0$ such that :

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} [M(\frac{| \alpha \Delta_u^n x_k + \beta \Delta_u^n y_k |}{\rho})]^{p_k} = 0.$$

Since $x, y \in [V, M, p, \Delta_u^n, s]_0$, there exists some positive ρ_1 and ρ_2 such that :

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} [M(\frac{| \Delta_u^n x_k |}{\rho_1})]^{p_k} = 0 \text{ and } \lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} [M(\frac{| \Delta_u^n y_k |}{\rho_2})]^{p_k} = 0.$$

Define $\rho = \max(2 \mid \alpha \mid \rho_1, 2 \mid \beta \mid \rho_2)$. Since M is nondecreasing and convex,

$$\begin{aligned}
&\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \alpha\Delta_u^n x_k + \beta\Delta_u^n y_k \mid}{\rho})]^{p_k} \\
\leq\ & \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \alpha\Delta_u^n x_k \mid}{\rho} + \frac{\mid \beta\Delta_u^n y_k \mid}{\rho})]^{p_k} \\
\leq\ & \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}\frac{1}{2^{p_k}}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho_1}) + M(\frac{\mid \Delta_u^n y_k \mid}{\rho_2})]^{p_k} \\
\leq\ & \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho_1}) + M(\frac{\mid \Delta_u^n y_k \mid}{\rho_2})]^{p_k} \\
\leq\ & K.\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho_1})]^{p_k} + K.\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n y_k \mid}{\rho_2})]^{p_k} \to 0,
\end{aligned}$$

as $n \to \infty$, where $K = \max(1, 2^{H-1})$, $H = \sup p_k$, so that $\alpha x + \beta y \in [V, M, p, \Delta_u^n, s]_0$. This completes the proof.

Theorem 2. *For any Orlicz function M and a bounded sequence $p = (p_k)$ of strictly positive real numbers, $[V, M, p, \Delta_u^n, s]_0$ is a total paranormed space with :*

$$g(x) = \inf\{\rho^{p_n/H} : (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho})]^{p_k})^{1/H} \leq 1,\ n = 1, 2, 3, \cdots\},$$

where $H = \max(1, \sup p_k)$.

Proof. Clearly $g(x) = g(-x)$. By using Theorem 2.1, for $\alpha = \beta = 1$, we get $g(x+y) \leq g(x) + g(y)$. Since $M(0) = 0$, we get $\inf\{\rho^{p_n/H}\} = 0$ for $x = 0$. Conversely, suppose $g(x) = 0$, then :

$$\inf\{\rho^{p_n/H} : (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho})]^{p_k})^{1/H} \leq 1\} = 0.$$

This implies that for a given $\epsilon > 0$, there exists some ρ_ϵ $(0 < \rho_\epsilon < \epsilon)$ such that :

$$(\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho_\epsilon})]^{p_k})^{1/H} \leq 1.$$

Thus,

$$(\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\epsilon})]^{p_k})^{1/H} \leq (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k \mid}{\rho_\epsilon})]^{p_k})^{1/H} \leq 1, \text{ for each } n.$$

Suppose that $x_{n_m} \neq 0$ for some $m \in I_n$, then $(\frac{\Delta_u^n x_{n_m}}{\epsilon}) \to \infty$. It follows that :

$$(\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_{n_m}\mid}{\epsilon})]^{p_k})^{1/H} \to \infty$$

which is a contradiction. Therefor $x_{n_m} = 0$ for all m. Finally we prove that scalar multiplication is continuous. Let μ be any complex number, then by definition,

$$g(\mu x) = \inf\{\rho^{p_n/H} : (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \mu\Delta_u^n x_k\mid}{\rho})]^{p_k})^{1/H} \leq 1,\ n = 1, 2, 3, \cdots\}.$$

Then
$g(\mu x) = \inf\{(\mid \mu \mid t)^{p_n/H} : (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{|\mu\Delta_u^n x_k|}{t})]^{p_k})^{1/H} \leq 1 \leq 1,\ n = 1, 2, 3, \cdots\}$,
where $t = \rho/ \mid \mu \mid$. Since $\mid \mu \mid^{p_n} \leq \max(1, \mid \mu \mid^{\sup p_n})$, we have

$$g(\mu x) \leq (\max(1, \mid \mu \mid^{\sup p_n}))^{1/H}. \inf\{(t)^{p_n/H}$$
$$: (\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k\mid}{t})]^{p_k})^{1/H} \leq 1,\ n = 1, 2, 3, \cdots\}$$

which converges to zero as x converges to zero in $[V, M, p, \Delta_u^n, s]_0$.

Now suppose $\mu_m \to 0$ and x is fixed in $[V, M, p, \Delta_u^n, s]_0$. For arbitrary $\epsilon > 0$, let N be a positive integer such that

$$\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k\mid}{\rho})]^{p_k} < (\epsilon/2)^H \text{ for some } \rho > 0 \text{ and all } n > N.$$

This implies that

$$\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \Delta_u^n x_k\mid}{\rho})]^{p_k} < \epsilon/2 \text{ for some } \rho > 0 \text{ and all } n > N.$$

Let $0 <\mid \mu \mid< 1$, using convexity of M, for $n > N$, we get

$$\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid \mu\Delta_u^n x_k\mid}{\rho})]^{p_k} < \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[\mid \mu \mid M(\frac{\mid \Delta_u^n x_k\mid}{\rho})]^{p_k} < (\epsilon/2)^H\ .$$

Since M is continuous everywhere in $[0, \infty)$, then for $n \leq N$,

$$f(t) = \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{\mid t\Delta_u^n x_k\mid}{\rho})]^{p_k}$$

is continuous at zero. So there exists $1 > \delta > 0$ such that $| f(t) | < (\epsilon/2)^H$ for $0 < t < \delta$.

Let K be such that $| \mu_m | < \delta$ for $m > K$ and $n \leq N$, then

$$(\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{| \mu_m \Delta_u^n x_k |}{\rho})]^{p_k})^{1/H} < \epsilon/2.$$

Thus

$$(\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}[M(\frac{| \mu_m \Delta_u^n x_k |}{\rho})]^{p_k})^{1/H} < \epsilon,$$

for $m > K$ and all n, so that $g(\mu x) \to 0$ $(\mu \to 0)$.

Theorem 3. *For any Orlicz function M which satisfies the Δ_2-condition, we have $[V, \lambda, \Delta_u^n, s] \subseteq [V, M, \Delta_u^n, s]$, where*
$[V, \lambda, \Delta_u^n, s] = \{x = (x_k) : \lim_n \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s}| \Delta_u^n x_k - le | = 0,$ for some $l \in \mathbb{C}\}$.

Proof. Let $x \in [V, \lambda, \Delta_u^n, s]$. Then

$$T_n = \frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s} | \Delta_u^n x_k - le | \to 0 \text{ as } n \to \infty, \text{ for some } l.$$

Let $\epsilon > 0$ and choose δ with $0 < \delta < 1$ such that $M(t) < \epsilon$ for $0 \leq t \leq \delta$. Write $y_k = | \Delta_u^n x_k - le |$ and consider

$$\frac{1}{\lambda_n}\sum_{k\in I_n} k^{-s} M(| y_k |) = \sum_1 + \sum_2,$$

where the first summation over $y_k \leq \delta$ and the second over $y_k > \delta$. Since M is continuous,

$$\sum_1 < \lambda_n \epsilon$$

and for $y_k < \delta$, we use the fact that $y_k < y_k/\delta < 1 + y_k/\delta$. Since M is nondecreasing and convex, it follows that

$$M(y_k) < M(1 + \delta^{-1} y_k) < \frac{1}{2}M(2) + \frac{1}{2}M(2\delta^{-1} y_k).$$

Since M satisfies the Δ_2-condition, there is a constant $K > 2$ such that $M(2\delta^{-1}y_k) \leq \frac{1}{2}K\delta^{-1}y_k M(2)$, therefor

$$\begin{aligned} M(y_k) &< \frac{1}{2}K\delta^{-1}y_k M(2) + \frac{1}{2}K\delta^{-1}y_k M(2) \\ &= K\delta^{-1}y_k M(2). \end{aligned}$$

Hence

$$\sum_{2} M(y_k) \le K\delta^{-1}M(2)\lambda_n T_n$$

which together with $\sum_1 \le \epsilon\lambda_n$ yields $[V, \lambda, \Delta_u^n, s] \subseteq [V, M, \Delta_u^n, s]$. This completes the proof.

The method of the proof of Theorem 3 shows that for any Orlicz function M which satisfies the Δ_2-condition, we have $[V, \lambda, \Delta_u^n, s]_0 \subseteq [V, M, \Delta_u^n, s]_0$ and $[V, \lambda, \Delta_u^n, s]_\infty \subseteq [V, M, \Delta_u^n, s]_\infty$, where

$$[V, \lambda, \Delta_u^n, s]_0 = \{x = (x_k) : \lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} \mid \Delta_u^n x_k \mid = 0\},$$

and

$$[V, \lambda, \Delta_u^n, s]_\infty = \{x = (x_k) : \sup_n \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} \mid \Delta_u^n x_k \mid < \infty\}.$$

Theorem 4. *Let $0 \le p_k \le q_k$ and (q_k/p_k) be bounded. Then $[V, M, q, \Delta_u^n, s] \subset [V, M, p, \Delta_u^n, s]$.*

Proof. The proof of Theorem 4 used the ideas similar to those used in proving Theorem 7 of Parashar and Choudhary [9].Mursaleen [7] introduced the concept of statistical convergence as follows :

A sequence $x = (x_k)$ is said to be $\lambda-$statistically convergent or $s_\lambda-$statistically convergent to L if for every $\epsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \mid \{k \in I_n : \mid x_k - L \mid \ge \epsilon\} \mid = 0,$$

where the vertical bars indicates the number of elements in the enclosed set. In this case we write $s_\lambda - \lim x = L$ or $x_k \to L \ (s_\lambda)$ and $s_\lambda = \{x : \exists L \in \mathbb{R}: s_\lambda - \lim x = L\}$.In a similar way, we say that a sequence $x = (x_k)$ is said to be $(\lambda, \Delta_u^n)-$statistically convergent or $s_\lambda(\Delta_u^n)-$statistically convergent to L if for every $\epsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \mid \{k \in I_n : \mid \Delta_u^n x_k - Le \mid \ge \epsilon\} \mid = 0,$$

where the vertical bars indicates the number of elements in the enclosed set. In this case we write $s_\lambda(\Delta_u^n) - \lim x = Le$ or $\Delta_u^n x_k \to Le \ (s_\lambda)$ and $s_\lambda(\Delta_u^n) = \{x : \exists L \in \mathbb{R}: s_\lambda(\Delta_u^n) - \lim x = Le\}$.

Theorem 5. *For any Orlicz function M, $[V, M, \Delta_u^n, s] \subset s_\lambda(\Delta_u^n)$.*

Proof. Let $x \in [V, M, \Delta_u^n, s]$ and $\epsilon > 0$. Then

$$\begin{aligned} \frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} M(\frac{| \Delta_u^n x_k - le |}{\rho}) &\geq \frac{1}{\lambda_n} \sum_{k \in I_n, |\Delta_u^n x_k - le| \geq \epsilon} M(\frac{| \Delta_u^n x_k - le |}{\rho}) \\ &\geq \frac{1}{\lambda_n} M(\epsilon/\rho). \mid \{k \in I_n : \mid \Delta_u^n x_k - le \mid \geq \epsilon\} \mid \end{aligned}$$

from which it follows that $x \in s_\lambda(\Delta_u^n)$.

To show that $s_\lambda(\Delta_u^n)$ strictly contain $[V, M, \Delta_u^n, s]$, we proceed as in [7]. We define $x = (x_k)$ by $(x_k) = k$ if $n - [\sqrt{\lambda_n}] + 1 \leq k \leq n$ and $(x_k) = 0$ otherwise. Then $x \notin l_\infty(\Delta_u^n, s)$ and for every ϵ $(0 < \epsilon \leq 1)$,

$$\frac{1}{\lambda_n} \mid \{k \in I_n : \mid \Delta_u^n x_k - 0 \mid \geq \epsilon\} \mid = \frac{[\sqrt{\lambda_n}]}{\lambda_n} \to 0 \text{ as } n \to \infty$$

i.e. $x \to 0$ $(s_\lambda(\Delta_u^n))$, where [ ] denotes the greatest integer function. On the other hand,

$$\frac{1}{\lambda_n} \sum_{k \in I_n} k^{-s} M(\frac{| \Delta_u^n x_k - 0 |}{\rho}) \to \infty \text{ as } n \to \infty$$

i.e. $x_k \nrightarrow 0$ $[V, M, \Delta_u^n, s]$. This completes the proof.

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