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A NEW APPROACH TO SOLVE SYSTEM OF SECOND ORDER NON-LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. In this paper a new method is introduced to solve system of second order non-linear ordinary differential equations. This method is based on homotopy perturbation technique. The effectiveness and convenience of the suggested method is illustrated with some examples.

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1. Introduction

In daily life, the physical phenomena and many scientific problems are formulated in the form of differential equations. Most problems are difficult to solve for their exact solutions. Therefore, several new techniques have been developed to obtain the analytical solutions which approximate the exact solutions. Such techniques are Adomian decomposition method [2], the variational iteration method [9] and the homotopy perturbation method (HPM) [10, 12-15]. These techniques have drawn the attention of scientists and engineers.

The HPM was proposed by Ji Huan He [10]. Further, He successfully applied this methods to asymptotology [12], limit cycle and bifurcation of non-linear problems [13], non-linear oscillators with discontinuities [14], non-linear wave equations [15]. Thus, He's method is universal and many mathematicians and engineers have applied it to solve various linear and non-linear problems. For example, this method is applied to the quadratic Riccati differential equation [1], non-linear Fredholm integral equations[20], axisymmetric flow over a stretching sheet [4], non-linear coupled systems of reaction-diffusion equations [7], non-linear schrödinger equations [6], non-linear polycrystalline solids [21], non-linear systems of partial differential equations [19], non-linear differential equations of fractional order [23], construction of solitary solutions and compacton-like solutions to partial differential equations [18], fractional IVPs [5], the Helmholtz equation [22], KdV equation [3] and other different type of problems of the various fields.

In the present paper, modified homotopy perturbation method is applied for the system of second order ordinary differential equations. To illustrate the effectiveness and convenience of the suggested procedure, few examples are considered.

2. Homotopy perturbation method

The homotopy perturbation method [8, 11, 16 - 17] is a combination of the classical perturbation technique and homotopy concept as used in topology. To explain the basic idea of homotopy perturbation method for solving non-linear differential equations, we consider the following non-linear ordinary differential equation

$$A(u) - f(r) = 0, r \in \Omega \tag{2.1}$$

subject to boundary condition

$$B\left(u, \frac{du}{dn}\right) = 0, r \in \Gamma, \tag{2.2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, Γ is the boundary of domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω.

The operator A can, generally speaking, be divided into two parts, a linear part L and a non-linear part N. Therefore, (2.1) can be written as follows

$$L(u) + N(u) - f(r) = 0. \tag{2.3}$$

By the homotopy technique, He constructed a homotopy $v(r, q) : \Omega \times [0, 1] \rightarrow R$ which satisfies

$$H(v, q) = (1 - q)L(v) - L(u_0) + qA(v) - f(r) = 0,$$

$$q \in [0, 1], r \in \Omega,$$

which is equivalent to

$$H(v, q) = L(v) - L(u_0) + qL(u_0) + q\left[N(v) - f(r)\right] = 0, \tag{2.4}$$

where $q \in [0, 1]$ is an embedding parameter, u_0 is an initial guess approximation of (2.1), which satisfies the boundary conditions. It follows from (2.4) that

$$H(v, 0) = L(v) - L(u_0) = 0, \tag{2.5}$$

$$H(v,1) = A(v) - f(r) = 0. \tag{2.6}$$

Thus, the changing process of q from zero to unity is just that of $v(r,q)$from $u_0(r)$ to $u(r)$. In topology, this is called deformation, and $L(v) - L(u_0), A(v) - f(r)$are called homotopic.

Here, we use the embedding parameter q as a small parameter and assume that the solution of (2.4) can be written as a power series in q

$$v = v_0 + qv_1 + q^2 v_2 + ... \tag{2.7}$$

Setting $q = 1$ we obtain the approximate solution of (2.1),

$$u = \lim_{q \to 1} v = v_0 + v_1 + v_2 + ... \tag{2.8}$$

The coupling of perturbation method and the homotopy method is called homotopy perturbation method, which has eliminated limitations of the traditional perturbation methods. In the other hand, this method can take the full advantage of perturbation techniques. The convergence of series (2.8) has been proved by He in his paper [8].

3. Modified homotopy perturbation method for system of second order ordinary differential equations

Consider the second order system of non-linear ordinary differential equations of the form

$$\begin{cases} x'' &= f(t,x,y,x',y'), \quad x(0) = A, \ x'(0) = B, \\ y'' &= g(t,x,y,x',y'), \quad y(0) = C, \ y'(0) = D, \end{cases} \tag{3.1}$$

where f and g are real functions and A , B , C , D are constants.

We construct the following general homotopy,

$$\begin{cases} v'' + u_0'' + pu_0'' + pf(t,v,w,v',w') = 0, \\ w'' + u_1'' + pu_1'' + pg(t,v,w,v',w') = 0, \end{cases} \tag{3.2}$$

where $p \in [0,1]$ is the embedding parameter and u_0 and u_1 are an initial guesses approximation of (3.1).

We assume that the solutions for (3.1) can be written as the power series in p as follows:

$$\begin{cases} v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + \ ... \\ w = w_0 + pw_1 + p^2 w_2 + p^3 w_3 + p^4 w_4 + \ ... \end{cases} \tag{3.3}$$

Setting $p=1$, we obtain the approximate solutions of (3.3),

$$x = \lim_{p\to 1} v = v_0 + v_1 + v_2 + ..., \quad (3.4)$$
$$y = \lim_{p\to 1} w = w_0 + w_1 + w_2 + ... \quad (3.5)$$

Using (3.3) in (3.2), we get

$$\begin{cases} \sum_{i=0}^{n} v_i'' + u_0'' + pu_0'' + f(t, \sum_{i=0}^{n} p^i v_i, \sum_{i=0}^{n} p^i w_i, \sum_{i=0}^{n} p^i v_i', \sum_{i=0}^{n} p^i w_i') = 0, \\ \sum_{i=0}^{n} w_i'' + u_1'' + pu_1'' + g(t, \sum_{i=0}^{n} p^i v_i, \sum_{i=0}^{n} p^i w_i, \sum_{i=0}^{n} p^i v_i', \sum_{i=0}^{n} p^i w_i') = 0. \end{cases} \quad (3.6)$$

Using Taylor's series with five variables on f and g and collecting the terms of like powers of p in (3.6), we obtain

$$p^0 : \begin{cases} v_0'' + u_0'' = 0, \quad v_0(0) = A, \; v_0'(0) = B, \\ w_0'' + u_1'' = 0, \quad w_0(0) = C, \; w_0'(0) = D, \end{cases} \quad (3.7)$$

$$p^1 : \begin{cases} v_1'' + u_0'' + f(t, v_0, \; w_0, v_0', w_0') = 0, \quad v_1(0) = 0, \; v_1'(0) = 0, \\ w_1'' + u_1'' + g(t, v_0, \; w_0, v_0', w_0') = 0, \quad w_1(0) = 0, \; w_1'(0) = 0, \end{cases} \quad (3.8)$$

$$p^2 : \begin{cases} v_2'' + f_{v_0'} v_1' + f_{w_0'} w_1' + f_{v_0} v_1 + f_{w_0} w_1 = 0, \quad v_2(0) = 0, \; v_2'(0) = 0, \\ w_2'' + g_{v_0'} v_1' + g_{w_0'} w_1' + g_{v_0} v_1 + g_{w_0} w_1 = 0, \quad w_2(0) = 0, \; w_2'(0) = 0, \end{cases} \quad (3.9)$$

$$p^3 : \begin{cases} v_3'' + \frac{1}{2} f_{v_0'^2} (v_1')^2 + f_{v_0' w_0'} v_1' w_1' + f_{v_0' v_0} v_1' v_1 + f_{v_0' w_0} v_1' w_1 + f_{v_0'} v_2' \\ + \frac{1}{2} f_{w_0'^2} (w_1')^2 + f_{w_0' v_0} w_1' v_1 + f_{w_0' w_0} w_1' w_1 + f_{w_0'} w_2' + \frac{1}{2} f_{v_0^2} v_1^2 \\ + f_{v_0 w_0} v_1 w_1 + f_{v_0} v_2 + \frac{1}{2} f_{w_0^2} w_1^2 + f_{w_0} w_2 = 0, \\ v_3(0) = 0, \; v_3'(0) = 0, \\ w_3'' + \frac{1}{2} g_{v_0'^2} (v_1')^2 + g_{v_0' w_0'} v_1' w_1' + g_{v_0' v_0} v_1' v_1 + g_{v_0' w_0} v_1' w_1 + g_{v_0'} v_2' \\ + \frac{1}{2} g_{w_0'^2} (w_1')^2 + g_{w_0' v_0} w_1' v_1 + g_{w_0' w_0} w_1' w_1 + g_{w_0'} w_2' + \frac{1}{2} g_{v_0^2} v_1^2 \\ + g_{v_0 w_0} v_1 w_1 + g_{v_0} v_2 + \frac{1}{2} g_{w_0^2} w_1^2 + g_{w_0} w_2 = 0, \\ w_3(0) = 0, \; w_3'(0) = 0, \end{cases} \quad (3.10)$$

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4. Applications

Now we apply the modified homotopy defined by (3.4 - 3.8) to solve system of second order ordinary differential equations.

Example 1. Consider the following system:

$$\begin{aligned} x'' - e^{\frac{1}{2}tx'} + e^{\frac{1}{3}y'} - 2 &= 0, \\ y'' + e^{3x-y'} - 3x' - 1 &= 0, \end{aligned}$$

subject to the initial conditions $x(0) = 0,\ x'(0) = 0,\ y(0) = 0,\ y'(0) = 0.$

In this case

$$f = -e^{\frac{1}{2}tv_0'} + e^{\frac{1}{3}w_0'} - 2,$$

and

$$g = e^{3v_0 - w_0'} - 3v_0' - 1,$$

According to the proposed homotopy, we have

$$p^0 : \begin{cases} v_0'' = 0, \\ v_0(0) = 0,\ v_0'(0) = 0, \\ w_0'' = 0, \\ w_0(0) = 0,\ w_0'(0) = 0, \end{cases}$$

$$p^1 : \begin{cases} v_1'' - 2 = 0, \\ v_1(0) = 0,\ v_1'(0) = 0, \\ w_1'' = 0, \\ w_1(0) = 0,\ w_1'(0) = 0, \end{cases}$$

$$p^2 : \begin{cases} v_2'' - t^2 = 0, \\ v_2(0) = 0,\ v_2'(0) = 0, \\ w_2'' + 3t^2 - 6t = 0, \\ w_2(0) = 0,\ w_2'(0) = 0, \end{cases}$$

$$p^3 : \begin{cases} v_3'' + t^2 - \frac{1}{3}t^3 - \frac{2}{3}t^4 = 0, \\ v_3(0) = 0,\ v_3'(0) = 0, \\ w_3'' - 3t^2 + \frac{19}{4}t^4 = 0, \\ w_3(0) = 0,\ w_3'(0) = 0, \end{cases}$$

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Corresponding solutions of the above system of linear second order ordinary differential equations are

$$\begin{aligned}
v_0 &= 0, \\
v_1 &= t^2, \\
v_1 &= t^2, v_1 = t^2, \\
v_3 &= -\frac{1}{12}t^4 + \frac{1}{60}t^5 + \frac{1}{45}t^6, \\
& . \\
& . \\
& .
\end{aligned}$$

and

$$\begin{aligned}
w_0 &= 0, \\
w_0 &= 0, \\
w_2 &= t^3 - \frac{1}{4}t^4, \\
w_2 &= t^3 - \frac{1}{4}t^4, \\
& . \\
& . \\
& .
\end{aligned}$$

Thus, form $v = v_0 + v_1 + v_2 + v_3 + ...$ and $w = w_0 + w_1 + w_2 + w_3 + ...$, we have $v = t^2 + \frac{1}{60}t^5 + \frac{1}{45}t^6 + ...$, and $w = t^3 - \frac{19}{120}t^6 + ...$, which converge to the exact solutions $x = t^2$, $y = t^3$.

Example 2. Now we consider the following system:

$$\begin{aligned}
x'' + x'^2 + y' &= 2t + 1, \\
y'' - x'y' &= 0\ ,
\end{aligned}$$

subject to the initial conditions $x(0) = 1,\ x'(0) = 2,\ y(0) = 0,\ y'(0) = 1.$

In this case

$$\begin{aligned}
f &= \left(v_0'\right)^2 + w_0' - 2t - 1, \\
g &= -v_0' w_0'.
\end{aligned}$$

We have according to the modified homotopy,

$$p^0 : \begin{cases} v_0'' = 0, \\ v_0(0) = 1,\ v_0'(0) = 2, \\ w_0'' = 0, \\ w_0(0) = 0,\ w_0'(0) = 1, \end{cases}$$

$$p^1:\begin{cases} v_1''-2t+4=0,\\ v_1(0)=0,\, v_1'(0)=0,\\ w_1''-2=0,\\ w_1(0)=0,\, w_1'(0)=0,\end{cases}$$
$$p^2:\begin{cases} 4t^2-14t+v_2''=0,\\ v_2(0)=0,\, v_2'(0)=0,\\ w_2''-t^2=0,\\ w_2(0)=0,\, w_2'(0)=0,\end{cases}$$
$$p^3:\begin{cases} w_3''+t^2-\frac{4}{3}t^3=0,\\ w_3(0)=0,\, w_3'(0)=0,\\ v_4''-115t^3+38t^4-\frac{52}{15}t^5=0,\\ v_4(0)=0,\, v_4'(0)=0,\end{cases}$$
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Corresponding solutions for the above system of linear second order ordinary differential equations are given by

$$\begin{aligned} w_2 &= t^3-\frac{1}{4}t^4,\\ v_1 &= -2t^2+\frac{1}{3}t^3,\\ v_2 &= \frac{7}{3}t^3-\frac{1}{3}t^4,\\ v_3 &= -\frac{11}{3}t^4+\frac{13}{20}t^5-\frac{1}{30}t^6,\\ & .\\ & .\\ & .\end{aligned}$$

and

$$\begin{aligned} w_0 &= t,\\ w_1 &= t^2,\\ w_2 &= \frac{1}{12}t^4,\\ w_3 &= -\frac{1}{12}t^4+\frac{1}{15}t^5,\\ & .\\ & .\\ & .\end{aligned}$$

Thus, form $v = v_0 + v_1 + v_2 + v_3 + ...$ and $w = w_0 + w_1 + w_2 + w_3 + ...$ we have $v = 1 + 2t - 2t^2 + \frac{8}{3}t^3 - 4t^4 + \frac{32}{5}t^5 - \frac{13}{10}t^6 + \frac{26}{315}t^7 + ...$, $w = t + t^2 - \frac{1}{360}t^6 + \frac{1}{315}t^7 + ...$, which converge to the exact solution $x = 1 + \ln(1 + 2t)$, $y = t + t^2$.

Example 3. For following system of differential equations:

$$\begin{aligned} x'' - 2xy' + x'y' &= 4e^{2t}, \\ y'' + xy + y &= (\cos t)\, e^{2t}, \end{aligned}$$

subject to the initial conditions $x(0) = 1,\ x'(0) = 2,\ \ y(0) = 1,\ y'(0) = 0,$

we have

$$f = -2v_0 w_0' + v_0' w_0' - 4e^{2t},$$

and

$$g = v_0 w_0 + w_0 - (\cos t)\, e^{2t}.$$

We have according to the proposed homotopy,

$$p^0 : \begin{cases} v_0'' = 0, \\ v_0(0) = 1,\ v_0'(0) = 2, \\ w_0'' = 0, \\ w_0(0) = 1,\ w_0'(0) = 0, \end{cases}$$

$$p^1 : \begin{cases} v_1'' - 8t - 8t^2 - \frac{16}{3}t^3 - \frac{8}{3}t^4 - \frac{16}{15}t^5 - \frac{16}{45}t^6 - 4 = 0, \\ v_1(0) = 0,\ v_1'(0) = 0, \\ w_1'' - \frac{3}{2}t^2 - \frac{1}{3}t^3 + \frac{7}{24}t^4 + \frac{19}{60}t^5 + \frac{13}{80}t^6 + 1 = 0, \\ w_1(0) = 0,\ w_1'(0) = 0, \end{cases}$$

$$p^2 : \begin{cases} v_2'' + 4t^2 - 2t^4 - \frac{1}{3}t^5 + \frac{7}{30}t^6 + \frac{19}{90}t^7 + \frac{13}{140}t^8 = 0, \\ v_2(0) = 0,\ v_2'(0) = 0, \\ w_2'' + t^2 + \frac{1}{3}t^3 + \frac{11}{12}t^4 + \frac{11}{20}t^5 + \frac{37}{360}t^6 - \frac{23}{2520}t^7 - \frac{293}{20\,160}t^8 - \frac{13}{2240}t^9 = 0, \\ w_2(0) = 0,\ w_2'(0) = 0, \end{cases}$$

$$p^3 : \begin{cases} v_3'' - 4t^2 + \frac{10}{3}t^4 + \frac{2}{3}t^5 + \frac{1}{2}t^6 + \frac{7}{45}t^7 - \frac{43}{1260}t^8 + \frac{41}{5040}t^9 - \frac{293}{45\,360}t^{10} + \frac{437}{50\,400}t^{11} = 0, \\ v_3(0) = 0,\ v_3'(0) = 0, \\ w_3'' - \frac{3}{2}t^4 - \frac{13}{15}t^5 - \frac{1}{9}t^6 - \frac{4}{315}t^7 + \frac{11}{1440}t^8 - \frac{1}{378}t^9 - \frac{4723}{453\,600}t^{10} + \frac{31\,951}{4989\,600}t^{11} = 0, \\ w_3(0) = 0,\ w_3'(0) = 0, \end{cases}$$

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Corresponding solutions of the above system of linear second order ordinary differential equations are

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\begin{aligned}
v_0 &= 1+2t,\\
v_1 &= 2t^2+\frac{4}{3}t^3+\frac{2}{3}t^4+\frac{4}{15}t^5+\frac{4}{45}t^6+\frac{8}{315}t^7+\frac{2}{315}t^8,\\
v_2 &= -\frac{1}{3}t^4+\frac{1}{15}t^6+\frac{1}{126}t^7-\frac{1}{240}t^8-\frac{19}{6480}t^9-\frac{13}{12\,600}t^{10},\\
v_3 &= \frac{1}{3}t^4-\frac{1}{9}t^6-\frac{1}{63}t^7-\frac{1}{112}t^8-\frac{7}{3240}t^9+\frac{43}{113\,400}t^{10}-\frac{41}{554\,400}t^{11}+\frac{293}{5987\,520}t^{12},\\
&\;.\\
&\;.\\
&\;.
\end{aligned}
$$

and

$$
\begin{aligned}
w_0 &= 1,\\
w_1 &= -\frac{1}{2}t^2+\frac{1}{8}t^4+\frac{1}{60}t^5-\frac{7}{720}t^6-\frac{19}{2520}t^7-\frac{13}{4480}t^8,\\
w_2 &= -\frac{1}{12}t^4-\frac{1}{60}t^5-\frac{11}{360}t^6-\frac{11}{840}t^7-\frac{37}{20\,160}t^8+\frac{23}{181\,440}t^9+\frac{293}{1814\,400}t^{10}+\frac{13}{246\,400}t^{11},\\
w_3 &= \frac{1}{20}t^6+\frac{13}{630}t^7+\frac{1}{504}t^8+\frac{1}{5670}t^9-\frac{11}{129\,600}t^{10}+\frac{1}{41\,580}t^{11},\\
&\;.\\
&\;.\\
&\;.
\end{aligned}
$$

Thus, form $v=v_0+v_1+v_2+v_3+...$ and $w=w_0+w_1+w_2+w_3+...$ we have

$v=1+2t+2t^2+\frac{4}{3}t^3+\frac{2}{3}t^4+...,\; w=1-\frac{1}{2}t^2+\frac{1}{24}t^4+\frac{7}{720}t^6+...,$

which converge to the exact solution $x=e^{2t}$, $y=\cos t$.

Example 4. At last we have

$$
\begin{aligned}
x''+2x'-y &= 2-te^{-t},\\
y''-x'+y' &= -2e^{-t}-1,
\end{aligned}
$$

subject to the initial conditions $x(0)=0,\; x'(0)=2,\; y(0)=1,\; y'(0)=0.$

In this case $f=f=2v_0'-w_0+\left(-2+te^{-t}\right),$ and $g=-v_0'+w_0'+2e^{-t}+1.$

We have according to the proposed homotopy,

$$
p^0:\begin{cases}
v_0''=0,\\
v_0(0)=0,\; v_0'(0)=2,\\
w_0''=0,\\
w_0(0)=1,\; w_0'(0)=0,
\end{cases}
$$

$$p^1:\begin{cases} v_1''+t-t^2+\frac{1}{2}t^3-\frac{1}{6}t^4+\frac{1}{24}t^5-\frac{1}{120}t^6+1=0,\\ v_1(0)=0,\ v_1'(0)=0,\\ w_1''-2t+t^2-\frac{1}{3}t^3+\frac{1}{12}t^4-\frac{1}{60}t^5+\frac{1}{360}t^6+1=0,\\ w_1(0)=0,\ w_1'(0)=0,\end{cases}$$

$$p^2:\begin{cases} v_2''-2t-\frac{1}{2}t^2+\frac{1}{3}t^3-\frac{1}{6}t^4+\frac{1}{20}t^5-\frac{1}{90}t^6+\frac{1}{504}t^7+\frac{1}{20\,160}t^8=0\\ v_2(0)=0,\ v_2'(0)=0,\\ w_2''+\frac{3}{2}t^2-\frac{2}{3}t^3+\frac{5}{24}t^4-\frac{1}{20}t^5+\frac{7}{720}t^6-\frac{1}{630}t^7=0\\ w_2(0)=0,\ w_2'(0)=0,\end{cases}$$

$$p^3:\begin{cases} v_3''+2t^2+\frac{1}{3}t^3-\frac{1}{24}t^4+\frac{1}{30}t^5-\frac{7}{720}t^6+\frac{1}{504}t^7-\frac{13}{40\,320}t^8-\frac{1}{30\,240}t^9=0,\\ v_3(0)=0,\ v_3'(0)=0,\\ w_3''-t^2-\frac{2}{3}t^3+\frac{1}{4}t^4-\frac{3}{40}t^5+\frac{1}{60}t^6-\frac{1}{336}t^7+\frac{1}{2240}t^8+\frac{1}{181\,440}t^9=0,\\ w_3(0)=0,\ w_3'(0)=0.\end{cases}$$

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Corresponding solutions of the above system of linear second order ordinary differential equations are

$$\begin{aligned} v_0 &= 2t,\\ v_1 &= -\frac{1}{2}t^2-\frac{1}{6}t^3+\frac{1}{12}t^4-\frac{1}{40}t^5+\frac{1}{180}t^6-\frac{1}{1008}t^7+\frac{1}{6720}t^8,\\ v_2 &= \frac{1}{3}t^3+\frac{1}{24}t^4-\frac{1}{60}t^5+\frac{1}{180}t^6-\frac{1}{840}t^7+\frac{1}{5040}t^8-\frac{1}{36\,288}t^9-\frac{1}{1814\,400}t^{10},\\ v_3 &= -\frac{1}{6}t^4-\frac{1}{60}t^5+\frac{1}{720}t^6-\frac{1}{1260}t^7+\frac{1}{5760}t^8-\frac{1}{36\,288}t^9+\frac{13}{3628\,800}t^{10}+\frac{1}{3326\,400}t^{11},\\ &\ \ \vdots \end{aligned}$$

and

$$\begin{aligned} w_0 &= 1,\\ w_1 &= -\frac{1}{2}t^2+\frac{1}{3}t^3-\frac{1}{12}t^4+\frac{1}{60}t^5-\frac{1}{360}t^6+\frac{1}{2520}t^7-\frac{1}{20\,160}t^8,\\ w_2 &= -\frac{1}{8}t^4+\frac{1}{30}t^5-\frac{1}{144}t^6+\frac{1}{840}t^7-\frac{1}{5760}t^8+\frac{1}{45\,360}t^9,\\ w_3 &= \frac{1}{12}t^4+\frac{1}{30}t^5-\frac{1}{120}t^6+\frac{1}{560}t^7-\frac{1}{3360}t^8+\frac{1}{24\,192}t^9-\frac{1}{201\,600}t^{10}-\frac{1}{19\,958\,400}t^{11},\\ &\ \ \vdots \end{aligned}$$

Thus, form $v = v_0 + v_1 + v_2 + v_3 + ...$ and $w = w_0 + w_1 + w_2 + w_3 + ...$ we have $v = 2t - \frac{1}{2}t^2 + \frac{1}{6}t^3 - \frac{1}{24}t^4 + ...,\ w = 1 - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \frac{1}{8}t^4 + ...,$
which converge to the exact solution
$x = t + 1 - e^{-t},\ y = e^{-t} + te^{-t}.$

5. Conclusion

In this paper, a modified homotopy method applied to solve the system of second order non-linear ordinary differential equations. Comparison of the results obtained by the proposed method with the exact solution reveals that the proposed method is very effective and convenient.

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