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ON APPROXIMATE AMENABILITY OF REES SEMIGROUP ALGEBRAS

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Abstract. Let $S = \mathcal{M}^o(G, P, I)$ be a Rees matrix semigroup with zero over a group G, we show that the approximate amenability of $\ell^1(S)$ is equivalent to its amenability whenever the group G is amenable and the index set I is finite.

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1. Introduction

In [8], Esslamzadeh introduced a new category of Banach algebras, l^1-Munn algebras, which he used as a tool in the study of semigroup algebras. He characterized amenable l^1-Munn algebras and also semisimple ones in this category. He also compared l^1-Munn algebras with some other well-known algebras and investigated some of their basic structural properties. In particular, he showed that the amenabilty of l^1-Munn algebra $\mathcal{M}(A, P, I, J)$ is equivalent to the amenability of the Banach algebra A whenever I and J are finite index sets and the sandwich matrix P is invertible.

In [5], Dales, Ghahramani, and Gronbaek introduced the concept of n-weak amenability for Banach algebras for $n \in$. They determined some relations between m- and n-weak amenability for general Banach algebras and for Banach algebras in various classes, and proved that, for each $n \in$, $(n + 2)$- weak amenability always implies n-weak amenability. Let A be a weakly amenable Banach algebra. Then it was proved in [5] that in the case where A is an ideal in its second dual $(A'',), A$ is necessarily $(2m - 1)$-weakly amenable for each $m \in$. The authors of [5] asked the following questions: (i) Is a weakly amenable Banach algebra necessarily 3-weakly amenable? (ii) Is a 2-weakly amenable Banach algebra necessarily 4-weakly amenable? A counter-example resolving question (i) was given by Zhang in [17], but it seems that question (ii) is still open.

It was also shown in [[5], Corollary 5.4] that for certain Banach space E the Banach algebra $\mathcal{N}(E)$ of nuclear operators on E is n-weakly amenable if and only if n is odd.

Another variation of the notion of amenability for Banach algebras was also introduced by Ghahramani and Loy in [10]. Let A be a Banach algebra and let X be a Banach A-bimodule. A derivation $D : A \to X$ is *approximately inner* if there is a net (x_α) in X such that

$$D(a) = lim_\alpha(a \cdot x_\alpha - x_\alpha \cdot a) \quad (a \in A),$$

the limit being taken in $(X, \|.\|)$. The Banach algebra A is *approximately amenable* if, for each Banach A-bimodule X, every continuous derivation $D : A \to X'$ is approximately inner.

The basic properties of approximately amenable Banach algebras were established in [10], see also [2]. Certainly every amenable Banach algebra is approximately amenable; a commutative, approximately amenable Banach algebra is weakly amenable; examples of commutative, approximately amenable Banach algebras which are not amenable were given in [[10], Example 6.1]. Characterizations of approximately amenable Banach algebras were also established in [10], they are analogous to the characterization of amenable Banach algebras as those with a bounded approximate diagonal.

A class of Banach algebras that was not considered in [5] is the Banach algebras on semigroups. In [15], Mewomo considered this class of Banach algebras by examining the n-weak amenability of some semigroup algebras. In particular, he showed that $l^1(S)$ is $(2k+1)$-weakly amenable for $k \in^+$ and a Rees matrix semigroup S.

In this paper, we shall continue our study on Rees matrix semigroup algebra with relation to its amenability and approximate amenability. We shall also extend the results in [8] on l^1-Munn algebras.

2. Preliminaries

First, we recall some standard notions; for further details, see [4], [11], and [3] .

Let A be an algebra. Let X be an A-bimodule. A *derivation* from A to X is a linear map $D : A \to X$ such that

$$D(ab) = Da \cdot b + a \cdot Db \quad (a, b \in A).$$

For example, $\delta_x : a \to a \cdot x - x \cdot a$ is a derivation; derivations of this form are the *inner derivations*.

Let A be a Banach algebra, and let X be an A-bimodule. Then X is a Banach A-bimodule if X is a Banach space and if there is a constant $k > 0$ such that

$$a \cdot x \leq k a x, \quad x \cdot a \leq k a x \quad (a \in A, x \in X).$$

By renorming X, we may suppose that $k = 1$. For example, A itself is Banach A-bimodule, and X', the dual space of a Banach A-bimodule X, is a Banach A-bimodule with respect to the module operations defined by

$$\langle x,\, a \cdot \lambda\rangle = \langle x \cdot a,\, \lambda\rangle, \quad \langle x,\, \lambda \cdot a\rangle = \langle a \cdot x,\, \lambda\rangle \quad (x \in X)$$

for $a \in A$ and $\lambda \in X'$; we say that X' is the *dual module* of X. Successively, the duals $X^{(n)}$ are Banach A-bimodules; in particular $A^{(n)}$ is a Banach A-bimodule for each $n \in$. We take $X^{(0)} = X$.

Let A be a Banach algebra, and let X be a Banach A-bimodule. Then $\mathcal{Z}^1(A,X)$ is the space of all continuous derivations from A into X, $\mathcal{N}^1(A,X)$ is the space of all inner derivations from A into X, and the first cohomology group of A with coefficients in X is the quotient space

$$\mathcal{H}^1(A,X) = \mathcal{Z}^1(A,X)/\mathcal{N}^1(A,X)\,.$$

The Banach algebra A is *amenable* if $\mathcal{H}^1(A,X') = \{0\}$ for each Banach A-bimodule X and *weakly amenable* if $\mathcal{H}^1(A,A') = \{0\}$. Further, as in [5], A is *n-weakly amenable* for $n \in$ if $\mathcal{H}^1(A,A^{(n)}) = \{0\}$, and A is *permanently weakly amenable* if it is n-weakly amenable for each $n \in$. For instance, each C^*-algebra is permanently weakly amenable [[5], Theorem 2.1].

Arens in [1] defined two products, and , on the bidual A'' of Banach algebra A; A'' is a Banach algebra with respect to each of these products, and each algebra contains A as a closed subalgebra. The products are called the *first* and *second Arens products* on A'', respectively. For the general theory of Arens products, see [4]-[6].

Let S be a non-empty set. Then

$$\ell^1(S) = \left\{ f \in^S \colon \sum_{s \in S} |f(s)| < \infty \right\},$$

with the norm $_1$ given by $\|f\|_1 = \sum_{s\in S} |f(s)|$ for $f \in \ell^1(S)$. We write δ_s for the characteristic function of $\{s\}$ when $s \in S$.

Now suppose that S is a semigroup. For $f, g \in \ell^1(S)$, we set

$$(f \star g)(t) = \left\{ \sum f(r)g(s) : r, s \in S,\, rs = t \right\} \quad (t \in S)$$

so that $f \star g \in \ell^1(S)$. It is standard that $(\ell^1(S), \star)$ is a Banach algebra, called the *semigroup algebra on S*. For a further discussion of this algebra, see [4], [6], for example. In particular, with $A = \ell^1(S)$, we identify A' with $C(\beta S)$, where βS is the Stone-Čech compactification of S, and $(A'',)$ with $(M(\beta S),)$, where $M(\beta S)$ is the space of regular Borel measures on βS of S; in this way, $(\beta S,)$ is a compact, right topological semigroup that is a subsemigroup of $(M(\beta S),)$ after the identification of $u \in \beta S$ with $\delta_u \in M(\beta S)$.

Let S be a semigroup, and let $o \in S$ be such that $so = os = o; (s \in S)$. Then o is a *zero* for the semigroup S. Suppose that $o \notin S$; set $S^o = S \cup \{o\}$, and define $so = os = o \quad (s \in S)$ and $o^2 = o$. Then S^o is a semigroup containing S as a subsemigroup; we say that S is formed by *adjoining a zero to S*.

We recall that S is a *right zero semigroup* if the product in S is such that

$$st = t \quad (s, t \in S) .$$

In this case, $f \star g = \varphi_S(f)g \;\; (f, g \in \ell^1(S))$.

Let S be a semigroup. we recall that S is regular if, for each $s \in S$, there exists $t \in S$ with $sts = s$. S is an inverse semigroup if for every $s \in S$ there is a unique $s^* \in S$ such that $ss^*s = s$ and $s^*ss^* = s^*$. An element $p \in S$ is an idempotent if $p^2 = p$; the set of idempotents of S is denoted by $E(S)$. Let S be a semigroup with a zero 0. Then an idempotent p is primitive if $p \neq 0$ or $q = 0$ whenever $q \in E(S)$ with $q \leq p$, where $\leq$ is partially ordered on $E(S)$ defined as $p \leq q$ if $p = pq = qp$ for every $p, q \in E(S)$. S is 0-simple if $S_{[2]} \neq \{0\}$ and the only ideals in S are $\{0\}$ and S, and S is completely 0-simple if it is 0-simple and contains a primitive idempotent.

3. Approximate amenability of l^1-Munn algebras

Let A be a unital algebra, let I and J be arbitrary index sets, and let $P = (a_{i,j})$ be a $J \times I$ nonzero matrix over A. Then $\mathcal{M}(A, P, I, J)$ the vector space of all $I \times J$ matrices over A is an algebra for the product

$$a \circ b = aPb \quad (a, b \in \mathcal{M}(A, P, I, J))$$

(in the sense of matrix products). This is the *Munn algebra over A with sandwich matrix P*, and it is denoted by

$$\mathcal{A} = \mathcal{M}(A, P, I, J) .$$

Now suppose that A is a unital Banach algebra and that each non-zero element in P has norm 1. Then $\mathcal{M}(A, P, I, J)$ is also a Banach algebra for the norm given by (3.1)

$$(a_{ij}) = \sum\{a_{ij} : i \in I \, j \in J \quad ((a_{ij}) \in \mathcal{M}(A, P, I, J)) . \qquad (1)$$

These Banach algebras are those defined by Esslamzadeh in [[8], Definition 3.1] called l^1-Munn algebra with the sandwich matrix P. When $J = I$, we denote $\mathcal{M}(A, P, I, J)$ by $\mathcal{M}(A, P, I)$, and when $J = I$ with P an identity $I \times I$ matrix over A, we denote

$\mathcal{M}(A, P, I, J)$ by $\mathcal{M}(A, I)$. Also we denote $\mathcal{M}(, I)$ simply by $\mathcal{M}(I)$ and in particular when $|I| = n < \infty, \mathcal{M}(I)$ is the algebra $\mathcal{M}_n$ of $n \times n$ complex matrices.

We may make the following assumptions if necessary : each non-zero element of P is invertible, and P has no zero rows or columns.

The following useful results are from [8] on l^1-Munn algebras.

Lemma 3.1 *Every $u \in \mathcal{M}(I)\hat{\otimes}A$ has a unique expression of the form* $u = \sum_{i,j\in I} \varepsilon_{ij} \otimes a_{ij}, \quad a_{ij} \in A.$

Lemma 3.2 $\mathcal{M}(A, I)$ *is isometrically algebra isomorphic to* $\mathcal{M}(I)\hat{\otimes}A$.

The next result is well-known for the case that I is finite, see [[6], Theorem 2.7]. The general case can be proved with the same technique and using Lemmas 3.1 and 3.2.

Theorem 3.3 *Let A be a unital Banach algebra.*
(i) The Banach algebra $\mathcal{M}(A, I)$ is amenable if and only if A is amenable.
(ii) The Banach algebra $\mathcal{M}(A, I)$ is weakly amenable if and only if A is weakly amenable.

We recall that a Banach algebra A is super-amenable if $\mathcal{H}^1(A, X) = \{0\}$ for each Banach A-bimodule X and that a diagonal operator $\pi : A\hat{\otimes}A \to A$ is defined as $\pi(a\hat{\otimes}b) = ab \quad (a, b \in A)$. $\mathbf{m} \in A\hat{\otimes}A$ is called a diagonal for A if $a \cdot \mathbf{m} - \mathbf{m} \cdot a = 0$ and $a\pi(\mathbf{m}) = a$. A is super-amenable if and only if A has a diagonal and $A\hat{\otimes}B$ is super-amenable if A and B are super amenable, see page 84 of [16] for further details. Since $\mathcal{M}_n$ has a diagonal, then it is super-amenable. With this, we have the next result.

Theorem 3.4 *let A be a super-amenable Banach algebra, then $\mathcal{M}(A, P, I, J)$ is super-amenable whenever I and J are finite and P is invertible.*

Proof. Since I and J are finite and P is invertible, then $\mathcal{M}(A, P, I, J)$ is topologically algebra isomorphic to $\mathcal{M}_n\hat{\otimes}A$ for $n = |I| = |J|$ by Lemma 3.2 above and Lemma 3.5 of [8], and so, it is super-amenable.

4. Approximate amenability of Ress semigroup algebras

Let S be a semigroup. It is not known in general when the semigroup algebra $\ell^1(S)$ is approximately amenable; partial result is given in [[9], Theorem 9.2]. Thus we cannot determine when $\ell^1(S)$ is approximately amenable. Some known structural implications of amenability of $\ell^1(S)$ for an arbitrary semigroup S are given below.

Theorem 4.1 *let S be a semigroup with $\ell^1(S)$ amenable. Then*
(i) $E(S)$ is finite [[3], Theorem 2]

(ii) S is regular [[3], Theorem 2]
(iii) $\ell^1(S)$ has an identity [[6], Corollary 10.6].

Here we give some special cases; we describe Rees semigroups, and show that, for each such semigroup S, the approximate amenability of $\ell^1(S)$ is equivalent to its amenability whenever the index set I is finite and the group G is amenable.

Rees semigroups are described in [[11], §3.2] and [[6], Chapter 3]. Indeed, let G be a group, and let I, J be arbitrary nonempty set; the zero adjoined to G is o. A *Rees semigroup* has the form $S = \mathcal{M}(G, P, I, J)$; here $P = (a_{ij})$ is a $J \times I$ matrix over G, the collection of $I \times J$ matrices with components in G. For $x \in G$, $i \in I$, and $j \in J$, let $(x)_{ij}$ be the element of $M(G^o, I, j)$ with x in the $(i, j)^{\text{th.}}$ place and o elsewhere. As a set, S consists of the collection of all these matrices $(x)_{ij}$. Multiplication in S is given by the formula

$$(x)_{ij}(y)_{k\ell} = (xa_{jk}y)_{i\ell} \quad (x, y \in G,\ i, k \in I, j, \ell \in J)\,;$$

it is shown in [[11], Lemma 3.2.2] that S is a semigroup.

Similarly, we have the semigroup $\mathcal{M}^o(G, P, I, J)$, where the elements of this semigroup are those of $\mathcal{M}(G, P, I, J)$, together with the element o, identified with the matrix that has o in each place (so that o is the zero of $\mathcal{M}^o(G, P, I, J)$), and the components of P are now allowed to belong to G^o. The matrix P is called the *sandwich matrix* in each case. The semigroup $\mathcal{M}^o(G, P, I, J)$ is a *Rees matrix semigroup with a zero over G*.

We write $\mathcal{M}^o(G, P, I)$ for $\mathcal{M}^o(G, P, I, I)$ in the case where $J = I$.

The above sandwich matrix P is *regular* if every row and column contains at least one entry in G; the semigroup $\mathcal{M}^o(G, P, I, J)$ is regular as a semigroup if and only if the sandwich matrix is regular.

For the Rees matrix semigroup $S = \mathcal{M}^o(G, P, I)$, suppose $P = (a_{ij})$, where $a_{ii} = e_G \quad (i \in I)$ and $a_{ij} = 0 \quad (i \neq j)$, so that $P = I_G(I)$ is the $I \times I$ identity matrix. Then we set $\mathcal{M}^o(G, P, I) = \mathcal{M}^o(G, I)$. With this notation, we have the next result.

Proposition 4.2 *let $S = \mathcal{M}^o(G, I)$ be a Rees matrix semigroup with a zero over a group G with index set I. Then $\ell^1(S)$ is amenable if and only if G is amenable and I is a finite set.*

Proof. Suppose the index set I is infinite. Since $\{(e)_{ii} : i \in I\} \subset E(S)$ where e is the identity element of G and $E(S)$ is the set of idempotents in S, then $E(S)$ is infinite since I is suppose to be infinite. Since S is inverse, then $\ell^1(S)$ is not amenable by using the remark on page 143 of [7].

As in [6], let $S = \mathcal{M}^o(G, P, I, J)$ be a Rees matrix semigroup with zero over a group G with index sets I and J and P a regular sandwich matrix. We set

$$N(P) = \{(j,k) \in I \times J : a_{jk} \neq 0\}$$

and

$$Z(P) = \{(j,k) \in I \times J : a_{jk} = 0\}.$$

For $i \in I$ and $j \in J$, let $e_{ij} = (e)_{ij}$ where e is the identity element of G. The elements e_{ij} are the matrix units of S. An idempotent other than o of $\mathcal{M}^o(G,P,I,J)$ has the form $(a_{jk}^{-1})_{kj}$, where $(j,k) \in N(P)$, and so

$$|E(S)| = |N(P)| + 1$$

if the index sets I and J are finite and $E(S)$ is infinite if I and J are infinite. In particular, in the case where $J = I$ and $P = I_G(I)$ and $S = \mathcal{M}^o(G,I)$, then

$$|E(S)| = |I| + 1$$

if I is finite and $E(S)$ is infinite if the index set I is infinite. With this, we give a generalization of proposition 4.2.

Proposition 4.3 *let $S = \mathcal{M}^o(G,P,I)$ be a Rees matrix semigroup with zero over a group G and a regular sandwich matrix P with index set I. Then $\ell^1(S)$ is amenable if and only if G is amenable and I is finite.*

Proof. Suppose the index set I is infinite, then $E(S)$ is infinite with the above explaination. Since $\ell^1(S)$ is inverse, then $\ell^1(S)$ is not amenable by the remark on page 143 of [7].

We next consider the approximate amenability of $\ell^1(S)$ for $S = \mathcal{M}^o(G,P,I,J)$.

Lemma 4.4 *Let S be any infinite set, then $\ell^1(S)$ is not approximately amenable.*

Proof. Suppose $\ell^1(S)$ is approximately amenable. Since S is infinite, there exists a continuous epimorphism $\varphi : \ell^1(S) \to \ell^1()$, and so $\ell^1()$ is approximately amenable using proposition 2.2 of [10]. This is a contradiction because $\ell^1()$ does not have a left approximate identity, so by [[10], Lemma 2.2], $\ell^1()$ is not approximately amenable.

Proposition 4.5 *Let $S = \mathcal{M}^o(G,P,I)$ be a Rees matrix semigroup with zero over a group G with an infinite index I. Then $\ell^1(S)$ is not approximately amenable.*

Proof. Clearly S is infinite if I is infinte. Thus the result follows from Lemma 4.4.

Theorem 4.6 *Let $S = \mathcal{M}^o(G, P, I)$ be a Rees matrix semigroup with a zero over a group G and a regular sandwich matrix P with index set I. Then the following are equivalent*
(i) $\ell^1(S)$ is amenable
(ii) $\ell^1(S)$ is approximately amenable
(iii) G is amenable and I is finite.

Proof. The implication (i) $\Rightarrow$ (ii) is clear, while the implications (iii) $\Leftrightarrow$ (i) is proposition 4.3. We only need to prove the implication (ii) $\Rightarrow$ (iii).
Suppose $\ell^1(S)$ is approximately amenable, then I is not infinite by proposition 4.5, so we conclude that I is finite. We finally prove that G is amenable. Since $\ell^1(S)$ is approximately amenable, then $\ell^1(S^1)$ is approximately amenable by [[10], Proposition 2.4], where $\ell^1(S^1)$ is the unitization of $\ell^1(S)$ and $S^1 = S \cup \{1\}$ such that $s \cdot 1 = 1 \cdot s = s \quad (s \in S^1)$ and $s \cdot t = st \quad (s, t \in S)$. And so by [[10], Theorem 2.1(b)], there is a net $(M_v) \subset (\ell^1(S^1)\hat{\otimes}\ell^1(S^1))^{**}$ such that for every $s \in S^1$,
$\delta_s \cdot M_v - M_v \cdot \delta_s \to 0$ and $\pi^{**}(M_v) = \delta_1$.
Let $i_o \in I$ be fixed and to each $\varphi \in \ell^\infty(G)$, we define $\tilde{\varphi} \in \ell^\infty(S^1 \times S^1)$ by

$$\tilde{\varphi}(s,t) = \begin{cases} \varphi(g) & \text{if } t = (g)_{i_o i_o}, \\ \\ 1 & \text{otherwise} \end{cases}$$

For each v, define $\langle \varphi, m_v \rangle = \langle \tilde{\varphi}, M_v \rangle$. Thus,

$$\langle 1, m_v \rangle = \langle \tilde{1}, M_v \rangle = \langle 1, \pi^{**}(M_v) \rangle = \langle 1, \delta_1 \rangle = 1.$$

For $g \in G$, we have

$$\left(\tilde{\varphi} \cdot \delta_{(g)i_o i_o}\right)(s,t) = \tilde{\varphi}\left(s, (g)_{i_o i_o} t\right) = \begin{cases} \varphi(gh) & \text{if } t = (h)_{i_o i_o}, \\ \\ 1 & \text{otherwise} \end{cases}$$

and

$$\varphi \tilde{\cdot} \delta_g(s,t) = \begin{cases} (\varphi \cdot \delta_g)(h) & \text{if } t = (h)_{i_o i_o}, \\ \\ 1 & \text{otherwise} \end{cases} = \begin{cases} \varphi(gh) & \text{if } t = (h)_{i_o i_o}, \\ \\ 1 & \text{otherwise} \end{cases}$$

Thus, $\tilde{\varphi} \cdot \delta_{(g)i_o i_o} = \varphi \tilde{\cdot} \delta_g$.
Similarly, $\delta_{(g)i_o i_o} \cdot \tilde{\varphi} = \tilde{\varphi}$.
And so, for each $g \in G, \varphi \in \ell^\infty(G)$ and v,

$$\langle \varphi \cdot \delta_g - \varphi, m_v \rangle = \langle \varphi \tilde{\cdot} \delta_g - \tilde{\varphi}, M_v \rangle$$

$$= \langle \tilde{\varphi} \cdot \delta_{(g)i_oi_o} - \delta_{(g)i_oi_o} \cdot \tilde{\varphi}, M_v \rangle$$
$$\langle \tilde{\varphi}, \delta_{(g)i_oi_o} \cdot M_v - M_v \cdot \delta_{(g)i_oi_o} \rangle$$
$$\leq \|\delta_{(g)i_oi_o} \cdot M_v - M_v \cdot \delta_{(g)i_oi_o}\| \|\varphi\|_\infty.$$

Thus, $\|\delta_g \cdot m_v - m_v\| \to 0$.
Let m be a w^*-cluster point of (m_v). By passing to a subnet, we may suppose that $m = w^* - lim_v m_v$. From $\|\delta_g \cdot m_v - m_v\| \to 0$, we have $\delta_g \cdot m = m$. And so, for every $\varphi \in \ell^\infty(G), g \in G$, we have

$$\langle \varphi \cdot \delta_g, m \rangle = \langle \varphi, \delta_g \cdot m \rangle = \langle \varphi, m \rangle.$$

By using [16, Theorem 1.1.9], it easily follows that G is amenable.

Remark 4.7 Theorem 4.6 shows that for a regular Rees matrix semigroup $S = \mathcal{M}^o(G, P, I)$ with zero over a group G, the approximate amenability of $\ell^1(S)$ is equivalent to its amenability in a case where G is amenable and I is finite.

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