Acta Universitatis Apulensis
ISSN: 1582-5329
No. 26/2011
pp. 155-166

SANDWICH THEOREMS FOR Φ-LIKE FUNCTIONS INVOLVING GENERALIZED INTEGRAL OPERATOR

Maslina Darus, Imran Faisal

Abstract. Having generalized integral operator, subordination theorems using standard concept of subordination are established. These results deduce some interesting sufficient conditions for analytic functions containing generalized integral operator satisfying the sandwich theorem for Φ-like functions. In fact, the results reduce to some well-known theorems studied by various authors.

Keywords: Φ-Like functions, subordination, superordination, Hadamard product.

2000 *Mathematics Subject Classification*: 30C45

1. Introduction and preliminaries

Univalent functions have been the topic of interests for many authors since the last 80 years or so. In fact, in recent years a remarkable advance knowledge in this field is seen everywhere in the literature. Almost every day and every week, results related to this area of studies appeared in numerous online journals. In this work, we will concentrate on Φ-like functions which perhaps was first introduced by Brickman [1] in 1973. He demonstrated a condition by which an analytic function can be proven to be univalent. After three years, a scientist named Ruscheweyh [16] introduced the general class of Φ-Like function. Then many other scientists continue to work in this direction.

Now we will state the definition given by the above said scientists. Let $\mathcal{H}$ be the class of functions analytic in $U = \{z : z \in C \quad |z| < 1\}$ and $\mathcal{A}$ be the subclass of $\mathcal{H}$ consisting of functions of the form

$$f(z) = a + a_2z^2 + a_3z^3 + ...$$

Let Φ be an analytic function in a domain containing $f(U)$, $\Phi(0) = 0$ and $\Phi'(0) > 0$. The function $f \in A$ is called Φ-like if

$$R(\frac{zf'(z)}{\Phi(f(z))} > 0, \quad z \in U.$$

This concept was introduced by Brickman [1] and established that a function $f \in A$ is univalent if and only if f is Φ-Like for some Φ.

Definition 1. *Let Φ be analytic function in a domain containing $f(U)$ such that $\Phi(0) = 0$, $\Phi'(0) = 1$ and $\Phi(\omega) \neq 0$ for $\omega \in f(U) \setminus \{0\}$. Let $q(z)$ be a fixed analytic function in U, $q(0) = 1$. The function $f \in A$ is called Φ-like with respect to q if*

$$\frac{zf'(z)}{\Phi(f(z))} \prec q(z), \quad z \in U.$$

We know that if functions f and g be analytic in U then f is called subordinate to g if there exist a Schwarz function $w(z)$ analytic in U such that $f(z) = g(w(z))$ where $z \in U$.

Then we denote this subordination by $f(z) \prec g(z)$ or simply $f \prec g$ but in a special case if g is univalent in U then above subordination is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$.

Let $\phi : C^3 \times U \to C$ and let h analytic in U. Assume that p,ϕ are analytic and univalent in U and p satisfies the differential superordination

$$h(z) \prec \phi(p(z), zp'(z), z^2p''(z); z). \tag{1}$$

Then p is called a solution of the differential superordination.

An analytic function q is called a subordinant if $q \prec p$ for all p satisfying equation (1). A univalent function q such that $p \prec q$ for all subordinants p of equation (1) is said to be the best subordinant.

Let $f \in \mathcal{A}$. Denote by $D^\lambda : \mathcal{A} \longrightarrow \mathcal{A}$ the operator defined by

$$D^\lambda = z/(1-z)^{\lambda+1} * f(z), \quad (\lambda > -1).$$

It is obvious that

$$D^0 f(z) = f(z), \quad D^1 f(z) = zf'(z)$$

and

$$D^\delta f(z) = z(z^{\delta-1} f(z))^\delta/\delta! \quad \delta \in \mathcal{N}_0.$$

The operator $D^\delta f$ is called the δth-order Ruscheweyh derivative of f.

Recently, Noor [2, 3] defined and studied an integral operator $I_n : \mathcal{A} \longrightarrow \mathcal{A}$ analogous to $D^\delta f$ as follows.

Let $f_n = z/(1-z)^{n+1}, (n \in \mathcal{N}_0$ and $f_n^{-1}(z)$ be defined such that

$$f_n(z) * f_n^{-1}(z) = z/(1-z)^2$$

Let $\mathcal{E}^-$ be the class of analytic functions, in U of the form

$$f(z) = 1/z - \sum_0^\infty a_n z^n \quad z \in U, a_n \geq 0.$$

Then

$$f_n(z) = f_n^{-1}(z) * f(z) = (z/(1-z)^{n+1})^{-1} * f(z).$$

We note that $I_0 f(z) = f(z), I_1 f(z) = zf'(z)$. The operator I_n is called the Noor integral of nth order of f (see [4, 5]), which is an important tool in defining several classes of analytic functions. In recent years, it has been shown that Noor integral operator has fundamental and significant applications in the geometric function theory.

For real or complex numbers a, b, c other than $0, -1, -2, ...$, the hypergeometric series is defined by

$$_2F_1(a, b; c; z) = \sum_0^\infty \frac{(a)_k (b)_k}{(c)_k (1)_k} z^k, \tag{2}$$

where $(x)_n$ is the pochhammer symbol defined by

$$(x)_n = \frac{\Gamma(n+x)}{\Gamma(x)} = \begin{cases} 1 & \text{if } n = 0 \\ x(x+1)(x+2)\cdots(x+n-1) & \text{if } n \in N \end{cases}$$

We note that the series (2) converges absolutely for all $z \in U$ so that it represents an analytic function in U. Also an incomplete beta function $\phi(a, c; z)$ is related to Gauss hypergeometric function $z_2F_1(a, b; c; z)$ as,

$$\phi(a, c; z) = z_2F_1(1, a; c; z)$$

and we note that $\phi(a, 1; z) = z/(1-z)^a$, where $\phi(a, 1; z)$ is a Koebe function. Here, $\phi(a, c; z)$ is a convolution operator defined by Carlson and Shaffer [6]. Further, Hohlov [7] introduced another convolution operator using $_2F_1(a, b; c; z)$.

Shukla and Shukla [8] then studied the mapping properties of a function f_μ given as the following

$$f_\mu(a, b, c, z) = (1-\mu) z_2F_1(a, b; c; z) + \mu z (z_2F_1(a, b; c; z))',$$

and investigated the geometric properties of an integral operator of the form

$$I(z) = \int_0^z \frac{f_\mu(t)}{t} dt.$$

Kim and Shon [9] considered linear operator $L_\mu : \mathcal{A} \longrightarrow \mathcal{A}$ defined by

$$L_\mu(a,b,c)f(z) = f_\mu(a,b,c)(z) * f(z).$$

Next we shall consider $(f_\mu)^{(-1)}$ given by

$$f_\mu(a,b,c)(z) * (f_\mu(a,b,c)(z))^{-1} = z/(1-z)^{\lambda+1}, \quad (\mu \geq 0, \lambda > -1),$$

where we obtain the following generalized linear operator:

$$I_\mu^\lambda(a,b,c)f(z) = (f_\mu(a,b,c)(z))^{-1} * f(z)$$

which is known as the generalized integral operator. Therefore, we can write $(f_\mu)^{-1}$ in the following form

$$(f_\mu(a,b,c)(z))^{-1} = \sum_{k=0}^{\infty} \frac{(\lambda+1)_k (c)_k}{(\mu k+1)(a)_k (b)_k} z^{k+1} \quad z \in U.$$

Thus

$$I_\mu^\lambda(a,b,c)f(z) = z + \sum_{k=0}^{\infty} \frac{(\lambda+1)_k (c)_k}{(\mu k+1)(a)_k (b)_k} a_{k+1} z^{k+1}.$$

Also it can easily be verified that

$$z(I_\mu^\lambda(a,b,c)f(z))' = (\lambda+1)I_\mu^{\lambda+1}(a,b,c)f(z) - \lambda(I_\mu^\lambda(a,b,c)f(z)),$$

$$z(I_\mu^\lambda(a+1,b,c)f(z))' = aI_\mu^\lambda(a,b,c)f(z) - (a-1)I_\mu^\lambda(a+1,b,c)f(z).$$

Definition 2. *Let $f \in A$ then $f \in \mathcal{S}^*$(the starlike subclass of A) if and only if for $z \in U$*

$$R(\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))} > o, \quad n \in N_0.$$

The authors obtain sufficient conditions for a function f containing generalized Integral operator of normalized analytic function by applying a method based on the differential subordination

$$q_1(z) \prec \frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))} \prec q_2(z).$$

To prove our subordination and superordination results, we need the following lemmas.

Lemma 1.[10] *Let q(z) be univalent in the unit disk U and θ, ϕ be analytic in a domain D containing $q(U)$ with $\phi(w) \neq 0$ when $w \in q(U)$,*

$$Q(z) = zq(z)\phi(q'(z)) \quad h(z) = \theta(q(z)) + Q(z).$$

Suppose that $Q(z)$ is starlike univalent in U and $R\left(\frac{zh'(z)}{Q(z)}\right) > 0$ for $z \in U$. If

$$\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z)),$$

then

$$p(z) \prec q(z),$$

and q is the best dominant.

Lemma 2.[11] *Let $q(z)$ be convex univalent in the unit disk U and ϑ,φ be analytic in a domain D containing $q(U)$. Suppose that $zq'(z)\varphi(q(z))$ is starlike univalent in U and $R(\frac{\vartheta' q(z)}{\vartheta q(z)} > 0$, $z \in U$.*

If $p(z) \in \mathcal{H}[q(0), 1] \bigcap Q$, with $p(U) \subseteq D$ and $\vartheta(p(z)) + zp'(z)\varphi p(z)$ is univalent in U and

$$\vartheta(q(z)) + zq'(z)\varphi q(z) \prec \vartheta(p(z)) + zp'(z)\varphi p(z),$$

then

$$q(z) \prec p(z),$$

and $q(z)$ is the best subordinant.

Definition 3.[12] *Denote by Q the set of all functions $f(z)$ that are analytic and injective on $\overline{U} - E(f)$ where $E(f) = \zeta \in \partial U : \lim_{z\to\zeta} f(z) = \infty$ and are such that $f'(\zeta) \neq 0$ for $\zeta \in \partial U - E(f)$.*

2. Sandwich Theorems.

By using lemmas 1 and 2, we prove the following subordination and superordination results.

Theorem 1. *Let $q(z) \neq 0$ be univalent in U such that $zq'(z)/q(z)$ is starlike univalent in U and*

$$R(1 + \frac{\alpha}{\gamma}q(z) + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}) > 0, \quad \alpha, \gamma \in \mathcal{C}, \quad \gamma \neq 0. \tag{3}$$

If $f \in A$ satisfies the subordination

$$\alpha(\frac{z(I_{\mu}^{\lambda}f(z))'}{\Phi(I_{\mu}^{\lambda}f(z))}) + \gamma(1 + \frac{z(I_{\mu}^{\lambda}f(z))''}{(I_{\mu}^{\lambda}f(z))'} - \frac{z\Phi'(I_{\mu}^{\lambda}f(z))}{\Phi(I_{\mu}^{\lambda}f(z))}) \prec \alpha q(z) + \gamma zq'(z)/q(z),$$

then

$$\frac{z(I_{\mu}^{\lambda}f(z))'}{\Phi(I_{\mu}^{\lambda}f(z))} \prec q(z),$$

and $q(z)$ is the best dominant.

Proof. Let

$$p(z) = \frac{z(I_{\mu}^{\lambda}f(z))'{}'}{\Phi(I_{\mu}^{\lambda}f(z))},$$

then after computation we have

$$zp'(z)/p(z) = 1 + \frac{z(I_{\mu}^{\lambda}f(z))''}{(I_{\mu}^{\lambda}f(z))'} - \frac{z\Phi'(I_{\mu}^{\lambda}f(z))}{\Phi(I_{\mu}^{\lambda}f(z))},$$

which yields the following subordination

$$\alpha p(z) + \gamma zp'(z)/p(z) \prec \alpha q(z) + \gamma zq'(z)/q(z), \quad \alpha, \gamma \in \mathcal{C}.$$

By setting

$$\theta(\omega) = \alpha\omega \quad \phi(\omega) = \gamma/\omega, \quad \gamma \neq 0,$$

it can be easily observed that $\theta(\omega)$ is analytic in C and $\phi(\omega)$ is analytic in $C \setminus \{0\}$ and that $\phi(\omega) \neq 0$ when $\omega \in C \setminus \{0\}$. Also by letting

$$Q(z) = zq'(z)\phi(q(z)) = \gamma zq'(z)/q(z),$$

and

$$h(z) = \theta(q(z)) + Q(z) = \alpha q(z) + \gamma zq'(z)/q(z),$$

we find that $Q(z)$ is starlike univalent in U and that

$$R(\frac{zh'(z)}{Q(z)} = R(1 + \frac{\alpha}{\gamma}q(z) + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}) > 0.$$

So by Lemma 1,we have $\frac{z(I_{\mu}^{\lambda}f(z))'}{\Phi(I_{\mu}^{\lambda}f(z))} \prec q(z)$.

In case $\Phi(\omega) = \omega$ in Theorem 1, then we have the following results.

Corollary 1. *Let $q(z) \neq 0$ be univalent in U. If q satisfies (3) and*

$$\alpha(\frac{z(I_{\mu}^{\lambda}f(z))'}{(I_{\mu}^{\lambda}f(z))}) + \gamma(1 + \frac{z(I_{\mu}^{\lambda}f(z))''}{(I_{\mu}^{\lambda}f(z))'} - \frac{z(I_{\mu}^{\lambda}f(z))'}{(I_{\mu}^{\lambda}f(z))}) \prec \alpha q(z) + \gamma z q'(z)/q(z),$$

then

$$\frac{z(I_{\mu}^{\lambda}f(z))'}{(I_{\mu}^{\lambda}f(z))} \prec q(z),$$

and $q(z)$ is the best dominant.

Corollary 2. *If $f \in \mathcal{A}$ and assume that (3) holds then*

$$1 + \frac{z(I_{\mu}^{\lambda}f(z))''}{(I_{\mu}^{\lambda}f(z))'} \prec \frac{1+Az}{1+Bz} + \frac{(A-B)z}{(1+Az)(1+Bz)},$$

implies

$$\frac{z(I_{\mu}^{\lambda}f(z))'}{(I_{\mu}^{\lambda}f(z))} \prec \frac{1+Az}{1+Bz}, \quad -1 \leq B \leq A \leq 1,$$

and $\frac{1+Az}{1+Bz}$ is the best dominant.

Proof. By setting $\alpha = \gamma = 1$ and $q(z) = 1 + Az/1 + Bz$ where $-1 \leq B \leq A \leq 1$.

Corollary 3. *If $f \in \mathcal{A}$ and assume that (3) holds then*

$$1 + \frac{z(I_{\mu}^{\lambda}f(z))''}{(I_{\mu}^{\lambda}f(z))'} \prec \frac{1+z}{1-z} + \frac{2z}{(1+z)(1-z)},$$

implies

$$\frac{z(I_{\mu}^{\lambda}f(z))'}{(I_{\mu}^{\lambda}f(z))} \prec \frac{1+z}{1-z},$$

and $\frac{1+z}{1-z}$ is the best dominant.

Proof. By setting $\alpha = \gamma = 1$ and $q(z) = 1 + z/1 - z$.

Theorem 2. *Let $q(z) \neq 0$ be convex univalent in the unit disk U. Suppose that*

$$R(\frac{\alpha}{\gamma}q(z)) > 0, \quad \alpha, \gamma \in C \, for \, z \in U, \tag{4}$$

and $zq'(z)/q(z)$ is starlike univalent in U. if,

$$\frac{z(I_{\mu}^{\lambda}f(z))'}{\Phi(I_{\mu}^{\lambda}f(z))} \in \mathcal{H}[q(0), 1] \bigcap Q, \quad f \in A,$$

$$\alpha(\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))})+\gamma(1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z\Phi'(I_\mu^\lambda f(z))}{\Phi(I_\mu^\lambda f(z))}),$$

is univalent is U and

$$q(z)+\gamma zq'(z)/q(z)\prec\alpha(\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))})+\gamma(1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z\Phi'(I_\mu^\lambda f(z))}{\Phi(I_\mu^\lambda f(z))}),$$

then

$$q(z)\prec\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))},$$

and q is the best subordinant.

Proof. Let

$$p(z)=\frac{z(I_\mu^\lambda f(z))'{}'}{\Phi(I_\mu^\lambda f(z))},$$

then after computation we get

$$zp'(z)/p(z)=1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z\Phi'(I_\mu^\lambda f(z))}{\Phi(I_\mu^\lambda f(z))},$$

this implies

$$\alpha q(z)+\gamma zq'(z)/q(z)\prec\alpha p(z)+\gamma zp'(z)/p(z),\quad \alpha,\gamma\in\mathcal{C}.$$

By setting

$$\vartheta(\omega)=\alpha\omega\quad \varphi(\omega)=\gamma/\omega,\quad \gamma\neq 0.$$

It can be easily observed that $\vartheta(\omega)$ is analytic in C and $\varphi(\omega)$ is analytic in $C\setminus\{0\}$ and that $\varphi(\omega)\neq 0$ when $\omega\in C\setminus\{0\}$. Also we obtain

$$R(\frac{\vartheta'(q(z))}{\varphi(q(z))})=R(\frac{\alpha}{\gamma}q(z))>0.$$

So by Lemma 2, we have

$$q(z)\prec\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))}.$$

When $\Phi(\omega)=\omega$ in Theorem 2, we obtain the following result

Corollary 4. *Let $q(z)\neq 0$ be univalent in U. If q satisfies (4) and*

$$\alpha q(z)+\gamma zq'(z)/q(z)\prec\alpha(\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))})+\gamma(1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))}),$$

then

$$q(z) \prec \frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))},$$

and $q(z)$ is the best dominant.

Theorem 3. *Let $q_1(z) \neq 0, q_2(z) \neq 0$ be convex univalent in the unit disk U satisfy (3) and (4) respectively. Suppose that and $zq_1'(z)/q_1(z)$, $zq_2'(z)/q_2(z)$ is starlike univalent in U. If,*

$$\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))} \in \mathcal{H}[q(0),1] \bigcap Q, \quad f \in A,$$

$$\alpha(\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))}) + \gamma(1 + \frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'} - \frac{z\Phi'(I_\mu^\lambda f(z))}{\Phi(I_\mu^\lambda f(z))}),$$

is univalent is U and

$$q_1(z)+\gamma\frac{zq_1'(z)}{q_1(z)} \prec \alpha(\frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))})+\gamma(1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z\Phi'(I_\mu^\lambda f(z))}{\Phi(I_\mu^\lambda f(z))}) \prec q_2(z)+\gamma\frac{zq_2'(z)}{q_2(z)},$$

then

$$q_1(z) \prec \frac{z(I_\mu^\lambda f(z))'}{\Phi(I_\mu^\lambda f(z))} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Applying simultaneously Theorem 1 and Theorem 2, we get the Sandwich result.

Corollary 5. *Let $q_1(z) \neq 0, q_2(z) \neq 0$ be convex univalent in the unit disk U satisfy (3) and (4) respectively. Suppose that and $zq_1'(z)/q_1(z)$, $zq_2'(z)/q_2(z)$ is starlike univalent in U. If,*

$$\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))} \in \mathcal{H}[q(0),1] \bigcap Q, \quad f \in A,$$

$$\alpha(\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))}) + \gamma(1 + \frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'} - \frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))}),$$

is univalent is U and

$$q_1(z)+\gamma\frac{zq_1'(z)}{q_1(z)} \prec \alpha(\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))})+\gamma(1+\frac{z(I_\mu^\lambda f(z))''}{(I_\mu^\lambda f(z))'}-\frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))}) \prec q_2(z)+\gamma\frac{zq_2'(z)}{q_2(z)},$$

then

$$q_1(z) \prec \frac{z(I_\mu^\lambda f(z))'}{(I_\mu^\lambda f(z))} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Note that Theorem 3 reduces to the following known result obtained by Ali et al. [13].

Corollary 6. *Let the statement of Theorem 3 holds true with $q_1(0) = q_2(0) = 1$. Then*

$$q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Proof. By setting $\Phi(\omega) = \omega, \quad \alpha = \gamma = 1$ and $n = 1$.

Corollary 7. *Let the statement of Theorem 3 holds true. Then*

$$q_1(z) \prec 1 + \frac{zf''(z)}{f'(z)} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Proof. By setting $\Phi(\omega) = \omega, \quad \alpha = \gamma = 1$ and $n = 0$.

Note also Theorem 3 reduces to the following known result obtained by Shanmugam et al. [14].

Corollary 8. *Let the statement of Theorem 3 holds true with $q_1(z) \neq 0$ and $q_2(z) \neq 0$, then*

$$q_1(z) \prec \frac{zf'(z)}{\Phi(f(z))} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Proof. By setting $\alpha = \gamma = 1$ and $n = 1$.

Moreover, Theorem 3 reduces to the following known result obtained by Shanmugam et al. [15]

Corollary 9. *Let the statement of Theorem 3 holds true with $q_1(0) = 1$ and $q_2(0) = 1$, then*

$$q_1(z) \prec \frac{zf'(z)}{\Phi(f(z))} \prec q_2(z),$$

and $q_1(z)$ is the best subordinant and $q_2(z)$ is the best dominant.

Proof. By setting $\alpha = \gamma = 1$ and $n = 1$.

Other work related to generalized integral operator can be found in ([17]-[19]).

Acknowledgement: The work here is fully supported by UKM-ST-06-FRGS0244-2010.

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Maslina Darus
School of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Bangi 43600 Selangor D. Ehsan, Malaysia
E-mail: *maslina@ukm.my*(Corresponding author)

Imran Faisal
School of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Bangi 43600 Selangor D. Ehsan, Malaysia
E-mail: *faisalmath@gmail.com*