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SPHERICAL IMAGES AND INVOLUTE OF SLANT HELICES IN EUCLIDEAN AND MINKOWSKI 3-SPACE

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Abstract. In this paper, we study slant helices in Euclidean 3-space and Minkowski 3-space. Firstly, we study the spherical image of slant helices in E^3. Then, making use of known properities of slant helices, we investigate spherical images of such curves in E_1^3. Furthermore, we obtain that the involute of the slant helix is a helix in E_1^3.

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1. Introduction

The helix is generally known as a curve in DNA double, carbon nanotubes, α-helices and collagen triple helix. In the geometry, the definition of helix is a curve that which tangent makes a constant angle with a fixed straight line called the axis of the general helix. The first two coordinates of helix provide circular motion while the third coordinate lifts the curve out of the plane. The shortest distance between two points on a cylinder (one not directly above the other) is a fractional turn of a helix. It is the reason why squirrels chasing one another up and around tree trunks follow helicalpaths. The most classic example of helices is a screw. The shape of screw likes stairs in minarets. Also in the nature, we can realize basic examples of helices such as nuts, bolts, horns. In animals helical structures usually appear in both mirror-image forms. Moreover, it is known that straight line ($\kappa = 0$) and circle ($\kappa = const.$, $\tau = 0$) are degenerate helices examples. In 1845 de Saint Venant proved the first condition for helices. This well known condition is "A necessary and sufficent condition that a curve be a general helix that the function

$$f = \frac{\tau}{\kappa}$$

is constant along the curve, where κ and τ denote the curvature and the torsion, respectively."

In Euclidean space, the slant helices are studied by Izumiya and Takeuchi, [1]. The slant helix is defined by property that its normal line makes a constant angle with a fixed direction. A necessary and sufficient condition that a curve be a slant helix is the function

$$\sigma(s) = (\frac{\kappa^2}{(\kappa^2+\tau^2)^{3/2}}(\frac{\kappa}{\tau})')(s)$$

is constant. Salkowski curves, anti-Salkowski curves and the curves of constant precessions are examples of slant helices. In [2] A.T. Ali defined k-slant helices and introduce some characterizations of this curve and in [3] M. Ergut, H.Balgetir and S. Aykurt obtain characterizations for this curve in E_1^3. A. T. Ali and R. Lopez studied slant helices, B_2 -slant helices and timelike curves of constant slope in Minkowski spaces, [4; 5]. The spherical image of tangent indicatrix and binormal indicatrix of slant helix and relations between a general helix and a slant helix were investigated by L. Kula, Y. Yayli, N. Ekmekci, K. Ilarslan in [6; 7]. Furthermore, the involute and evolute curves were investigeted by B. Bükcü and M. K. Karacan, [8].

In this paper, we study the spherical image of principal normal of slant helices and obtain the differential equation in E^3. We show that the spherical image of principal normal of slant helices is a circle. Then, making use of known properities of slant helices, we investigate spherical images of such curves in E_1^3. Furthermore, we obtain that the involute of the slant helix is a helix in E_1^3.

2. Preliminaries

In Euclidean 3-space E^3, the usual metric is

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. The curve $\alpha : I \subset R \to E^3$, $\alpha = \alpha(s)$ in E is called unit speed if $\langle \alpha'(s), \alpha'(s) \rangle = 1$ for each $s \in I$. $T(s) = \alpha'(s)$ is a unit tangent vector and $\kappa(s) = \| \alpha''(s) \|$ is the curvature of α at s. If $\kappa(s) \neq 0$, then the unit principal normal vector $N(s)$ of the curve α at s is given by $\alpha''(s) = \kappa(s)N(s)$. The unit vector $B(s) = T(s) \wedge N(s)$ is called the unit binormal vector of α at s. The Frenet formula for α is given by

$$\begin{aligned} T'(s) &= \kappa(s)N(s), \\ N'(s) &= -\kappa(s)T(s) + \tau(s)B(s), \\ B'(s) &= -\tau(s)N(s), \end{aligned}$$

and $\langle T, T \rangle = \langle N, N \rangle = \langle B, B \rangle = 1$, $\langle T, N \rangle = \langle N, B \rangle = \langle B, T \rangle = 0$, [2].

The slant helix in E^3 is defined by property that its normal line makes a constant angle with a fixed direction. A necessary and sufficent condition that a curve be a slant helix is the function

$$\sigma(s) = (\frac{\kappa^2}{(\kappa^2+\tau^2)^{3/2}}(\frac{\kappa}{\tau})')(s) \tag{2.1}$$

is constant, [1].

Let α be unit speed regular curve in Euclidean 3-space with Frenet vectors T, N and B. The unit tangent vectors along the curve $\alpha(s)$ generate a curve $\alpha_T = T(t)$ on the sphere of radius 1 about the origin. The curve α_T is called the spherical indicatrix of T or more commonly, α_T is called tangent indicatrix of the curve α. If $\alpha = \alpha(s)$ is a natural representation of α, then $\alpha_T = T(s)$ will be a representation of α_T. Similarly one considers the principal normal indicatrix $\alpha_N = N(s)$ and binormal indicatrix $\alpha_B = B(s)$.

If the Frenet frame of the principal normal indicatrix $\alpha_N = N$ of a space curve in E^3 is $\{T_N, N_N, B_N\}$, then we have Frenet formula:

$$\begin{bmatrix} T_N'(s_N) \\ N_N'(s_N) \\ B_N'(s_N) \end{bmatrix} = \begin{bmatrix} 0 & \kappa_N & 0 \\ -\kappa_N & 0 & \tau_N \\ 0 & -\tau_N & 0 \end{bmatrix} \begin{bmatrix} T_N \\ N_N \\ B_N \end{bmatrix}, \tag{2.2}$$

where

$$\begin{aligned} T_N &= \frac{-T+fB}{\sqrt{1+f^2}}, \\ N_N &= \frac{\sigma}{\sqrt{1+\sigma^2}}\left[\frac{fT+B}{\sqrt{1+f^2}} - \frac{N}{\sigma}\right], \\ B_N &= \frac{\sigma}{\sqrt{1+\sigma^2}}\left[\frac{fT+B}{\sqrt{1+f^2}} + \sigma N\right], \end{aligned}$$

and

$$\begin{gathered} s_N = \int \kappa(s)\sqrt{1+f^2(s)}ds, \\ \kappa_N = \sqrt{1+\sigma^2}, \\ \tau_N = \Gamma\sqrt{1+\sigma^2}, \\ f = \frac{\tau}{\kappa}, \\ \sigma = \frac{f'(s)}{\kappa(s)\left(1+f^2(s)\right)^{3/2}} \end{gathered} \tag{2.3}$$

where

$$\Gamma = \frac{\sigma'(s)}{\kappa(s)\sqrt{1+f^2(s)}\,(1+\sigma^2(s))^{3/2}},$$

s_N is natural representation of the principal normal indicatrix of the curve α and κ_N, τ_N are the curvature and torsion of α_N , respectively. Therefore, we have

$$\Gamma = \frac{\tau_N}{\kappa_N},$$

[2].

The Minkowski 3- space E_1^3 is the real vector space E^3 endowed with the standard flat Lorentzian metric given by

$$\langle , \rangle = -dx_1^2 + dx_2^2 + dx_3^2,$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E_1^3. If $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are arbitrary vectors in E_1^3, we define the (Lorentzian) vector product of u and v as the following

$$u \wedge v = \begin{vmatrix} -i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix},$$

[9].

The vector v in E_1^3 called a spacelike vector, null vector or a timelike vector if $\langle v, v\rangle > 0$ or $v = 0, \langle v, v\rangle = 0$ or $\langle v, v\rangle < 0$, respectively. For $v \in E_1^3$, the norm of the vector v defined by

$$\|v\| = \sqrt{|\langle v, v\rangle|}$$

and v is called a unit vector if $\|v\| = 1$, [8]. Given a unit speed curve γ in Minkowski space E_1^3 it is possible to define a Frenet frame $\{T(s), N(s), B(s)\}$ associated for each point s. Here T, N and B are the tangent, principal normal and binormal vector field, respectively, [9].

Let γ be a nonnull curve and $\{T(s), N(s), B(s)\}$ are Frenet vector fields, then the Frenet formulas are as follows

$$\begin{aligned} T' &= \kappa N, \\ N' &= (\varepsilon_B)\kappa T + \tau B, \\ B' &= (\varepsilon_T)\tau N, \end{aligned} \tag{2.4}$$

and

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ (\varepsilon_B)\kappa & 0 & \tau \\ 0 & (\varepsilon_T)\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

where $\varepsilon_v = \langle v, v\rangle$ and κ, τ are curvature function and torsion function of the curve γ, respectively, [10].

Let $\{T_N, N_N, B_N\}$ be the Frenet frame of the principal normal indicatrix $\gamma_N = N$ of a nonnull curve in E_1^3, then we get the Frenet equations

$$\begin{bmatrix} T_N'(s_N) \\ N_N'(s_N) \\ B_N'(s_N) \end{bmatrix} = \begin{bmatrix} 0 & \varepsilon_1\kappa_N(s_N) & 0 \\ -\varepsilon_0\kappa_N(s_N) & 0 & -\varepsilon_0\varepsilon_1\tau_N(s_N) \\ 0 & -\varepsilon_1\tau_N(s_N) & 0 \end{bmatrix} \begin{bmatrix} T_N(s_N) \\ N_N(s_N) \\ B_N(s_N) \end{bmatrix}, \tag{2.5}$$

where if $\varepsilon_2 = \begin{cases} 1, & \text{if } \varepsilon_1 F^2 - 1 > 0, \\ -1, & \text{if } \varepsilon_1 F^2 - 1 < 0, \end{cases}$, the vector fields T_N, N_N and B_N are

$$\begin{aligned} T_N &= -\frac{\varepsilon_0\left(T + \varepsilon_1 F B\right)}{\sqrt{|1-\varepsilon_1 F^2|}}, \\ N_N &= \frac{\varepsilon_0\varepsilon_1\varepsilon_2\sigma}{\sqrt{|1+\varepsilon_0\varepsilon_2\sigma^2|}}\left[\frac{FT+\varepsilon_1 B}{\sqrt{1-\varepsilon_1F^2}} + \frac{N}{\sigma}\right], \\ B_N &= \frac{\sigma}{\sqrt{|1+\varepsilon_0\varepsilon_2\sigma^2|}}\left[\frac{FT+\varepsilon_1 B}{\sqrt{1-\varepsilon_1F^2}} + \varepsilon_1\varepsilon_2\sigma N\right], \end{aligned}$$

where

$$\kappa_N = \sqrt{|1+\varepsilon_0\varepsilon_2\sigma^2|},$$

$$\tau_N = \frac{\left[\kappa^3(1-\varepsilon_1F^2)\left[F\kappa'' - \varepsilon_1\tau'' + \varepsilon_0\left(1-\varepsilon_1\right)\kappa^3 F\left(1-\varepsilon_1F^2\right)\right] + 3\varepsilon_0\kappa^2(F')(-\varepsilon_1\kappa\kappa' + \tau\tau')\right]}{\left(1-\varepsilon_1F^2\right)^3\left(1+\varepsilon_0\varepsilon_2\sigma^2\right)},$$

$$\Gamma = \frac{\sigma'(s)}{\kappa(s)\sqrt{|1-\varepsilon_1F^2(s)|}\left|1+\varepsilon_0\varepsilon_2\sigma^2(s)\right|^{3/2}}$$

and

$$F = \frac{\tau}{\kappa},$$

$$\sigma = \frac{F'(s)}{\kappa(s)\left|1-F^2(s)\right|^{3/2}},$$

[3].

A unit speed curve γ in E_1^3 is called a slant helix if there exists a constant vector field U such that the function $\langle N(s), U\rangle$ is constant.

Let γ be a unit speed timelike curve in E_1^3. Then γ is a slant helix if and only if either one the next two functions

$$\frac{\kappa^2}{(\tau^2-\kappa^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)' \text{ or } \frac{\kappa^2}{(\kappa^2-\tau^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)' \tag{2.7}$$

is constant everywhere $\tau^2 - \kappa^2$ does not vanish.

Let γ be a unit speed spacelike curve in E_1^3. If the normal vector of γ is spacelike, then γ is a slant helix if and only if (2.1) is constant. If the normal vector of γ is timelike, then γ is a slant helix if and only if the function

$$\frac{\kappa^2}{(\tau^2+\kappa^2)^{3/2}}(\frac{\tau}{\kappa})' \tag{2.8}$$

is constant. Furthermore, any spacelike curve with null normal vector is a slant curve, [4].

3. The spherical image of the principal normal indicatrix of slant helices in E^3

Theorem 3.1. *Let a unit speed curve α in E^3 be a slant helix. The spherical image of the principal normal indicatrix of α is a circle.*

Proof. Let $\{T(s), N(s), B(s)\}$ be the moving frame along α. Then the Frenet formula of α is

$$\begin{aligned} T'(s) &= \kappa(s)N(s), \\ N'(s) &= -\kappa(s)T(s) + \tau(s)B(s), \\ B'(s) &= -\tau(s)N(s), \end{aligned}$$

If the Frenet frame of the principal normal indicatrix $\alpha_N = N$ of a curve α is $\{T_N, N_N, B_N\}$, then we have the equations (2.2,2.3). Since the curve α is slant helix, we know that the function in E^3

$$\sigma = \frac{f'(s)}{\kappa(s)\left(1+f^2(s)\right)^{3/2}}$$

is constant. If we consider this in the equation

$$\Gamma = \frac{\sigma'(s)}{\kappa(s)\sqrt{1+f^2(s)}\left(1+\sigma^2(s)\right)^{3/2}}$$

then we have $\Gamma = 0$ and $\tau_N = 0$, $\kappa_N = const.$, therefore we obtain that the spherical image of the principal normal indicatrix of α is a circle.

Theorem 3.2. *Let α be a unit speed slant helix in E^3 with Frenet vectors T, N and B. If the principal normal indicatrix of α is $\alpha_N = N$, then the equation*

$$\phi' N \left(\cos\phi T - \sin\phi B\right)^{-1}$$

is constant where $\phi' = d\phi/ds$.

Proof. Let the Frenet-Serret frames of the curve $\alpha_N = N$ be $\{T_N, N_N, B_N\}$ and the curvature of α_N be κ_N, here

$$\kappa_N = \|D_{T_N} T_N\|$$

and

$$\alpha(s_N) = N(s)$$

where s_N is the parameter of the principal normal indicatrix $\alpha_N = N$. Then

$$T_N = -\frac{\kappa}{\|W\|}T + \frac{\tau}{\|W\|}B,$$

$$D_{T_N} T_N = \frac{1}{\|W\|}\left(\phi' \sin\phi T - \|W\| N + \phi' \cos\phi B\right)$$

where $W = -\kappa T + \tau B$ is Darboux vector and $\phi' = d\phi/ds$. From the equations (2.3), we have

$$\kappa_N = \sqrt{\left(\frac{\phi'}{\|W\|}\right)^2 + 1} = \sqrt{1 + \sigma^2}$$

Since the curve α is slant helix and $\sigma = const.$, then we have

$$\left(\frac{\phi'}{\|W\|}\right)^2$$

is constant. Therefore

$$\frac{\phi' N}{\kappa \cos\phi N + \tau \sin\phi N} = const.$$

$$\frac{\phi' N}{\cos\phi T - \sin\phi B} = const.$$

This completes the proof.

4. Spherical image of slant helices in E_1^3

Theorem 4.1. *Let a unit speed nonnull curve* γ *in* E_1^3 *be a slant helix. The spherical image of the principal normal indicatrix of* γ *is a circle.*

Proof. Let the Frenet frame of γ be $\{T(s), N(s), B(s)\}$. If the Frenet frame of the principal normal indicatrix $\gamma_N = N$ of a curve γ is $\{T_N, N_N, B_N\}$, then we have the equations (2.5,2.6). Since the curve γ is slant helix in E_1^3, the function

$$\sigma = \frac{F'(s)}{\kappa(s)\left(1+F^2(s)\right)^{3/2}}$$

is constant. If we consider this in the equation

$$\Gamma = \frac{\sigma'(s)}{\kappa(s)\sqrt{1+F^2(s)}\left(1+\sigma^2(s)\right)^{3/2}}$$

then we have $\Gamma = 0$ and $\tau_N = 0$, $\kappa_N = const.$, therefore we obtain that the spherical image of the principal normal indicatrix of γ is a circle.

Theorem 4.2. *Let a unit speed nonnull curve γ in E_1^3 be a slant helix. The spherical image of the tangent indicatrix of γ is a spherical helix.*

Proof. Let γ be a unit speed timelike curve with a spacelike principal normal in E_1^3 and the Frenet-Serret frames of γ be $\{T, N, B\}$. Then the Frenet formula is

$$\begin{aligned} T' &= \kappa N, \\ N' &= \kappa T + \tau B, \\ B' &= -\tau N, \end{aligned} \tag{4.1}$$

where T is timelike, N and B are spacelike. In this case

$$\langle T, T\rangle = -1, \langle N, N\rangle = \langle B, B\rangle = 1, \langle T, N\rangle = \langle N, B\rangle = \langle B, T\rangle = 0.$$

If we define the curvature and the torsion of the tangent indicatrix of γ by κ_T and τ_T , respectively, then we have

$$\kappa_T = \frac{\sqrt{\kappa^2 - \tau^2}}{\kappa}$$

and

$$\tau_T = \frac{\kappa}{\kappa^2 - \tau^2}\left(\frac{\tau}{\kappa}\right)'.$$

Thus, we have

$$\frac{\tau_T}{\kappa_T} = \frac{\kappa^2}{\left(\kappa^2 - \tau^2\right)^{3/2}}\left(\frac{\tau}{\kappa}\right)'.$$

Since the equations (2.7) is constant, the last equation $\frac{\tau_T}{\kappa_T}$ is costant. According to the definition of the helix, the spherical image of the tangent indicatrix of γ is a spherical helix.

Secondly, let γ be a unit speed spacelike curve with a spacelike principal normal in E_1^3. The Frenet formula of γ is

$$\begin{aligned} T' &= \kappa N, \\ N' &= -\kappa T + \tau B, \\ B' &= \tau N, \end{aligned} \tag{4.2}$$

where T and N are spacelike, B is timelike and

$$\langle T, T\rangle = \langle N, N\rangle = 1, \langle B, B\rangle = -1, \langle T, N\rangle = \langle N, B\rangle = \langle B, T\rangle = 0.$$

If we define the curvature and the torsion of the tangent indicatrix of γ by κ_T and τ_T , respectively, then we have

$$\kappa_T = \frac{\sqrt{\tau^2 - \kappa^2}}{\kappa}$$

and

$$\tau_T = -\frac{\kappa}{\tau^2 - \kappa^2}\left(\frac{\tau}{\kappa}\right)'.$$

From the last equations, we have

$$\frac{\tau_T}{\kappa_T} = -\frac{\kappa^2}{(\tau^2 - \kappa^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)'.$$

Since the last equation is constant, the tangent indicatrix of γ is a spherical helix.

Let γ be a unit speed spacelike curve with a timelike principal normal in E_1^3. Then

$$\begin{aligned} T' &= \kappa N, \\ N' &= \kappa T + \tau B, \\ B' &= \tau N, \end{aligned} \tag{4.3}$$

where T, N, B are tangent, principal normal and binormal vectors of γ. Here, T and B are spacelike, N is timelike and

$$\langle T, T\rangle = \langle B, B\rangle = 1, \langle N, N\rangle = -1, \langle T, N\rangle = \langle N, B\rangle = \langle B, T\rangle = 0.$$

After some calculations, we have

$$\kappa_T = \frac{\sqrt{\kappa^2 + \tau^2}}{\kappa},$$

$$\tau_T = \frac{\kappa}{\kappa^2 + \tau^2}\left(\frac{\tau}{\kappa}\right)'$$

and

$$\frac{\tau_T}{\kappa_T} = \frac{\kappa^2}{(\kappa^2 + \tau^2)^{3/2}} \left(\frac{\tau}{\kappa}\right)'.$$

Since the equation (2.8), the tangent indicatrix of γ is a spherical helix.

Theorem 4.3. *Let a unit speed nonnull curve γ in E_1^3 be a slant helix. The spherical image of the binormal indicatrix of γ is a spherical helix.*

Proof. Let γ be a unit speed timelike curve in E_1^3 and $\{T, N, B\}$ be the Frenet-Serret frames of the curve γ. From the equation (4.1), we have the curvature and the torsion of the binormal indicatrix of γ by, respectively

$$\kappa_B = \frac{\sqrt{\tau^2 - \kappa^2}}{\kappa}$$

and

$$\tau_B = \frac{\tau}{\tau^2 - \kappa^2} \left(\frac{\kappa}{\tau}\right)'.$$

Then, we have

$$\frac{\tau_B}{\kappa_B} = -\frac{\kappa^2}{(\tau^2 - \kappa^2)^{3/2}} \left(\frac{\tau}{\kappa}\right)'.$$

Since the equations (2.7) is constant, we can show easily that the binormal indicatrix of γ is a spherical helix.

Let γ be a unit speed spacelike curve with a spacelike principal normal or a unit speed spacelike curve with a timelike principal normal in E_1^3. The spherical images of the binormal indicatricies of γ is a spherical helix. From the Theorem 4.2, we can show easily that the proof is hold.

5. The involute of the slant helix in E_1^3

In this section, we study the involute of the unit speed spacelike slant helix γ with a timelike principal normal and show that the involute of γ is a helix in E_1^3.

Theorem 5.1. *Let a timelike curve β with a spacelike principal normal be the involute of the unit speed spacelike curve γ with a timelike principal normal in E_1^3. Then, the curve γ is a slant helix if and only if the curve β is a helix.*

Proof. Let the Frenet-Serret frames of the curves γ and β be $\{T,N,B\}$ and $\{T^*,N^*,B^*\}$, respectively. Then, we know that

$$\begin{aligned} T^* &= \frac{1}{\sqrt{\kappa^2+\tau^2}}N, \\ N^* &= \frac{\kappa}{\sqrt{\kappa^2+\tau^2}}T - \frac{\tau}{\sqrt{\kappa^2+\tau^2}}B, \\ B^* &= -\frac{\tau}{\sqrt{\kappa^2+\tau^2}}T - \frac{\kappa}{\sqrt{\kappa^2+\tau^2}}B, \end{aligned}$$

is hold.

If we take the curvature of the curve β by κ^* and the torsion of β by τ^*, we know that

$$\kappa^* = \frac{\sqrt{\kappa^2+\tau^2}}{|c-s|\,\kappa}$$

and

$$\tau^* = \frac{\kappa}{|c-s|\,(\kappa^2+\tau^2)}\left(\frac{\tau}{\kappa}\right)'$$

is given in [8]. Therefore, we have

$$\frac{\tau^*}{\kappa^*} = \frac{\kappa^2}{(\kappa^2+\tau^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)'.$$

This means that the curve β is a helix.

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