Acta Universitatis Apulensis No. 30/2012
ISSN: 1582-5329 pp. 209-219

GROWTH OF ITERATED ENTIRE FUNCTIONS IN TERMS OF ITS MAXIMUM TERM

Ratan Kumar Dutta

Abstract. In this paper we consider relative iteration of entire functions and study comparative growth of the maximum term of iterated entire functions with that of the maximum term of the related functions.

2000 *Mathematics Subject Classification*: 30D20.

1. Introduction and Definitions

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function. Then $M(r, f) = \max_{|z|=r} |f(z)|$ and $\mu(r, f) = \max_n |a_n| r^n$ are respectively called the maximum modulus and maximum term of $f(z)$ on $|z| = r$.

Following Sato [5], we write $log^{[0]}x = x$, $exp^{[0]}x = x$ and for positive integer $m \geq 1$, let $log^{[m]}x = log(log^{[m-1]}x)$, $exp^{[m]}x = exp(exp^{[m-1]}x)$.

The numbers ρ_f and λ_f defined by

$$\rho_f = \limsup_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log r}$$

and

$$\lambda_f = \liminf_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log r}$$

are respectively called the order and the lower order of $f(z)$.

A simple but useful relation between $M(r, f)$ and $\mu(r, f)$ is the following theorem.

Theorem 1. [7] *For $0 \leq r < R$,*

$$\mu(r, f) \leq M(r, f) \leq \frac{R}{R-r} \mu(R, f).$$

Taking $R = 2r$, for all sufficiently large values of r,

$$\mu(r, f) \leq M(r, f) \leq 2\mu(2r, f). \tag{1}$$

Taking two times logarithms in (1) it is easy to verify that

$$\rho_f = \limsup_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log r}$$

and

$$\lambda_f = \liminf_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log r}.$$

According to Lahiri and Banerjee [3] if $f(z)$ and $g(z)$ be entire functions then the iteration of f with respect to g is defined as follows:

$$\begin{aligned} f_1(z) &= f(z) \\ f_2(z) &= f(g(z)) = f(g_1(z)) \\ f_3(z) &= f(g(f(z))) = f(g_2(z)) = f(g(f_1(z))) \\ &\ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots \\ f_n(z) &= f(g(f........(f(z) \text{ or } g(z))........)), \\ &\quad \text{according as } n \text{ is odd or even,} \end{aligned}$$

and so

$$\begin{aligned} g_1(z) &= g(z) \\ g_2(z) &= g(f(z)) = g(f_1(z)) \\ g_3(z) &= g(f_2(z)) = g(f(g(z))) \\ &\ldots\ldots \qquad \ldots\ldots \\ g_n(z) &= g(f_{n-1}(z)) = g(f(g_{n-2}(z))). \end{aligned}$$

Clearly all $f_n(z)$ and $g_n(z)$ are entire functions.

Definition 1. [6] *Let $g(z)$ be an entire function of finite lower order λ_g. A function $\lambda_g(r)$ is called a lower proximate order of $g(z)$ relative to $\mu(r,g)$ if*

(i) $\lambda_g(r)$ is real, continuous and piecewise differentiable for all sufficiently large values of r,

(ii) $\lim_{r\to\infty} \lambda_g(r) = \lambda_g$,

(iii) $\lim_{r\to\infty} r \log r \lambda_g'(r) = 0$,

and (iv) $\liminf_{r\to\infty} \frac{\log \mu(r,g)}{r^{\lambda_g(r)}} = 1$.

Proposition 1. [4] *For $\delta(>0)$ the function $r^{\lambda_g+\delta-\lambda_g(r)}$ is an increasing function of r.*

In this paper we study growth properties of the maximum term of iterated entire functions as compared to the growth of the maximum term of the related function to generalisc some earlier results. Throughout the paper we denote by $f(z)$, $g(z)$ etc. non-constant entire functions of order (lower order) $\rho_f(\lambda_f)$, $\rho_g(\lambda_g)$ etc. We do not explain the standard notations and definitions of the theory of entire functions as those are available in [2], [8], [9].

2. Lemmas

The following lemmas will be needed in the sequel.

Lemma 1. [1] *If f and g are any two entire functions, for all sufficiently large values of r,*

$$M\left(\frac{1}{8}M\left(\frac{r}{2},g\right)-|g(0)|,f\right)\leq M(r,fog)\leq M(M(r,g),f)$$

Lemma 2. *If λ_g be finite, then*

$$\liminf_{r\to\infty}\frac{\log M(r,g)}{\log\mu(r,g)}\leq 2^{\lambda_g}.$$

The proof of the lemma is an immediate consequence of Theorem 2.16 of Lahiri and Sharma [4] but still for the sake of completeness we give the proof in details.

Proof of Lemma 2. From Definition 1,

$$\liminf_{r\to\infty}\frac{\log\mu(r,g)}{r^{\lambda_g(r)}}=1.$$

So, for given $\varepsilon(0<\varepsilon<1)$ it follows that

$$\log\mu(r,g)<(1+\varepsilon)r^{\lambda_g(r)}\tag{2}$$

for a sequence of values of r tending to infinity.

Therefore, for a sequence of values of r tending to infinity, we get from (1)

$$\begin{aligned}\log M(r,g) &\leq \log 2\mu(2r,g)\\ &\leq \log 2+(1+\varepsilon)(2r)^{\lambda_g(2r)},\quad\text{using (2)}\\ &= \log 2+(1+\varepsilon)\frac{(2r)^{\lambda_g+\delta}}{(2r)^{\lambda_g+\delta-\lambda_g(2r)}},\end{aligned}$$

where $\delta(>0)$ is arbitrary.

Now by Proposition 1, it follows that for a sequence of values of r tending to infinity

$$\begin{aligned}\log M(r,g) &< \log 2+(1+\varepsilon)\frac{(2r)^{\lambda_g+\delta}}{r^{\lambda_g+\delta-\lambda_g(r)}}\\ &= \log 2+(1+\varepsilon)2^{\lambda_g+\delta}r^{\lambda_g(r)}.\end{aligned}$$

Again since $\liminf_{r\to\infty} \frac{\log \mu(r,g)}{r^{\lambda_g(r)}} = 1$, for all sufficiently large values of r

$$(1-\varepsilon)r^{\lambda_g(r)} < \log \mu(r,g).$$

Thus for a sequence of values of r tending to infinity, we have

$$\log M(r,g) \leq \log 2 + \frac{1+\varepsilon}{1-\varepsilon} 2^{\lambda_g+\delta} \log \mu(r,g).$$

Since $\varepsilon,\ \delta > 0$ be arbitrary, we have

$$\liminf_{r\to\infty} \frac{\log M(r,g)}{\log \mu(r,g)} \leq 2^{\lambda_g}.$$

This proves the lemma.

Lemma 3. *If ρ_f and ρ_g are finite, then for any $\varepsilon > 0$,*

$$\log^{[n]} \mu(r,f_n) \leq \begin{cases} (\rho_f+\varepsilon)\log M(r,g) + O(1) & \text{when } n \text{ is even} \\ (\rho_g+\varepsilon)\log M(r,f) + O(1) & \text{when } n \text{ is odd} \end{cases}$$

for all sufficiently large values of r.

Proof. First suppose that n is even. Then in view of (1) and by Lemma 1 it follows that for all sufficiently large values of r,

$$\begin{array}{rcl} \mu(r,f_n) & \leq & M(r,f_n) \\ & \leq & M(M(r,g_{n-1}),f) \\ \text{i.e., } \log\ \mu(r,f_n) & \leq & \log M(M(r,g_{n-1}),f) \\ & \leq & [M(r,g_{n-1})]^{\rho_f+\varepsilon}. \\ \text{So, } \log^{[2]}\ \mu(r,f_n) & \leq & (\rho_f+\varepsilon)\log M(r,g(f_{n-2})) \\ & \leq & (\rho_f+\varepsilon)[M(r,f_{n-2})]^{\rho_g+\varepsilon}. \\ \text{i.e., } \log^{[3]}\ \mu(r,f_n) & \leq & (\rho_g+\varepsilon)\log M(r,f_{n-2}) + O(1). \\ \ldots\ \ \ldots & & \ldots\ \ \ldots \\ \ldots\ \ \ldots & & \ldots\ \ \ldots \\ \text{Therefore } \log^{[n]} \mu(r,f_n) & \leq & (\rho_f+\varepsilon)\log M(r,g) + O(1). \end{array}$$

Similarly if n is odd then for all sufficiently large values of r

$$\log^{[n]} \mu(r,f_n) \leq (\rho_g+\varepsilon)\log M(r,f) + O(1).$$

This proves the lemma.

Lemma 4. *If λ_f, λ_g are non-zero finite, then*

$$\log^{[n]} \mu(r, f_n) > \begin{cases} (\lambda_f - \varepsilon) \log M(r, g) + O(1) & \text{when } n \text{ is even} \\ (\lambda_g - \varepsilon) \log M(r, f) + O(1) & \text{when } n \text{ is odd.} \end{cases}$$

Proof. First suppose that n is even. Let $\epsilon(> 0)$ be such that $\epsilon < min\{\lambda_f, \lambda_g\}$. Now we have from {[7], p-113} for all sufficiently large values of r,

$$\begin{aligned} \mu(r, fog) &> e^{[M(r,g)]^{\lambda_f - \varepsilon}}. \\ \text{So,} \qquad \log \mu(r, fog) &> [M(r, g)]^{\lambda_f - \varepsilon}. \end{aligned} \tag{3}$$

Now

$$\begin{aligned} \log \mu(r, f_n) &= \log \mu(r, f(g_{n-1})) \\ &> [M(r, g_{n-1})]^{\lambda_f - \varepsilon} \qquad \text{using (3)} \\ &\geq [\mu(r, g_{n-1})]^{\lambda_f - \varepsilon} \qquad \text{from (1).} \\ \log^{[2]} \mu(r, f_n) &> (\lambda_f - \varepsilon) \log \mu(r, g(f_{n-2})) \\ &> (\lambda_f - \varepsilon)[M(r, f_{n-2})]^{\lambda_g - \varepsilon} \qquad \text{using (3).} \end{aligned}$$

$$\begin{aligned} \log^{[3]} \mu(r, f_n) &> (\lambda_g - \varepsilon) \log[\mu(r, f_{n-2})] + O(1) \\ &> (\lambda_g - \varepsilon)[M(r, g_{n-3})]^{\lambda_f - \varepsilon} + O(1). \end{aligned}$$

Taking repeated logarithms

$$\begin{aligned} log^{[n-1]} \mu(r, f_n) &\geq (\lambda_g - \varepsilon)[M(r, g)]^{\lambda_f - \varepsilon} + O(1). \\ \log^{[n]} \mu(r, f_n) &\geq (\lambda_f - \varepsilon) \log M(r, g) + O(1). \end{aligned}$$

Similarly,

$$\log^{[n]} \mu(r, f_n) \geq (\lambda_g - \varepsilon) \log M(r, f) + O(1) \qquad \text{when n is odd.}$$

This proves the lemma.

3. Theorems

Theorem 2. *If ρ_f and ρ_g are finite, then*

$$\liminf_{r \to \infty} \frac{\log^{[n]} \mu(r, f_n)}{\log \mu(r, g)} \leq \rho_f 2^{\lambda_g}$$

when n is even.

Proof. We have from Lemma 3 for all sufficiently large values of r,

$$\begin{aligned}\log^{[n]}\mu(r,f_n) &\le (\rho_f+\varepsilon)\log M(r,g)+O(1).\\ \liminf_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,g)} &\le (\rho_f+\varepsilon)\liminf_{r\to\infty}\frac{\log M(r,g)}{\log\mu(r,g)}.\end{aligned}$$

Since $\varepsilon>0$ is arbitrary, we get from Lemma 2,

$$\liminf_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,g)}\le \rho_f 2^{\lambda_g}.$$

This proves the theorem.

Theorem 3. *Under the assumptions of Theorem 2 if n is odd then*

$$\liminf_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}\le \rho_g 2^{\lambda_f}.$$

Theorem 4. *Let $f(z)$ and $g(z)$ be entire functions of finite order with $\rho_g<\lambda_f$ and n is even then*

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}=0.$$

Proof. When n is even then we have from Lemma 3 for all sufficiently large values of r,

$$\begin{aligned}\log^{[n]}\mu(r,f_n) &\le (\rho_f+\varepsilon)\log M(r,g)+O(1)\\ &\le (\rho_f+\varepsilon)r^{\rho_g+\varepsilon}+O(1). \end{aligned}\tag{4}$$

Also from definition of lower order we have for $r\ge r_0$,

$$\log\mu(r,f)\ge r^{\lambda_f-\varepsilon}.\tag{5}$$

So from (4) and (5) we get for $r\ge r_0$,

$$\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}\le\frac{(\rho_f+\varepsilon)r^{\rho_g+\varepsilon}+O(1)}{r^{\lambda_f-\varepsilon}}.$$

Since $\lambda_f>\rho_g$, we can choose $\varepsilon>0$ such that $\lambda_f-\varepsilon>\rho_g+\varepsilon$ then

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}=0.$$

This proves the theorem.

Note 1. If we take $\rho_g < \rho_f$ in Theorem 4, the result is still valid.

Theorem 5. *Let $f(z)$ and $g(z)$ be entire functions of finite order with $\rho_f < \lambda_g$ and n is odd then*
$$\limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,g)} = 0.$$

Theorem 6. *Let $f(z)$ and $g(z)$ be entire functions of finite order with $\lambda_g > \rho_f$ and n is even then*
$$\limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)} = \infty.$$

Proof. When n is even then from Lemma 4 we have for all sufficiently large values of r and $0 < \varepsilon < \min\{\lambda_f,\lambda_g\}$,
$$\begin{aligned} \log^{[n]}\mu(r,f_n) &> (\lambda_f - \varepsilon)\log M(r,g) + O(1) \\ &\geq (\lambda_f - \varepsilon) r^{\lambda_g - \varepsilon} + O(1). \end{aligned} \tag{6}$$

Also for all sufficiently large values of r,
$$\log\mu(r,f) \leq r^{\rho_f + \varepsilon}. \tag{7}$$

Therefore from (6) and (7) we get for $r \geq r_0$,
$$\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)} \geq \frac{(\lambda_f - \varepsilon) r^{\lambda_g - \varepsilon} + O(1)}{r^{\rho_f + \varepsilon}}.$$

Since $\lambda_g > \rho_f$, we can choose $\varepsilon > 0$ such that $\lambda_g - \varepsilon > \rho_f + \varepsilon$,
$$\limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)} = \infty.$$

This proves the theorem.

Theorem 7. *Let $f(z)$ and $g(z)$ be entire functions of finite order with $\lambda_f > \rho_g$ and n is odd then*
$$\limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,g)} = \infty.$$

Theorem 8. *Let $f(z)$ and $g(z)$ be transcendental entire functions of non zero finite order then*
$$\limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)} = \infty = \limsup_{r\to\infty} \frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,g)}.$$

Proof. First we consider n is even then from (6) we have for sufficiently large values of r,

$$\log^{[n]}\mu(r,f_n) > (\lambda_f-\varepsilon)r^{\lambda_g-\varepsilon}+O(1) \tag{8}$$

where $0<\varepsilon<\min\{\lambda_f,\lambda_g\}$ and from (7)

$$\log^{[2]}\mu(r,f) \leq (\rho_f+\varepsilon)\log r. \tag{9}$$

So from (8) and (9) we get,

$$\frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)} \geq \frac{(\lambda_f-\varepsilon)r^{\lambda_g-\varepsilon}+O(1)}{(\rho_f+\varepsilon)\log r}.$$

Since $\varepsilon>0$ is arbitrary,

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)}=\infty.$$

Also when n is odd then from Lemma 4 we have for sufficiently large values of r and $0<\varepsilon<\min\{\lambda_f,\lambda_g\}$,

$$\begin{aligned}\log^{[n]}\mu(r,f_n) &> (\lambda_g-\varepsilon)\log\mu(r,f)+O(1)\\ &\geq (\lambda_g-\varepsilon)r^{\lambda_f-\varepsilon}+O(1).\end{aligned} \tag{10}$$

So from (9) and (10) we get,

$$\frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)} \geq \frac{(\lambda_g-\varepsilon)r^{\lambda_f-\varepsilon}+O(1)}{(\rho_f+\varepsilon)\log r}.$$

Since $\varepsilon>0$ is arbitrary,

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)}=\infty.$$

Similarly we have

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log^{[2]}\mu(r,g)}=\infty.$$

This proves the theorem.

Note 2. If we take one more logarithm of the numerator then the expression in Theorem 8 is finite. Thus we shall prove this theorem.

Theorem 9. *Let $f(z)$ and $g(z)$ be transcendental entire functions of finite order then*

$$\begin{aligned}
(i)\quad \limsup_{r\to\infty} \frac{\log^{[n+1]}\mu(r,f_n)}{\log^{[2]}\mu(r,g)} &\le \frac{\rho_g}{\lambda_g}, \quad \textit{when } n \textit{ is even and } \lambda_g>0,\\
(ii)\quad \limsup_{r\to\infty} \frac{\log^{[n+1]}\mu(r,f_n)}{\log^{[2]}\mu(r,f)} &\le \frac{\rho_f}{\lambda_f}, \quad \textit{when } n \textit{ is odd and } \lambda_f>0.
\end{aligned}$$

Proof. First we consider n is even then from Lemma 3 we have for sufficiently large values of r,

$$\begin{aligned}
\log^{[n+1]}\mu(r,f_n) &\le \log^{[2]} M(r,g)+O(1)\\
&\le (\rho_g+\varepsilon)\log r+O(1). \qquad (11)
\end{aligned}$$

Also we have for $r\ge r_0$ and $0<\varepsilon<\lambda_g$,

$$\log^{[2]}\mu(r,f)\ge(\lambda_g-\varepsilon)\log r. \qquad (12)$$

Therefore from (11) and (12) we get for $r\ge r_0$ and $0<\varepsilon<\lambda_g$,

$$\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}\le\frac{(\rho_g+\varepsilon)\log r+O(1)}{(\lambda_g-\varepsilon)\log r}.$$

Since $\varepsilon>0$ is arbitrary,

$$\limsup_{r\to\infty}\frac{\log^{[n]}\mu(r,f_n)}{\log\mu(r,f)}\le\frac{\rho_g}{\lambda_g}.$$

Similarly for odd n we get second part of the theorem.
This proves the theorem.

Theorem 10. *If f,g and h are three non constant entire functions of finite order and $\rho_f<\lambda_h$, then*

$$\lim_{r\to\infty}\frac{\log^{[n-2]}\mu(r,f_n)}{\log^{[n-2]}\mu(r,h_n)}=0$$

for n is even and $h_n(z)=h(g(h........(h(\ g(z))........))$.

Proof. When n is even then from Lemma 3 we have for all large values of r,

$$\begin{aligned}
\log^{[n]}\mu(r,f_n) &\le (\rho_f+2\varepsilon)\log M(r,g)\\
&\le \log[M(r,g)]^{\rho_f+2\varepsilon}.
\end{aligned}$$

Therefore

$$\log^{[n-2]} \mu(r, f_n) \leq e^{[M(r,g)]^{\rho_f+2\varepsilon}}. \tag{13}$$

Also from Lemma 4 we have for sufficiently large values of r and $0 < \varepsilon < \frac{1}{4}(\lambda_h - \rho_f)$,

$$\log^{[n-2]} \mu(r, h_n) > e^{[M(r,g)]^{\lambda_h-2\varepsilon}}. \tag{14}$$

So from (13) and (14) we obtain,

$$\begin{aligned}\frac{\log^{[n-2]} \mu(r, f_n)}{\log^{[n-2]} \mu(r, h_n)} &\leq \frac{e^{[M(r,g)]^{\rho_f+2\varepsilon}}}{e^{[M(r,g)]^{\lambda_h-2\varepsilon}}} \\ &\leq \frac{1}{e^{[M(r,g)]^{\lambda_h-2\varepsilon}-[M(r,g)]^{\rho_f+2\varepsilon}}}.\end{aligned}$$

Since $0 < \varepsilon < \frac{1}{4}(\lambda_h - \rho_f)$ is arbitrary and g is non constant,

$$\lim_{r\to\infty} \frac{\log^{[n-2]} \mu(r, f_n)}{\log^{[n-2]} \mu(r, h_n)} = 0.$$

This proves the theorem.

Theorem 11. *If f, g and h are three entire functions with non zero lower order and finite order also $\rho_h < \lambda_g$ then*

$$\lim_{r\to\infty} \frac{\log^{[n]} \mu(r, f_n)}{\log^{[n]} \mu(r, f_n')} = \infty$$

for n is even and $f_n'(z) = f(h(f........(f(h(z))........))$.

Proof. When n is even then from Lemma 3 we have for $r > r_0$,

$$\log^{[n]} \mu(r, f_n') \leq (\rho_f + \varepsilon) r^{\rho_h+\varepsilon}. \tag{15}$$

Hence from (6) and (15) we have for sufficiently large values of r and $0 < \varepsilon < \min\{\lambda_f,\lambda_h\}$,

$$\frac{\log^{[n]} \mu(r, f_n)}{\log^{[n]} \mu(r, f_n')} \geq \frac{(\lambda_f - \varepsilon) r^{\lambda_g-\varepsilon} + O(1)}{(\rho_f + \varepsilon) r^{\rho_h+\varepsilon} + O(1)}.$$

Since $\rho_h < \lambda_g$, we can choose $\varepsilon > 0$ such that $\rho_h + \varepsilon < \lambda_g - \varepsilon$,

$$\lim_{r\to\infty} \frac{\log^{[n]} \mu(r, f_n)}{\log^{[n]} \mu(r, f_n')} = \infty$$

This proves the theorem.

References

[1] J. Clunie, *The composition of entire and meromorphic functions*, Mathematical essays dedicated to A. J. Macintyre, Ohio Univ. Press, (1970), 75-92.

[2] W. K. Hayman, *Meromorphic Functions*, Oxford University Press, 1964.

[3] B. K. Lahiri and D. Banerjee, *Relative fix points of entire functions*, *J. Indian Acad. Math.*, Volume 19, Number 1, (1997), 87-97.

[4] I. Lahiri and D. K. Sarma, *Maximum term of composite entire functions*, *Banyan Mathematical Journal*, Volume 1, (1994), 35–48.

[5] D. Sato, *On the rate of growth of entire functions of fast growth*, *Bull. Amer. Math. Soc.*, Volume 69, (1963), 411-414.

[6] S. M. Saha, *A note on lower proximate orders*, *J. Indian Math. Soc.,* Volume 12, (1948), 31-32.

[7] A. P. Singh, *On Maximum term of composition of entire functions*, *Proc. Nat. Acad. Sci. India*, Volume 59(A), (1989), 103-115.

[8] G. Valiron, *Lectures on the General Theory of Integral functions*, Chelsea Publishing Company, 1949.

[9] C. C. Yang and H. X. Yi, *Uniqueness Theory of Meromorphic Functions*, Kluwer Academic Publishers and Science Press, Beijing, 2003.

Ratan Kumar Dutta
Patulia High School
Patulia, Kolkat-700119
West Bengal, India
email: *ratan_3128@yahoo.com*