ON SOME LINEAR D-CONNECTIONS ON THE TOTAL SPACE E OF A VECTOR BUNDLE $\xi = (E, \pi, M)$

Adrian Lupu

Abstract. Using the theory introduced by R. Miron and M. Anastasiei [2] and some results from the theory given by P. Stavre [4], [5], [6], we will obtain in this paper the results from section 2. Based on [4]-[6], we will obtain other results in a future paper.

2000 Mathematics Subject Classification: 53C60.

Keywords and phrases: linear d-connections on the total space of a vector bundle, $\overset{(12)}{D}$ operators, w conjugations.

1. Introduction

Let us consider $\xi = (E, \pi, M)$ a vector bundle with the base space $M_n = (M, [A], \mathbb{R}^n)$ a C^∞ differentiable, n-dimensional, paracompact manifold, with the m-dimensional fiber type. We obtain the total space E, with the structure $E_{n+m} = (E, [\mathcal{A}], \mathbb{R}^{n+m})$ of C^∞-differentiable, $(n+m)$-dimensional, paracompact manifold ([2]).

Let us consider an almost sympectic structure w, on E whose restriction to the vertical subspace is nondegenerate. It results that $(n+m)$ must be an even number and that m must also be an even number. Therefore n is an even number. In these conditions, there is a nonlinear connection, $\underset{(w)}{N}$, given by:

$$w(hY, vZ) = 0, \quad X, Z \in \mathcal{X}(E). \tag{1}$$

It results the decomposition:

$$w = hw + vw, \tag{2}$$

where:

$$(hw)(X,Z) = w(hX,hZ), \quad (vw)(X,Z) = w(vX,vZ), \quad X,Z \in \mathcal{X}(E)$$

and h, v - the horizontal and vertical projectors associated to $\underset{(w)}{N}$. In a similar way, if we have $\underset{(w)}{N}$ we will have an horizontal distribution $H : u \in E \to H_uE$ such that $T_uE = H_uE \oplus V_uE$. In the followings we will use the notions and the notations from [2].

Since E_{n+m} is C^∞-differentiable and paracompact it results that there are linear connections, $\{D\}$, on E. The linear connections D, on E, which have the remarkable geometric property that regarding parallel transport, preserve the horizontal distribution H and the vertical one, V, have an important role. This property is important for analytical mechanics and theoretical physics. Such connections D, are called linear d-connections (or remarkable connections). It results:

Proposition 1. [2] *A linear connection D, on E, for which is fixed a nonlinear connection N, of h and v projectors is a linear d-connection iff:*

$$hD_XvY = 0, \quad vD_XhY = 0. \tag{3}$$

In the followings we will consider $N = \underset{(w)}{N}$, without notice that. Into a local base, adapted to N: $\left\{\frac{\delta}{\delta x^i}, \frac{\partial}{\partial y^a}\right\}$ $(i = \overline{1,n},\ a = \overline{1,m})$ we can write:

$$D^h_{\frac{\delta}{\delta x^k}} \frac{\delta}{\delta x^j} = \Gamma^i_{jk}(x,y)\frac{\delta}{\delta x^i}, \quad D^h_{\frac{\delta}{\delta x^k}} \frac{\partial}{\partial y^a} = \Gamma^{n+b}_{n+a\ \ k}(x,y)\frac{\partial}{\partial y^b} \tag{4}$$

$$D^v_{\frac{\partial}{\partial y^b}} \frac{\delta}{\delta x^j} = \Gamma^i_{jn+b}\frac{\delta}{\delta x^i}, \quad D^u_{\frac{\partial}{\partial y^b}} \frac{\partial}{\partial y^a} = \Gamma^{n+c}_{n+a\ \ n+b}(x,y)\frac{\partial}{\partial y^c}, \tag{5}$$

where $(\Gamma^i_{jk}, \Gamma^{n+b}_{n+a\ \ k}) = D^h$ are the local coefficients of the h-covariant derivative and $(\Gamma^i_{jn+b}, \Gamma^{n+c}_{n+a\ \ n+b}) = D^v$ are the local coefficients of the v-covariant derivative $(i,j,k = \overline{1,n},\ a,b,c = \overline{1,m})$.

Now we can write the d-tensor field of torsion, which characterize the torsion T of a linear d-connection and the d-tensor field of curvature, which characterize the curvature tensor of one linear d-connection [2].

1. Special operators $\overset{(12)}{D} w$, $\overset{(21)}{D} w$

Let us consider $\overset{(1)}{D}$, a d-linear connection on E, given in the adapter base to $N = \underset{(w)}{N}$, by the local coefficients:

$$\overset{(1)}{D}{}^h = (\overset{(1)}{\Gamma}{}^i_{jk}, \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ k}), \quad \overset{(1)}{D}{}^v = (\overset{(1)}{\Gamma}{}^i_{j\ n+b}, \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ n+b})$$

$\overset{(2)}{D}$, a linear d-connection, given by the local coefficients:

$$\overset{(2)}{D}{}^h = (\overset{(2)}{\Gamma}{}^i_{jk}, \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ k}), \quad \overset{(2)}{D}{}^v = (\overset{(2)}{\Gamma}{}^i_{j\ n+b}, \overset{(2)}{\Gamma}{}^{n+a}_{n+c\ n+b})$$

given by:

$$\overset{(2)}{\Gamma}{}^i_{jk} = \overset{(1)}{\Gamma}{}^i_{jk} + w^{ir} \overset{(1)}{D}{}^h_k w_{rj}, \quad \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ k} = \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ k} + w^{n+c\ n+b} \overset{(1)}{D}{}^h_k w_{n+b\ n+a} \tag{1}$$

$$\overset{(2)}{\Gamma}{}^i_{j\ n+b} = \overset{(1)}{\Gamma}{}^i_{j\ n+b} + w^{ir} \overset{(1)}{D}{}^v_{n+b} w_{rj}, \tag{2}$$

$$\overset{(2)}{\Gamma}{}^{n+c}_{n+a\ n+b} = \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ n+b} + w^{n+c\ n+d} \overset{(1)}{D}{}^v_{n+b} w_{n+d\ n+a}$$

where:

$$w_{ir} = w\left(\frac{\delta}{\delta x^r}, \frac{\delta}{\delta x^i}\right), \quad w_{n+a\ n+b} = w\left(\frac{\partial}{\partial y^b}, \frac{\partial}{\partial y^a}\right) \tag{3}$$

$$w_{r\ n+b} = w\left(\frac{\delta}{\delta x^r}, \frac{\partial}{\partial y^b}\right) = 0, \quad w^{\alpha\beta} w_{\beta\gamma} = \delta^\alpha_\gamma. \tag{4}$$

We have:

$$w = \frac{1}{2} w_{ik} dx^i \wedge dx^k + \frac{1}{2} w_{n+a\ n+b} \delta y^a \wedge \delta y^b \tag{5}$$

where $(dx^i, \delta y^a)$ is a local base, dual to the local adapted base $\left(\frac{\delta}{\delta x^r}, \frac{\partial}{\partial y^b}\right)$.

Let us consider:

$$\overset{(21)}{\tau}{}^i_{jk} \overset{def}{=} \overset{(2)}{\Gamma}{}^i_{jk} - \overset{(1)}{\Gamma}{}^i_{jk}, \quad \overset{(21)}{\tau}{}^{n+c}_{n+a\ k} \overset{def}{=} \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ k} - \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ k} \tag{6}$$

$$\overset{(21)}{\tau}{}^{i}_{j\ n+b} \overset{def}{=} \overset{(2)}{\Gamma}{}^{i}_{j\ n+b} - \overset{(1)}{\Gamma}{}^{i}_{j\ n+b}, \qquad \overset{(21)}{\tau}{}^{n+c}_{n+a\ n+b} \overset{def}{=} \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ n+b} - \overset{(1)}{\Gamma}{}^{n+c}_{n+a\ n+b}. \tag{7}$$

From (1), (2), (6), (7) it results:

$$0 = \overset{(1)}{D}{}^{h}_{k} w_{jl} - \overset{(21)}{\tau}{}^{r}_{jk} w_{rl} \tag{8}$$

$$0 = \overset{(1)}{D}{}^{h}_{k} w_{n+a\ n+b} - \overset{(21)}{\tau}{}^{n+c}_{n+a\ k} w_{n+c\ n+b} \tag{9}$$

$$0 = \overset{(1)}{D}{}^{v}_{n+c} w_{jl} - \overset{(21)}{\tau}{}^{r}_{j\ n+c} w_{rl} \tag{10}$$

$$0 = \overset{(1)}{D}{}^{v}_{n+c} w_{n+a\ n+b} - \overset{(21)}{\tau}{}^{n+d}_{n+a\ n+c} w_{n+d\ n+b}. \tag{11}$$

It results:

Proposition 1. *Let us consider an almost symplectic structure w on E, and $\overset{(1)}{D}$ a linear d-connection with its local coefficients $\overset{(1)}{D}{}^{h}$, $\overset{(1)}{D}{}^{v}$. Then (1), (2) are equivalent with (8)-(11).*

Let's give an interpretation for (8)-(11). In [6] P. Stavre has introduced the operators: $\overset{(21)}{D} w$, $\overset{(12)}{D} w$ by:

$$(\overset{(12)}{D}{}_X w)(Y,Z) = (\overset{(1)}{D}{}_X w)(Y,Z) - w(Y, \overset{(21)}{\tau}(X,Z)) \tag{12}$$

$$(\overset{(21)}{D}{}_X w)(Y,Z) = (\overset{(2)}{D}{}_X w)(Y,Z) - w(Y, \overset{(12)}{\tau}(X,Z)) \tag{13}$$

and has studied their general properties, where:

$$\overset{(21)}{\tau}(X,Z) = \overset{(2)}{D}{}_X Z - \overset{(1)}{D}{}_X Z, \quad \overset{(12)}{\tau}(X,Z) = \overset{(1)}{D}{}_X Z - \overset{(2)}{D}{}_X Z. \tag{14}$$

In the general case we have:

$$(\overset{(12)}{D}{}_X w)(Y,Z) \neq (\overset{(12)}{D}{}_X w)(Z,Y), \tag{15}$$

$$(\overset{(12)}{D}{}_X w)(Y,Z) \neq -(\overset{(12)}{D}{}_X w)(Z,Y)$$

$$\overset{(12)}{D}{}_X w \neq \overset{(21)}{D}{}_X w. \tag{16}$$

The notion of almost simplectic conjugation of two linear connections $\overset{(1)}{D}$, $\overset{(2)}{D}$ is also introduced in [6]. It has been proved that this is equivalent with the relation:

$$(\overset{(12)}{D}_X w)(Y,Z) = -2\beta(X)w(Y,Z) \tag{17}$$

where β is a certain 1-form. An ample theory is obtained starting from here. In this case we write $\overset{(2)}{D} \overset{\omega}{\sim} \overset{(1)}{D}$.

Taking into account [5] we shall use the w-conjugation theory for two linear d-connections $\overset{(1)}{D}$, $\overset{(2)}{D}$ on the total space E, equipped with an almost simplectiv structure w.

Definition 1. *If $\overset{(12)}{D}_X w = 0$ then we will say that $\overset{(1)}{D}$ and $\overset{(2)}{D}$ are (w)-absolute conjugated. We shall write: $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$.*

Proposition 2. *Let us consider $\overset{(1)}{D}$, $\overset{(2)}{D}$ two linear d-connection on E, with respect to $N = \underset{(w)}{N}$. Then the relation $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ is characterized by:*

$$\overset{(12)}{D}{}^h_X hw = 0, \quad \overset{(12)}{D}{}^h_X vw = 0 \tag{18}$$

$$\overset{(12)}{D}{}^v_X hw = 0, \quad \overset{(12)}{D}{}^v_X vw = 0. \tag{19}$$

Proof. From (1), (2) section 1 and from the definition of $\overset{(12)}{D} w$, since $\overset{(1)}{D}$, $\overset{(2)}{D}$ are linear d-connections, it results that $\overset{(12)}{D} w = 0$ is characterized only by (18), (19).

From Proposition 2 and from (1), (2) section 2 it results:

Proposition 3. *A d-linear connection $\overset{(2)}{D}$ such that $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ is well-defined by (1), (2) and conversely.*

A characterization of a d-linear connection $\overset{(2)}{D}$, defined by (1), (2), is given in that way:

Proposition 4. *The relation* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ *is symmetric i.e.:*

$$\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D} \Leftrightarrow \overset{(1)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(2)}{D} \tag{20}$$

Proof. If we have $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ then we will have (1), (2) and conversely. By direct calculus, it results:

$$\overset{(2)}{D}{}^h_k w_{ij} = \overset{(12)}{\tau}{}^r_{ik} w_{rj} \tag{21}$$

$$\overset{(2)}{D}{}^h_k w_{n+a\ n+b} = \overset{(12)}{\tau}{}^{n+c}_{n+a\ k} w_{n+c\ n+b} \tag{22}$$

$$\overset{(2)}{D}{}^v_{n+b} w_{ij} = \overset{(12)}{\tau}{}^r_{i\ n+b} w_{rj} \tag{23}$$

$$\overset{(2)}{D}{}^v_{n+c} w_{n+a\ n+b} = \overset{(12)}{\tau}{}^{n+d}_{n+a\ n+c} w_{n+d\ n+b} \tag{24}$$

and therefore $\overset{(12)}{D} w = 0$. Hence $\overset{(1)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(2)}{D}$.

Like a corollary, it results:

Proposition 5. *If we have (1), (2) then we will have:*

$$\overset{(1)}{\Gamma}{}^i_{jk} = \overset{(2)}{\Gamma}{}^i_{jk} + w^{ir} \overset{(2)}{D}{}^h_k w_{rj} \tag{25}$$

$$\overset{(1)}{\Gamma}{}^{n+c}_{n+a\ k} = \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ k} + w^{n+c\ n+d} \overset{(2)}{D}{}^h_k w_{n+d\ n+a} \tag{26}$$

$$\overset{(1)}{\Gamma}{}^i_{j\ n+b} = \overset{(2)}{\Gamma}{}^i_{j\ n+b} + w^{ir} \overset{(2)}{D}{}^v_{n+b} w_{rj} \tag{27}$$

$$\overset{(1)}{\Gamma}{}^{n+c}_{n+a\ n+b} = \overset{(2)}{\Gamma}{}^{n+c}_{n+a\ n+b} + w^{n+c\ n+d} \overset{(2)}{D}{}^v_{n+b} w_{n+d\ n+a}. \tag{28}$$

Proposition 6. *Let us consider* $\overset{(1)}{D}, \overset{(2)}{D}, \overset{(1)}{D} \neq \overset{(2)}{D}$ *two linear d-connections on* E*, with respect to* $N = \underset{(w)}{N}$*. If* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ *then* $\overset{(1)}{D}, \overset{(2)}{D}$ *won't be* w*-compatible.*

Proof. A linear d-connection D, on E, is w-compatible ($Dw = 0$) iff ([2]):

$$D^h_X hw = 0, \quad D^h_X vw = 0 \tag{29}$$

$$D^v_X hw = 0, \quad D^v_X vw = 0 \tag{30}$$

Let us consider $\overset{(1)}{D}$, $\overset{(2)}{D}$ two linear d-connections. If $\overset{(1)}{D}$ is w-compatible and $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ then we will have (1), (2) with $\overset{(1)}{D}w = 0$. It results $\overset{(2)}{D} = \overset{(1)}{D}$ which is a contradiction.

In the same way, if $\overset{(2)}{D}$ had been w-compatible and $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$, taking into account of (25)-(28) it would have resulted $\overset{(2)}{D} = \overset{(1)}{D}$ which is a contradiction.

Therefore, it will not exist a d-linear connection $\overset{(2)}{D}$ on E so that $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$ if $\overset{(1)}{D}$ is a linear, w-compatible connection on E.

Let us consider a linear connection D on E, with the torsion:

$$T(X,Y) = D_X Y - D_Y X - [X,Y], \quad X, Y \in \mathcal{X}(E).$$

We will denote the coefficients of T in a local base $\{\underset{(\alpha)}{X}\}$ $(\alpha = \overline{1, n+m})$ by:

$$T(\underset{(\beta)}{X}\underset{(\alpha)}{X}) = T^{\sigma}_{\alpha\beta}\underset{(\sigma)}{X}, \quad T^{\sigma}_{\alpha\beta} = -T^{\sigma}_{\beta\alpha}. \tag{31}$$

With these notations, if D is a linear d-connection on E and we choose the adapted local base $\left\{ \underset{(r)}{X} = \frac{\delta}{\delta x^r}; \underset{(n+a)}{X} = \frac{\partial}{\partial y^{\alpha}} \right\}$ then T will be characterized by the d-tensorial fields of the local components:

$$T^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{kj}, \quad T^{n+a}_{jk} = R^{n+a}_{jk} \tag{32}$$

$$T^i_{j\ n+b} = \Gamma^i_{j\ n+b}, \quad T^{n+a}_{j\ n+b} = \frac{\partial N^a_j}{\partial y^b} - \Gamma^{n+a}_{n+b\ j} \tag{33}$$

$$T^i_{n+a\ n+b} = 0, \quad T^{n+c}_{n+a\ n+b} = \Gamma^{n+c}_{n+a\ n+b} - \Gamma^{n+c}_{n+b\ n+a} \tag{34}$$

where ([2]):

$$R^{n+a}_{jk} = \frac{\delta N^a_k}{\delta x^j} - \frac{\delta N^a_j}{\delta x^k} \tag{35}$$

A problem which appears is to establish on what conditions two linear d-connections, w-absolute conjugated, $\overset{(1)}{D}$ and $\overset{(2)}{D}$ have the same torsion: $\overset{(2)}{T} = \overset{(1)}{T}$.

Proposition 7. *Let us consider* $\overset{(1)}{D}, \overset{(2)}{D}$ *two linear d-connections on* E *such that* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$. *Then* $\overset{(1)}{T} = \overset{(2)}{T}$ *iff:*

$$\overset{(1)}{D}{}^h_k w_{lj} - \overset{(1)}{D}{}^h_j w_{lk} = 0 \tag{36}$$

$$\overset{(1)}{D}{}^h_k w_{n+d\ n+a} = 0 \tag{37}$$

$$\overset{(1)}{D}{}^v_{n+b} w_{lj} = 0 \tag{38}$$

$$\overset{(1)}{D}{}^v_{n+b} w_{n+d\ n+a} - \overset{(1)}{D}{}^v_{n+a} w_{n+d\ n+b} = 0. \tag{39}$$

Proof. Because $\overset{(1)}{D}, \overset{(2)}{D}$ are linear d-connections on E, their torsions $\overset{(1)}{T}, \overset{(2)}{T}$ in the local, adapted base have the components (32)-(35).

We already have:

$$\overset{(1)}{T}{}^{n+a}_{jk} = \overset{(2)}{T}{}^{n+a}_{jk} = R^{n+a}_{jk} \tag{40}$$

$$\overset{(1)}{T}{}^i_{n+a\ n+b} = \overset{(2)}{T}{}^i_{n+a\ n+b} = 0 \tag{41}$$

From the condition $\overset{(1)}{T}{}^i_{jk} = \overset{(2)}{T}{}^i_{jk}$ and from (1) we obtain the equivalent relation:

$$\overset{(1)}{D}{}^h_k w_{lj} - \overset{(1)}{D}{}^h_j w_{lk} = 0 \tag{42}$$

From (1) it will also result the equivalent relation:

$$\overset{(1)}{D}{}^h_k w_{n+d\ n+a} = 0 \tag{43}$$

if we have the condition:

$$\overset{(1)}{T}{}^{n+a}_{j\ n+b} = \overset{(2)}{T}{}^{n+a}_{j\ n+b}.$$

From (2) we will obtain the equivalent relation:

$$\overset{(1)}{D}{}^v_{n+b} w_{lj} = 0 \tag{44}$$

if we have $\overset{(1)}{T}{}^i_{j\ n+b} = \overset{(2)}{T}{}^i_{j\ n+b}$.

Furthermore from (2) we obtain the equivalent relation

$$\overset{(1)}{D}{}^v_{n+b} w_{n+d\ n+a} - \overset{(1)}{D}{}^v_{n+a} w_{n+d\ n+b} = 0 \tag{45}$$

if we have the condition:

$$\overset{(1)}{T}{}^{n+c}_{n+a\ n+b} = \overset{(2)}{T}{}^{n+c}_{n+a\ n+b}.$$

We obtain thus (36)-(39) and conversely.

Like a corollary we get, equivalent:

Proposition 8. *Let us consider* $\overset{(1)}{D}$, $\overset{(2)}{D}$ *two linear d-connections on* E, *so that* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$. *Then* $\overset{(1)}{T} = \overset{(2)}{T}$ *iff:*

$$\overset{(2)}{D}{}^h_k w_{ij} - \overset{(2)}{D}{}^h_j w_{ik} = 0 \tag{46}$$

$$\overset{(2)}{D}{}^h_k w_{n+a\ n+b} = 0 \tag{47}$$

$$\overset{(2)}{D}{}^v_{n+b} w_{ij} = 0 \tag{48}$$

$$\overset{(2)}{D}{}^v_{n+b} w_{n+c\ n+a} - \overset{(2)}{D}{}^v_{n+a} w_{n+d\ n+b} = 0. \tag{49}$$

Definition 2. *A linear d-connection on* E *will be called* w*-semicompatible if we have:*

$$D^h_k w_{n+a\ n+b} = 0, \quad D^h_X vw = 0 \tag{50}$$

$$D^v_{n+b} w_{ij} = 0, \quad D^v_X hw = 0. \tag{51}$$

Definition 3. *A linear d-connection* D, *on* E, *will be called* w*-semi Codazzi connection if we have:*

$$D^h_k w_{ij} = D^h_j w_{ik} \tag{52}$$

$$D^v_{n+b} w_{n+c\ n+a} = D^v_{n+a} w_{n+c\ n+b} \tag{53}$$

We have:

Proposition 9. *Let us consider* $\overset{(1)}{D}$, $\overset{(2)}{D}$ *two linear d-connections on* E *such that* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$. *Then* $\overset{(1)}{T} = \overset{(2)}{T}$ *iff* $\overset{(1)}{D}$ *is* w*-semicompatible and it is* w*-semi Codazzi.*

Proposition 10. *Let us consider* $\overset{(1)}{D}$, $\overset{(2)}{D}$ *two linear d-connections on* E *such that* $\overset{(2)}{D} \underset{abs}{\overset{w}{\sim}} \overset{(1)}{D}$. *Then* $\overset{(1)}{T} = \overset{(2)}{T}$ *iff* $\overset{(2)}{D}$ *is w-semi compatible and w-semi Codazzi.*

If the distribution H is integrable then we will have:

$$R^{n+a}_{jk} = 0.$$

More general results and the relation between the curvature tensors $\overset{(1)}{R}$, $\overset{(2)}{R}$ will be given in a future paper.

I would like to thank Professor P. Stavre for his support during the preparation of this paper.

References

[1] A. Lupu, *The w-conjugation in the vector bundle*, RIGA Conference, Braşov, June 2007.

[2] R. Miron, M. Anastasiei, *Fibrate vectoriale. Spaţii Lagrange. Aplicaţii în teoria relativităţii*, Ed. Academiei, 1987.

[3] A. Nastaselu, A. Lupu, *On some structures* F, F^*, National Seminar of Finsler Lagrange Spaces, Braşov, 2004.

[4] P. Stavre, *Fibrate vectoriale*, vol. I, II, III, Ed. Universitaria, Craiova, 2004-2006.

[5] P. Stavre, *Aprofundări în geometria diferenţială. Aplicaţii la spaţii Norden şi la teoria relativităţii*, vol. III, Ed. Universitaria, Craiova, 2006.

[6] P. Stavre, *Introducere în teoria structurilor geometrice conjugate*, Ed. Matrix, Bucureşti, 2007.

[7] P. Stavre, *On Some Remarkable Results of R. Miron*, Libertas Math., tom XXIV, Arlington, Texas, 2004, p. 51-58.

Author:

Adrian Lupu
Technical High School "Decebal",
Drobeta Turnu Severin, Romania
email: adilupudts@yahoo.ca