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APPLICATION OF THE DIRECT METHOD TO A MICROCONVECTION MODEL

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Abstract. A linear model of natural convection under microgravity conditions for a binary liquid layer in the presence of the Soret effect is investigated analytically using a general method for treating two-point eigenvalue problems depending on several physical parameters. The secular equation, which allows us to obtain the neutral curve, is obtained and discussed for different values of the parameters.

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1. Introduction

The crystal growth, measurement of thermophysical properties, fluid flows, complex plasmas are a few fields where the microgravity conditions occur.

The term "microconvection" was introduced to characterize non-solenoidal flows driven by density changes with the temperature. Then it was extended to convective motions of weakly compressible liquids. The first model of microconvection concerned the convection in a closed cavity under low gravity, with the density depending on the temperature only [8].

In the natural convection the patterns are due to the buoyancy effect: the temperature induces density variations in fluids. So, when heated the fluids become buoyant and tend to rise, while when cooled due to the gravity they tend to come down.

Theoretically the motion can be induced by temperature differences even in the absence of gravity. At low gravity, small variations of the thermophysical properties of the medium can influence the natural convection. For a single component fluid, the model of natural convection under microgravity conditions was studied in [1].

We are concerned with a class of microconvection models with strong Soret effect in a binary liquid layer. Assume that an infinite horizontal layer of this binary fluid of thickness d is bounded by two impermeable walls on which the normal heat flux is specified. The gravity is constant g. Then the nondimensional governing conduction-convection

equations and the boundary conditions are [1]

$$\begin{aligned} &\nabla \mathbf{v} = S\Delta T + Le\Delta C,\\ &Pr^{-1}\epsilon \mathbf{v}\nabla \mathbf{v} = -\nabla p" + \Delta \mathbf{v} - \frac{G(T+C)\mathbf{k}}{1+\epsilon(T+C)},\\ &\epsilon \mathbf{v}\nabla T = \Delta T,\\ &\epsilon \mathbf{v}\nabla C = Le\Delta(C-\sigma T), \end{aligned} \tag{1}$$

$$u = w = 0, \;\; T_z = -1, \;\; C_z = \sigma T_z, \;\; S = 1 - Le\sigma, \quad \text{at } z = 0 \text{ and } 1. \tag{2}$$

where $p'' = p - \rho_0 g\mathbf{kx} - (\eta/3)\nabla \mathbf{v}$, $\mathbf{v} = (u, w)$ is the velocity field, T is the temperature, C is the concentration, ϵ is the Boussinesq parameter, Le is the Lewis number, σ is the separation ratio and G stands for the Galileo number. In addition, ∇ and Δ are the nabla and Laplace operators respectively and $\mathbf{k}$ is the unit vector in the upwards vertical direction.

The direct method based on the characteristic equation is one of the most simple methods to treat two-point problems for linear ordinary differential equations with constant coefficients. It was first systematically applied to hydrodynamic stability theory by one of the authors (A.G.) and then extensively used by her group e.g. [2]-[7]. By means of it we write the general form of the solution of the two-point problem for the governing differential equations in terms of these roots. Further introduction of the general solution into the boundary conditions leads to the secular equation. The neutral manifolds, in particular the neutral curves, separate the domain of stability from the domain of instability. Their determination is our aim.

2. The eigenvalue problem

Take the following mechanical equilibrium

$$u_0 = w_0 = 0, \;\; T_{0z} = -1, C_{0z} = -\sigma.$$

as the basic state, decompose the perturbed fields into a sum of basic and perturbation fields, namely $(u, v, T, C) = (u_0, v_0, T_0, C_0) + (u', v', T', C')$, introduce the perturbation stream function Ψ' such that

$$u' = \Psi'_z + ST'_x + LeC'_x, w' = -\Psi'_x + ST'_z + LeC'_z,$$

where the subscripts stand for the differentiation, and substitute the normal mode perturbations

$$(\Psi', T', C') = (\Psi'(z), T'(z), C'(z))exp(-iax)$$

into the equations (1) linearized about this equilibrium to get the two-point eigenvalue problem [1]

$$\begin{cases} (D^2 - a^2)^2\Psi' + iaG'(T' + C') = 0, \\ \\ -\epsilon(ia\Psi' + SDT' + LeDC') = (D^2 - a^2)T', \\ \\ \epsilon\sigma(ia\Psi' + SDT' + LeDC') = Le(D^2 - a^2)(C' - \sigma T'), \end{cases} \tag{3}$$

where $D = \dfrac{d}{dz}$, $G' = \dfrac{G}{1 + \epsilon(T_0 + C_0)^2}$. [1] Assume that G' is constant.

The boundary conditions read [1]

$$D\Psi' = ia(ST' + LeC'), \Psi' = DT' = DC' = 0, \text{ at } z = 0 \text{ and } 1. \tag{4}$$

The unknown functions Ψ', T', C' are the eigenvectors of the problem (3)-(4) corresponding to the eigenvalue Le.

Using the last two equations from (3) and the boundary conditions $DT' = DC' = 0$ at $z = 0$ and 1, we have

$$\begin{cases} (D^2 - a^2)U = 0, \\ DU = 0 \text{ at } z = 0 \text{ and } 1, \end{cases}$$

where $U = \sigma(1 - Le)T' + LeC'$. This implies $U = 0$, $\forall z \in [0, 1]$, whence the following relationship between the unknown functions T' and C'

$$C' = \frac{\sigma(Le - 1)}{Le}T'. \tag{5}$$

In order to write (3), (4) into a more convenient form we use (5) in the first two equations (3) and eliminate Ψ' between them to obtain

$$Le(D^2 - a^2)^3T' + \varepsilon Le(1 - \sigma)D(D^2 - a^2)^2T' + a^2\varepsilon G'[Le + \sigma(Le - 1)]T' = 0, \tag{6}$$

or, since, from physical reasons, $Le \neq 0$ and letting $a_1 = \varepsilon(1 - \sigma)$, $a_2 = a^2\varepsilon G'[1 + \sigma(1 - 1/Le)]$,

$$(D^2 - a^2)^3T' + a_1D(D^2 - a^2)^2T' + a_2T' = 0. \tag{7}$$

The boundary conditions (4) written in T' only read

$$DT' = (D^2 - a^2)T' = D^3T' = 0 \quad \text{at } z = 0 \text{ and } 1. \tag{8}$$

Thus (3)-(4) is equivalent to the two-point eigenvalue problem (6),(7) depending on three parameters: a, a_1, a_2. In order to solve it we investigate the multiplicity m_i of the roots λ_i of the characteristic equation associated with the six order ordinary differential equation (6). Then, we write the corresponding form of the general solution of (6) and introduce it into (7) to get the secular equation. Then, the secular and characteristic equation are solved simultaneously to yield the solution $Le = Le(a, \varepsilon, \sigma)$ of the eigenvalue problem (6), (7).

Since irrespective of m_i the general solution of (6) is a sum of products of polynomials in z by $e^{\lambda_i z}$, it follows that for every fixed wave number, $a > 0$, the set of eigenvalues of the problem (3)-(44) is discrete and is situated on the half-axis $Le > 0$. As a varies, while other parameters are kept fixed, the smallest eigenvalue generates the neutral curve $Le = Le(a)$.

3.The general secular equation

The algebraic equation associated with equation (6) reads

$$(\lambda^2 - a^2)^3 + a_1\lambda(\lambda^2 - a^2)^2 + a_2 = 0. \tag{9}$$

This a six order linear algebraic equation with the roots λ_i, $i = 1, .., 6$. In this section we treat the general case, i.e. when (9) has six mutually distinct roots. Then the general solution of (7) has the form

$$T'(z) = \sum_{i=1}^{6} A_i e^{\lambda_i z}, \tag{10}$$

where $A_i, i = 1...6$ are constants. Then from (5) and from $(3)_2$, we obtain

$$C'(z) = \frac{\sigma(Le-1)}{Le}\sum_{i=1}^{6} A_i e^{\lambda_i z}, \Psi' = \frac{i}{\varepsilon a}\sum_{i=1}^{6}[\varepsilon(1-\sigma)\lambda_i + (\lambda_i^2 - a^2)]A_i e^{\lambda_i z}. \tag{11}$$

The boundary conditions (11) lead to the following linear system of algebraic equations

$$\begin{cases} \sum\limits_{i=1}^{6} A_i\lambda_i = 0, & \sum\limits_{i=1}^{6} A_i\lambda_i e^{\lambda_i} = 0, \\ \sum\limits_{i=1}^{6}(\lambda_i^2 - a^2)A_i = 0, & \sum\limits_{i=1}^{6}(\lambda_i^2 - a^2)A_i e^{\lambda_i} = 0, \\ \sum\limits_{i=1}^{6} \lambda_i^3 A_i = 0, & \sum\limits_{i=1}^{6} \lambda_i^3 A_i e^{\lambda_i} = 0. \end{cases} \tag{12}$$

Imposing the condition that the determinant of the system (11) to vanish we obtain the secular equation

$$\begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ \lambda_1 e^{\lambda_1} & \lambda_2 e^{\lambda_2} & \lambda_3 e^{\lambda_3} & \lambda_4 e^{\lambda_4} & \lambda_5 e^{\lambda_5} & \lambda_6 e^{\lambda_6} \\ \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & \mu_6 \\ \mu_1 e^{\lambda_1} & \mu_2 e^{\lambda_2} & \mu_3 e^{\lambda_3} & \mu_4 e^{\lambda_4} & \mu_5 e^{\lambda_5} & \mu_6 e^{\lambda_6} \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & \lambda_4^3 & \lambda_5^3 & \lambda_6^3 \\ \lambda_1^3 e^{\lambda_1} & \lambda_2^3 e^{\lambda_2} & \lambda_3^3 e^{\lambda_3} & \lambda_4^3 e^{\lambda_4} & \lambda_5^3 e^{\lambda_5} & \lambda_6^3 e^{\lambda_6} \end{vmatrix} = 0, \tag{13}$$

where $\mu_i = \lambda_i^2 - a^2, i = 1...6$. The solution $a_1 = a_1(a, a_2)$, i.e. $Le = Le(a, \varepsilon, \sigma)$, of (13) is obtained numerically by solving the system consisting of (13) and the six equations (9) for λ_1, ..., λ_6.

4. Case of multiple roots of (9)

For various values of the physical parameters, equation (9) can have multiple roots. In these cases, the form of the general solution of (6) and, consequently, the form of the secular equation will change accordingly.

The algebraic equation (9) has the simplified form

$$\lambda^6 + a_1\lambda^5 - 3a^2\lambda^4 - 2a_1a^2\lambda^3 + 3a^4\lambda^2 + a_1a^4\lambda + a_2 - a^6 = 0. \tag{14}$$

Let us prove that (14) has no solutions of third order algebraic multiplicity. Using the notations $\mu = \dfrac{\lambda}{a}$, $6b = \dfrac{a_1}{a}$, $d = \dfrac{a_2 - a^6}{a^6}$, we rewrite (14)

$$\mu^6 + 6b\mu^5 - 3\mu^4 + 12b\mu^3 + 3\mu^2 + 6b\mu + d = 0. \tag{α_1}$$

A root $\mu_1 = \mu_2 = \mu_3$ is a common root for (α_1) and for the first and the second derivatives of (α_1),i.e. (α_2), (α_3), where

$$\mu^5 + 5b\mu^4 - 2\mu^3 - 6b\mu^2 + \mu + b = 0, \tag{α_2}$$

$$5\mu^4 + 20b\mu^3 - 6\mu^2 - 12b\mu + 1 = 0. \tag{α_3}$$

In order to obtain a root of (α_1), (α_2), (α_3) we performed algebraic combinations of (α_i), $i = 1, 2, 3$ and we found a possible root $\mu = \dfrac{(25b^3 + 3)b}{375b^4 + 46b^2 + 1}$. If this root is a common root for (α_i), $i = 1, 2, 3$ then the following relation must be satisfied

$$b(1 + 92b^2 + 3640b^4 + 78450b^6 + 965000b^8 + 6375000b^{10} + 17578125b^{12}) = 0.$$

This way, (14) has solutions of third order algebraic multiplicity only for $b = 0$, i.e. $a_1 = 0$.

This case is treated in the next section.

5. Case $\sigma = 1$

Subcase $Le = 0.5$. The eigenvalue problem becomes

$$\begin{cases} (D^2 - a^2)^2\Psi' + iaG'(T' + C') = 0, \\ -\varepsilon(ia\Psi' + LeDT' + LeDC') = (D^2 - a^2)T', \\ \varepsilon(ia\Psi' + LeDT' + LeDC') = Le(D^2 - a^2)(C' - T'), \end{cases} \tag{15}$$

and, with (5), $C' = -T'$. In this case, equations (15) lead to

$$(D^2 - a^2)^3 T' = 0, \tag{16}$$

such that the characteristic equation has the form $(\lambda^2 - a^2)^3 = 0$. Therefore, the secular equation is no longer (13) and we must derived it from the beginning.

The general solution of (16) has the form

$$T'(z) = (A_0 + A_1 z + A_2 z^2)\cosh(az) + (B_0 + B_1 z + B_2 z^2)\sinh(az). \qquad (17)$$

The expression of the unknown function Ψ' is found from $-i\varepsilon a\Psi' = (D^2 - a^2)T'$,

$$\Psi' = -\frac{2}{i\varepsilon a}[A_2 + (B_1 + 2B_2 z)a]\cosh(az) + [(A_1 + A_2 z)a + B_2]\sinh(az).$$

Taking into account the boundary condition, the secular equation has the form

$$16a^6 \sinh a(\sinh^2 a - a^2) = 0. \qquad (18)$$

Since this equation is satisfied only for $a = 0$, it follows that $T' = C' = \Psi' = 0$. Hence, there are no secular points $(Le, a, \sigma) = (0.5, a, 1)$.

Subcase $Le \neq 0.5$, $G' = \dfrac{a^4 Le}{\varepsilon(2Le - 1)}$ **(**$d = 0$**).** The equations (3) imply

$$(D^2 - a^2)^3 T' + a^6 T' = 0, \qquad (19)$$

so the characteristic equation reads $(\lambda^2 - a^2)^3 + a^6 = 0$. The solutions of the characteristic equation are

$$\lambda_1 = -\lambda_4 = \frac{a}{2}\sqrt{6 + 2i\sqrt{3}}, \lambda_2 = -\lambda_5 = \frac{a}{2}\sqrt{6 - 2i\sqrt{3}}, \lambda_3 = \lambda_6 = 0,$$

so, two of the sheets of the hypersurface defined by (9) coalesce. The general solution of (19) has the form

$$T'(z) = A + Bz + \sum_{i=1}^{2} A_i \cosh(\lambda_i z) + B_i \sinh(\lambda_i z). \qquad (20)$$

such that, with (5) we get $C' = \dfrac{Le - 1}{Le}(A + Bz + \sum\limits_{i=1}^{2} A_i \cosh(\lambda_i z) + B_i \sinh(\lambda_i z))$ and

$$\Psi'(z) = \frac{a}{i\varepsilon}(A + Bz) - \frac{1}{\varepsilon i a}\sum_{i=1}^{2} A_i \mu_i \cosh(\lambda_i z) + B_i \mu_i \sinh(\lambda_i z).$$

If we denote $d_i = \cosh\lambda_i - 1$, it leads to the secular equation

$$\begin{vmatrix} \lambda_1\sinh\lambda_1 & \lambda_2\sinh\lambda_2 & \lambda_1 d_1 & \lambda_2 d_2 \\ \mu_1 d_1 & \mu_2 d_2 & \mu_1\sinh\lambda_1 & \mu_2\sinh\lambda_2 \\ 0 & 0 & \lambda_1^3 & \lambda_2^3 \\ \lambda_1^3\sinh\lambda_1 & \lambda_2^3\sinh\lambda_2 & \lambda_1^3 d_1 & \lambda_2^3 d_2 \end{vmatrix} = 0, \tag{21}$$

equivalent to $\sinh\dfrac{\lambda_1}{2}\sinh\dfrac{\lambda_2}{2}\Big(\lambda_2^3\mu_1\cosh\dfrac{\lambda_2}{2}\sinh\dfrac{\lambda_1}{2} - \lambda_1^3\mu_2\cosh\dfrac{\lambda_1}{2}\sinh\dfrac{\lambda_2}{2}\Big) = 0.$

Let $\varepsilon_{1,2} = \dfrac{-1 \mp i\sqrt{3}}{2}$ be two third order roots of 1. Then $\mu_{1,2} = -\varepsilon_{1,2}a^2$ and $\lambda_1 = a\sqrt[4]{3}\sqrt{-i\varepsilon_2}$, $\lambda_2 = a\sqrt[4]{3}\sqrt{i\varepsilon_1}$. It is immediate that (21) admits the unique solution $a = 0$. This implies $T' = C' = \Psi' = 0$. Thus the neutral curve does not contain secular pointsof the type $(Le, \sigma, \varepsilon, G') = (Le, 1, \varepsilon, \dfrac{a^4 Le}{\varepsilon(2Le-1)})$.

Subcase $Le \neq 0.5$, $G' \neq \dfrac{a^4 Le}{\varepsilon(2Le-1)}(a_2 \neq 0)$.

The ordinary differential equation satisfied by the unknown function T' is

$$(D^2 - a^2)^3 T' + a_3 T' = 0, \tag{22}$$

where $a_3 = \dfrac{\varepsilon a^2 G'(2Le-1)}{Le}$.

The corresponding characteristic equation $(\lambda^2 - a^2)^3 + a_3 = 0$ has the following roots:

$$\begin{aligned} &\lambda_3 = \sqrt{a^2 - \sqrt[3]{-a_3}}, \lambda_1 = \sqrt{a^2 + \sqrt[3]{-a_3}\varepsilon_1}, \\ &\lambda_2 = \sqrt{a^2 + \sqrt[3]{-a_3}\varepsilon_2}, \lambda_4 = -\lambda_1, \lambda_5 = -\lambda_2, \lambda_6 = -\lambda_3. \end{aligned} \tag{23}$$

For $G = a^6$, we obtain the previous case.

Since, in this case, ε, a,G' are different from zero, the conditions imposed to the parameters, imply that the roots of the characteristic equation are distinct, so, we can write the general solution of (22)

$$T'(z) = \sum_{i=1}^{3} A_i\cosh(\lambda_i z) + B_i\sinh(\lambda_i z). \tag{24}$$

From (3) it follows

$$C'(z) = \frac{Le-1}{Le}\sum_{i=1}^{3} A_i\cosh(\lambda_i z) + B_i\sinh(\lambda_i z),$$
$$\Psi'(z) = -\frac{1}{i\varepsilon a}\sum_{i=1}^{3} A_i\mu_i\cosh(\lambda_i z) + B_i\mu_i\sinh(\lambda_i z).$$

Sustituting (24) into the boundary conditions (8) we obtained the secular equation

$$\begin{vmatrix} 0 & 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1\sinh(\lambda_1) & \lambda_2\sinh(\lambda_2) & \lambda_3\sinh(\lambda_3) & \lambda_1\cosh(\lambda_1) & \lambda_2\cosh(\lambda_2) & \lambda_3\cosh(\lambda_3) \\ \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 0 \\ \mu_1\cosh(\lambda_1) & \mu_2\cosh(\lambda_2) & \mu_3\cosh(\lambda_3) & \mu_1\sinh(\frac{\lambda_1}{2}) & \mu_2\sinh(\lambda_2) & \mu_3\sinh(\lambda_3) \\ 0 & 0 & 0 & \lambda_1^3 & \lambda_2^3 & \lambda_3^3 \\ \lambda_1^3\sinh(\frac{\lambda_1}{2}) & \lambda_2^3\sinh(\frac{\lambda_2}{2}) & \lambda_3^3\sinh(\frac{\lambda_3}{2}) & \lambda_1^3\cosh(\frac{\lambda_1}{2}) & \lambda_2^3\cosh(\frac{\lambda_2}{2}) & \lambda_3^3\cosh(\lambda_3) \end{vmatrix} = 0, \tag{25}$$

which can be simplified taking into account that the eigenvalue problem is symmetric with respect to $z = 0.5$ [6](the boundary conditions are the same on the lower and the upper surface). The simplification is obtained by performing the change of variable $x = z - 0.5$.

Then the eigenvalue problem is the same as (3),(4), with the only difference that the boundary conditions are taken at $x = \pm 0.5$ and the unknown functions depend on x. In particular, now $D \equiv \dfrac{d}{dx}$. Since the characteristic equation keeps its form, the general solution T' is given by (24) and imposing the boundary conditions (4) at $x = \pm 0.5$ it follows that the secular equation has the form $\Delta = \Delta_e \cdot \Delta_o = 0$[6], where

$$\Delta_e = \begin{vmatrix} \lambda_1\sinh(\frac{\lambda_1}{2}) & \lambda_2\sinh(\frac{\lambda_2}{2}) & \lambda_3\sinh(\frac{\lambda_3}{2}) \\ \mu_1\cosh(\frac{\lambda_1}{2}) & \mu_2\cosh(\frac{\lambda_2}{2}) & \mu_3\cosh(\frac{\lambda_3}{2}) \\ \lambda_1^3\sinh(\frac{\lambda_1}{2}) & \lambda_2^3\sinh(\frac{\lambda_2}{2}) & \lambda_3^3\sinh(\frac{\lambda_3}{2}) \end{vmatrix} =0$$

and

$$\Delta_o = \begin{vmatrix} \lambda_1\cosh(\frac{\lambda_1}{2}) & \lambda_2\cosh(\frac{\lambda_2}{2}) & \lambda_3\cosh(\frac{\lambda_3}{2}) \\ \mu_1\sinh(\frac{\lambda_1}{2}) & \mu_2\sinh(\frac{\lambda_2}{2}) & \mu_3\sinh(\frac{\lambda_3}{2}) \\ \lambda_1^3\cosh(\frac{\lambda_1}{2}) & \lambda_2^3\cosh(\frac{\lambda_2}{2}) & \lambda_3^3\cosh(\frac{\lambda_3}{2}) \end{vmatrix} = 0.$$

Numerical evaluations [6] showed that the best eigenvalue corresponds to $\Delta_e = 0$. In order to simplify the secular equation, column i is divided by $\cosh(\lambda_i)$, $i = 1, 2, 3$, for $\cosh(\lambda_i) \neq 0$.

Then the simplified form of the secular equation is

$$\begin{aligned}&\mu_1 \tanh(\frac{\lambda_2}{2})\tanh(\frac{\lambda_3}{2})\lambda_2\lambda_3(\mu_2-\mu_3)+\mu_2\tanh(\frac{\lambda_3}{2})\tanh(\frac{\lambda_1}{2})\lambda_3\lambda_1(\mu_3-\mu_1)+\\&+\mu_3\tanh(\frac{\lambda_1}{2})\tanh(\frac{\lambda_2}{2})\lambda_1\lambda_2(\mu_1-\mu_2)=0\end{aligned} \tag{26}$$

It is only $\cosh(\frac{\lambda_1}{2})$ that can be equal to zero. This occurs when λ_1 is a pure imaginary solution of the characteristic equation and, consequently, $\cosh(\frac{\lambda_1}{2})$ becomes a cosine function which vanish for an argument of the form $\frac{(2n+1)\pi}{2}$. In this way, the solution of $\cosh(\frac{\lambda_1}{2}) = 0$ has the form $\lambda_1^2 = -(2n+1)^2\pi^2$, $n \in \mathbb{N}$ [6]. Then, the equation

$$Le = \left(2 - \frac{[(2n+1)^2\pi^2 + a^2]^3}{\varepsilon a^2 G'}\right)^{-1} \tag{27}$$

define the secular hypersurface.

Subcase $\sigma = \frac{Le}{1-Le}$**,** $Le \neq 0.5 (a_2 = 0)$**.** The stability is governed by the following boundary value problem

$$\begin{aligned}&(D^2-a^2)^3T' + \frac{\varepsilon(1-2Le)}{1-Le}D(D^2-a^2)^2T' = 0,\\&DT' = (D^2-a^2)T' = D^3T' = 0 \quad \text{at } z = 0,1.\end{aligned} \tag{28}$$

In this case the general solution of (28) reads

$$T'(z) = -C'(z) = (A+Bz)\cosh(az) + (C+Dz)\sinh(az) + \sum_{i=1}^{2} A_i e^{\lambda_i z},$$

and leads to

$$\begin{aligned}\Psi'(z) = &\Big[-\frac{1}{ia}L(B+Ca) - \frac{2D}{i\varepsilon}\Big]\cosh(az) + iLBz\sinh(az) + \Big[\frac{-1}{ia}L(Aa+D)-\\&-\frac{2B}{i\varepsilon}\Big]\sinh(az) + iLDz\cosh(az) + \sum_{i=1}^{2}\Big[\frac{-1}{ia}L\lambda_i - \frac{1}{i\varepsilon a}\mu_i\Big]A_i e^{\lambda_i z}.\end{aligned}$$

where $\lambda_{1,2}$ are roots of the equation $\lambda^2+\dfrac{\varepsilon(1-2Le)}{1-Le}\lambda-a^2=0$, $\lambda_{3,4}=-\lambda_{5,6}=a$ and $L=\varepsilon\dfrac{1-2Le}{1-Le}$. Then the secular equation reads

$$\begin{vmatrix} \lambda_1 & \lambda_2 & 0 & 0 & 1 & 0 \\ \lambda_1 e^{\lambda_2} & \lambda_2 e^{\lambda_2} & 1 & a\sinh a & \cosh a & \sinh a + a\cosh a \\ 0 & 0 & 0 & 0 & L & 2a \\ 0 & 0 & 0 & (2+L)a\sinh a & L\cosh a & (2+L)a\cosh a + L\sinh a \\ \lambda_1^3 & \lambda_2^3 & 0 & 2a^2 & a^2 & 0 \\ \lambda_1^3 e^{\lambda_1} & \lambda_2^3 e^{\lambda_2} & a^2 & 2a^2\cosh a + a^3\sinh a & a^2\cosh a & 3a^2\sinh a + a^3\cosh a \end{vmatrix} = 0.$$

6. Conclusions

The direct method was applied in order to determine the secular equation in a problem of natural convection under microgravity conditions for a binary liquid layer in the presence of the Soret effect. It is shown that there are not multiple roots of order greater or equal to three of the characteristic equation. Then four particular cases were treated and the simplified secular equations were obtained in each of these cases.

Taking into consideration that the governing eigenvalue problem depends on four parameters the investigation of the bifurcation of manifolds was a difficult problem.

When the characteristic equation has double roots it is possible to obtain false secular points, that is why all this cases remains to be investigated. In a future work we shall obtain a complete analytical caracterization of this case.

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