HYBRID GA-SQP ALGORITHMS FOR THREE-DIMENSIONAL BOUNDARY DETECTION PROBLEMS IN POTENTIAL CORROSION DAMAGE

Nicolae S Mera

Abstract. In this paper we consider the identification of the geometric structure of the boundary of the solution domain for the three-dimensional Laplace equation. Cauchy data consisting of boundary measurements of currents and voltages on the remainder of the boundary are used to determine the material loss caused by corrosion. This problem arrise in the early detection of corrosion in aging aircraft components. The domain identification problem is considered as a variational problem to minimize a defect functional, which utilises some additional data on certain known parts of the boundary. A sequential quadratic programming (SQP) optimisation algorithm and a real coded genetic algorithm (GA) are combined in order to obtain a fast algorithm that can deal with the multimodal objective function. The unknown boundary is parameterized using B-splines. The Laplace equation is discretised using the method of fundamental solutions (MFS). Numerical results are presented and discussed for four hybrid algorithms and several test examples.

2000 *Mathematics Subject Classification*: 65N21, 68T20, 90C56.

1. INTRODUCTION

We consider the nonlinear problem of determining quantitative information about corrosion occurring on an inaccessible part of the boundary of a specimen using current flux and voltage measurements. Non-destructive testing methods for rapid and reliable corrosion detection in complex metallic assemblies is an on-going challenge. The early detection of corrosion in aging aircraft has thus both economic and safety implications. The life of an aircraft can be considerably extended by early detection of corrosion and by taking corrective

action to remove and arrest corrosion of structure. As the age of an aircraft increases, the importance of detection of corrosion increases because there is an increased probability of an interaction between the corrosion and other forms of damage such as fatigue cracks. The detection of corrosion without disassembly of the aircraft prevents the possibility of aggravating the issue by removing effective sealant.

The most common technique for detection corrosion in aircraft is visual inspection for surface distortions or pillowing of the outer skin. However, special non destructive techniques must be applied when regions are partially or completely inaccessible for inspection due to the overlying structure. In airplanes, corrosion may occur in relatively inaccessible locations, such as within the lap joints of the skin of an airplane as is shown schematically in Figure 1(a). Two aluminum panels A and B overlap and corrosion may occur at the joint. One side of the aircraft skin is not accessible for inspection, so any damage on this side has to be identified from measurements available on the accessible side.

An electric voltage is applied to the aircraft skin, on the upper part and

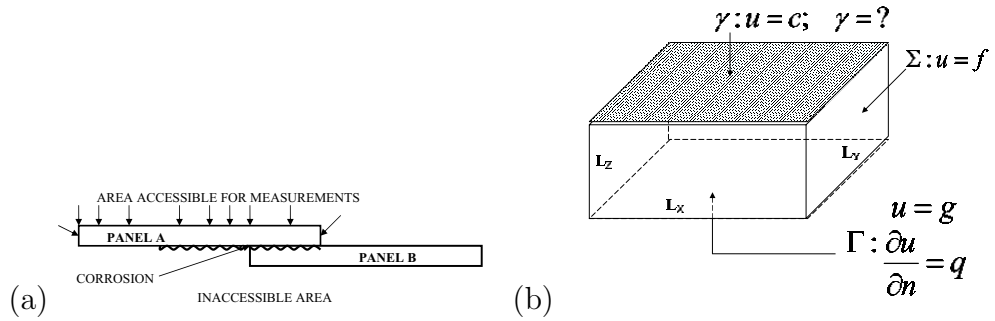


Figure 1: (a) Schematic drawing of two overlapping panels in an aircraft skin with corrosion occurring at the joint and (b) Problem formulation: the domain and the boundary conditions of the 3D problem considered.

side boundaries of the panel A in Figure 1(a) and the resulting voltage pattern is recorded using electronic sensors placed on the accessible part of the aircraft component. Analysis is performed to determine material loss on the bottom part of the panel A and between the panels. It should be noted that the thickness of the aluminum panels in aircraft is several order of magnitude smaller than their width or length and the curvatures encountered are not very large in most cases. Therefore the panel investigated can be modeled as a thin plate.

In this paper the problem of boundary detection is reformulated as an optimisation problem and the unknown boundary is sought in a B-splines parametric form. Thus the problem is regularized by using the function specification method, i.e. by assuming a parametric form of the unknown function and reducing the problem to the determination of a reduced number of parameters, i.e. the control points of the B-splines surface. We note that once the problem has been formulated as an optimisation problem then various optimisation algorithms may be used in order to locate the optimum of the objective function. The efficiency of a particular optimisation method clearly depends on the form of the objective function. In the problem considered in this paper, the objective function has a complex nonlinear and nonmonotonic structure. Both heuristic methods such as genetic algorithms (GAs) see Mera *et al.* [7] and deterministic methods such as sequential quadratic programming (SQP), see Mera and Lesnic [8] have been successfully applied to optimise this function. Evolutionary algorithms search the whole space being able to deal with multi-modal functions and discontinuities. Deterministic, gradient based optimisation methods do not search the full parameter space and can tend to converge toward local extrema of the fitness function, which is clearly unsatisfactory for problems where the fitness varies nonmonotonically with the parameters. On the other hand in vicinity of local optima the rates of convergence exhibited by the gradient based algorithms clearly outperform those of evolutionary algorithms, see Mera *et al.* [9]. Therefore it is the purpose of this paper to formulate hybrid algorithms that incorporate the advantages of both deterministic and heuristic optimisation algorithms.

2. MATHEMATICAL FORMULATION

In this paper the EIT is used to determine material loss occurring on the inaccessible portion $\gamma \subset \partial\Omega$ of the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{R}^3$ by measuring voltages and currents (Cauchy data) on an accessible portion of the boundary. Therefore, we consider the Laplace equation in a three-dimensional damaged finite plate Ω for the potential u, namely,

$$\nabla^2 u = 0, \qquad \text{in} \quad \Omega \tag{1}$$

subject to the boundary conditions

$$u = g, \quad \frac{\partial u}{\partial n} = q \quad \text{on} \quad \Gamma \tag{2}$$

$$\alpha_0 u + \beta_0 \frac{\partial u}{\partial n} = f \quad \text{on} \quad \Sigma \tag{3}$$

$$\alpha_1 u + \beta_1 \frac{\partial u}{\partial n} = h \quad \text{on} \quad \gamma \tag{4}$$

where $f, g, h, q, \alpha_0, \beta_0, \alpha_1$ and β_1 are prescribed functions and n is the outward normal to the boundary $\partial\Omega$. In eqns (2)−(4) the boundaries γ, Γ and Σ are disjoint and $\gamma \cup \Gamma \cup \Sigma = \partial\Omega$. The problem investigated consists of determining the shape of the unknown boundary γ given the boundary data specified by eqns (2)−(4). This problems occurs in several contexts, such as corrosion detection by electrostatic measurements, see Kaup and Santosa [5]. Several computational results for the two-dimensional case can be found in the literature, using, for example, Tikhonov regularization in connection with the L-curve method, see Kaup *et al.* [6]. Charton *et al.*[1] proposed a variational technique based on parameterising the unknown boundary using the function specification method. For the three dimensional case a sequential quadratic programming method of solution was developed in Mera and Lesnic [8] while the same problem was investigated using a genetic algorithm in Mera et al [7].

3. THE METHOD OF SOLUTION

3.1 Reformulation as an optimisation problem

The boundary detection problem given by eqns (1)-(4) can be reformulated as an optimisation problem. For a given possible solution γ for the unknown boundary, the following direct problem is solved:

$$\nabla^2 u = 0, \qquad \text{in} \quad \Omega \tag{5}$$

$$u = g, \qquad \text{on} \quad \Gamma \tag{6}$$

$$\alpha_0 u + \beta_0 \frac{\partial u}{\partial n} = f \quad \text{on} \quad \Sigma \tag{7}$$

$$\alpha_1 u + \beta_1 \frac{\partial u}{\partial n} = h \quad \text{on} \quad \gamma \tag{8}$$

and the current flux on the known boundary $u'_{calc} = \left.\frac{\partial u}{\partial n}\right|_{\Gamma}$ is evaluated. The solution of the problem may be found by minimising the functional

$$J(\gamma) = \|u'_{calc} - q\|_{L^2(\Gamma)} \tag{9}$$

where q is the measured current flux on the boundary Γ. The boundary γ can be parameterised in different forms and the parameters characterising the shape of the boundary are determined by minimising the functional (9). In this paper we have used B-splines to parameterise the unknown surface γ.

3.2 The parametrisation of the boundary by B-splines surfaces

The unknown boundary γ is sought in the parametric form

$$\gamma(v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,k}(v)N_{j,l}(w)\mathbf{P}_{ij}, \qquad (v,w) \in [0,1]\times[0,1] \tag{10}$$

where $N_{i,k}$ and $N_{j,l}$ are the k^{th} and l^{th} degree B-spline basis functions given by

$$N_{i,0}(v) = \chi_{[v_i, v_{i+1}]}; \quad N_{i,k}(v) = \frac{v - v_i}{v_{i+k} - v_i}N_{i,k-1} + \frac{v_{i+k+1} - v}{v_{i+k+1} - v_{i+1}}N_{i+1,k-1} \tag{11}$$

$$N_{j,0}(w) = \chi_{[w_j, w_{j+1}]}; \quad N_{j,l}(w) = \frac{w - w_j}{w_{j+l} - w_j}N_{j,l-1} + \frac{w_{j+l+1} - w}{w_{j+l+1} - w_{j+1}}N_{j+1,l-1} \tag{12}$$

defined on the nonperiodic, nonuniform knot vectors $V = (v_i)_{i=0,n+k+1}$ and $W = (w_i)_{i=0,m+l+1}$, see Piegl and Tiller [13]. The points $\mathbf{P}_{ij}$ are the control points of the B-spline surface γ and they determine the shape of this boundary. Thus, the problem of boundary detection is reduced to determining the $(m+1)(n+1)$ control points $\mathbf{P}_{ij}$. The x and y coordinates of the control points are considered to be known while the z coordinate is to be detected. In the problem considered in this paper we assume that the intersection between the boundary γ and Σ is known, i.e. we know the curves that form the boundary of the surface γ but we don't know it's internal shape. Thus, the control points $\mathbf{P}_{ij}$ with $i \in \{0, n\}$ or $j \in \{0, m\}$ are known and the only unknowns are the z coordinates of the remaining control points. Thus, the problem of boundary identification is reduced to determining the components of the matrix

$$\mathbb{Z}\left(z_{P_{ij}}\right)_{i=\overline{1,n-1},j=\overline{1,m-1}} = \begin{bmatrix} z_{P_{11}} & z_{P_{12}} & \dots & z_{P_{1,m-2}} & z_{P_{1,m-1}} \\ \dots & \dots & \dots & \dots & \dots \\ z_{P_{n-1,1}} & z_{P_{n-1,2}} & \dots & z_{P_{n-1,m-2}} & z_{P_{n-1,m-1}} \end{bmatrix} \tag{13}$$

It should be noted that the ill-posed problem of identifying the boundary of the domain is regularized by using the function specification method, i.e. by assuming a pre-defined shape and reducing the problem to the determination of the parameters of the B-spline surface. In the more general case when no parametric shape is assumed for the boundary γ then the problem is ill-posed and regularization terms must be included in the objective functional (9). By using such regularization terms that penalize non-smooth solutions, and choosing the appropriate regularization parameter, a stable numerical solution can be obtained.

3.3 The method of fundamental solutions

A meshless method, namely the method of fundamental solution, see Fairweather and Karageorghis [2], is used to solve the direct problem (5)-(8) and to evaluate the objective function (9) for a given boundary γ. The numerical solution is developed by using the fundamental solution of the Laplace equation as a basis function.

In order to introduce the method of fundamental solutions, we consider the entire domain Ω and we consider a set of M nodal points $\underline{x}_i$, $i = \overline{1, M}$ outside Ω. The fundamental solution of the three-dimensional Laplace eqn.(5) is given by:

$$F(\underline{x}) = \frac{1}{4\pi\|\underline{x}\|} \tag{14}$$

Then the fundamental solutions centered at the nodal points $\underline{x}_i$, $i = \overline{1, M}$ given by

$$\phi_i(\underline{x}) = F(\underline{x} - \underline{x}_i) = \frac{1}{4\pi\|\underline{x} - \underline{x}_i\|} \tag{15}$$

satisfy the partial differential eqn.(5). The method of fundamental solutions is based on the fact that an approximation to the solution of the direct problem (5)-(8) can be expanded in terms of these basis functions, ϕ_i, i.e. the solution is sought in the form:

$$u^*(\underline{x}) = \sum_{i=1}^{M} \alpha_i \phi_i(\underline{x}) \tag{16}$$

where α_i are unknown coefficients. For this choice of basis functions the approximated solution u^* already satisfies the Laplace eqn.(5) and the coefficients α_i, $i = \overline{1, M}$, are determined such that u^* satisfies the boundary conditions (6)-(8).

We consider N collocation points $\underline{x}_j$, $j = \overline{1, N}$ on the boundary of the domain Ω. Using the given values of the voltage (or current flux) at these N collocation points, then the resulting system of linear algebraic equations for the unknown coefficients α_i is obtained by collocating (16) at the points $\underline{x}_j$, $j = \overline{1, N}$ and a system of linear algebraic equations,

$$A\underline{\alpha} = \underline{b} \tag{17}$$

is obtained, where the vector $\underline{\alpha}$ contains the unknown coefficients α_i, $i = \overline{1, M}$. It should be noted that in order to uniquely determine the coefficients α_i, $i = \overline{1, M}$ the number of collocation points N must be greater or equal to the number of nodal points M.

However, the system of linear algebraic equations is ill-conditioned and cannot be solved by direct methods, such as the least squares methods, since such an approach would produce a highly unstable solution. Most standard numerical methods cannot achieve good accuracy in solving the matrix eqn.(17) due to the large value of the condition number of the matrix A which increases dramatically with respect to increasing the total number of collocation points and nodal points. Several regularization methods have been developed for solving these kind of ill-conditioned problems, see Hansen [4]. In this study we use the standard Tikhonov regularization method, see Tikhonov *et al.* [21].

The Tikhonov regularized solution $\underline{\alpha}_\lambda$ for eqn.(17) is defined as the solution of the following least squares problem:

$$\min_{\underline{\alpha}} \{ \|A\underline{\alpha} - \underline{b}\| + \lambda^2 \|\underline{\alpha}\| \} \tag{18}$$

where $\|\cdot\|$ denotes the usual Euclidean norm and λ is the regularization parameter.

The choice of a suitable value of the the regularization parameter λ is crucial for the accuracy of the final numerical solution and there are several methods for determining an optimal value for this parameter, such as the L-curve method, Hansen [3], the discrepancy principle, Tikhonov *et al.* [14], etc. In this study we use the L-curve method, i.e. we define the curve

$$L = \{ (\ln(\|\underline{\alpha}_\lambda\|), \ln(\|A\underline{\alpha}_\lambda - \underline{b}\|)), \lambda > 0 \} \tag{19}$$

This curve is known as the L-curve and a suitable regularization parameter λ is one that corresponds to a regularised solution near the "corner", i.e. the

point of maximum curvature of the L-curve, see Hansen [3].

For the problem of boundary detection, the MFS is particularly suitable since the geometry of the system changes for every possible solution tested during the optimisation process. Further, MFS does not require any meshing of the domain or the boundary in order to calculate the boundary data. This reduces the computational effort and eliminates the important perturbations due to changes in the mesh. MFS also is known to be very efficient from the point of view of the computational time required to solve a linear direct problem such as (5)-(8). The MFS can be also used to compute values very near to, or even on the boundaries, and there is no singularity in the solution, as the nodal points are always placed outside the solution domain. The MFS is also suitable for high-dimensional problems and irregular domains.

3.4 Optimisation algorithms

The boundary identification problem (1)-(4) has been solved in Mera *et al.* [9] by using two different algorithms, one deterministic, gradient based optimisation algorithm and one heuristic, evolutionary search algorithm. We consider again the same algorithms, namely, as a gradient based optimisation algorithm we use the sequential quadratic programming optimisation algorithm E04UCF from the NAG Libraries, see NAG [12] and for the evolutionary search we consider a float number encoded genetic algorithm proposed in Mera *et al.* [7] with population size $n_{pop} = 10$, number of offspring $n_{child} = 15$, uniform arithmetic crossover, crossover probability $p_c = 0.65$, tournament selection, tournament size $k = 2$, tournament probability $p_t = 0.8$, non-uniform mutation, mutation probability $p_m = 0.5$, elitism, elitism parameter $n_e = 2$. The float number encoding of the variables enables natural implementation of fine local tuning processes. Indeed the non-uniform mutation operator is an operator that enables fine local tuning and causes the GA to search the space uniformly initially and locally at later stages.

In Mera *et al.* [9] it was found that accurate results are obtained for various shapes of the boundary γ both by the SQP and the GA methods, provided care is taken when specifying the initial guesses for the SQP algorithm. One major disadvantage of gradient methods is the convergence toward local optima. For example, one cannot expect the objective functional (9) to be convex and twice continuously differentiable, which would ensure the convergence of gradient based method toward the absolute minimum. Since the functional may have a complex structure, with multiple ridges and valleys, the gradi-

ent methods may fail to locate the global optimum. It should be noted that similar results have been obtained for the two-dimensional case. It was found in Charton *et al.* [1] that the numerical solutions obtained by the Gradient Projection Method (GPM) or Nelder-Mead Simplex Method (NMNS) depend on the initial guess specified. For some initial guesses, these gradient based methods converge toward local optima which are far from the real solution for the boundary γ. On the other hand, the genetic algorithm, starting with a random initial population, was always found to converge to the global optimum. In general, it in known that genetic algorithms have the ability to escape local optima and therefore they are particularly suitable for nonlinear functions with complex, highly structured landscape which can appear in shape optimisation problems. One disadvantage of genetic algorithms, in comparison with gradient methods, is the increased amount of computational time required for the evolutionary search process.

In order to combine the advantages of the two optimisation methods we consider the following four hybrid algorithms.

H1 - Several GA generations are performed in order to identify the most promising areas and then the SQP optimisation algorithm is applied using as an initial guess the best individual found by the GA. It should be noted that in this approach the GA is used to specify a good initial guess for the SQP algorithm.

H2 - A SQP operator is introduced in the GA scheme. The effect of this operator is equivalent to executing 5 SQP steps on the selected individual. In the H2 algorithm during every generation the SQP operator is applied to the best individual in the population, i.e. every generation 5 SQP steps are executed starting from the best individual and the result is introduced back in the GA population.

H3 - The same SQP operator as in H2 algorithm is used but it is applied to all members of the GA population, once every 10 generations.

H4 - Every generation, the SQP algorithm is applied to the best GA individual for an unlimited number of steps, until convergence is obtained but the result is not introduced back in the GA population in order to avoid premature convergence to local optima.

These four hybrid algorithm are tested on several test examples in order to identify their performance and limitations.

4. NUMERICAL RESULTS

In order to test the convergence and the stability of the methods proposed, we use as the solution domain the thin plate illustrated in Figure 1(b). The lengths of the domain are taken to be $L_x = L_y = 1.0$ and $L_z = 0.1$ althought it was found that stable computations can be performed up to $L_z = 0.5$, see Mera and Lesnic [8]. The unknown boundary γ is assumed to be maintained at constant voltage, say $u = c$, while a Dirichelet boundary condition is prescribed on Σ and Cauchy boundary conditions are prescribed on Γ. The method of fundamental solution, see section 3.3, is used to solve the direct problem (5)-(8) and to evaluate the objective function (9) for a given boundary γ. In all the results presented in this paper, $N = 150$ collocation points have been uniformly distributed on the boundary $\partial\Omega$ and $M = 150$ nodal point have been distributed on the boundary of a cube of side length $L = 3.0$ that includes in the interior the solution domain Ω. In this section we consider the retrieval of the boundary γ given by various shapes parameterized by B-splines as described in section 3.2. For the numerical test examples considered in this paper we use cubic B-splines, i.e. $k = l = 3$ and we set $m = n = 4$, i.e. the surface γ is given by $(m+1)(n+1) = 25$ control points. The knot vectors are taken to be

$$U = V = (0, 0, 0, 0, \frac{1}{2}, 1, 1, 1, 1) \tag{20}$$

The x and y coordinates of the control points are given by

$$x_{P_{i,j}} = -\frac{L_x}{2} + \frac{i}{n} L_x, \quad i = \overline{0, n}, j = \overline{0, m} \tag{21}$$

$$y_{P_{i,j}} = -\frac{L_y}{2} + \frac{j}{m} L_y, \quad i = \overline{0, n}, j = \overline{0, m} \tag{22}$$

The z coordinates of the control points $\mathbf{P}_{ij}$ with $i \in \{0, n\}$ or $j \in \{0, m\}$ are fixed to $z = 0.5 L_z$, since $\gamma \bigcap \Sigma$ is assumed to be known, and the z components of the other control points, i.e. the components of the matrix $\mathbb{Z}$ given by eqn.(13) are to be determined. The boundary data g was perturbed with $s = 1\%$ noise in order to simulate the inherent measurment errors. For every result presented in this paper for the GA, the algorithm was run 5 times for

various sequences of random numbers and the result presented is the average of the five runs.

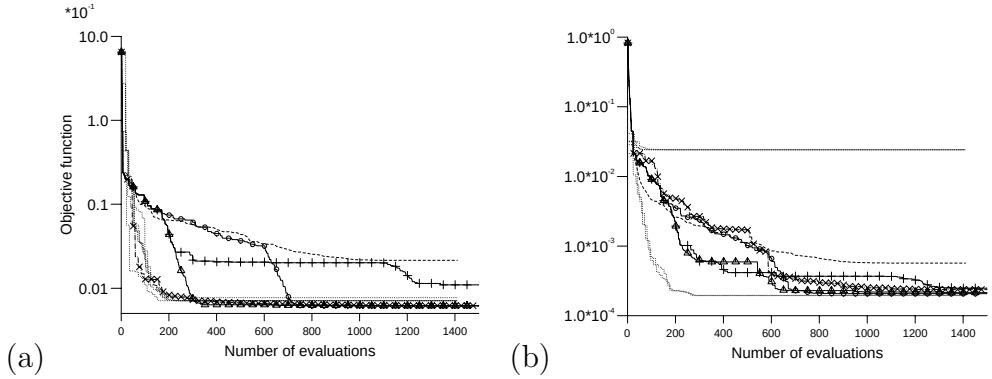
The first test example considered is a unimodal, symmetrical test example given by eqn.(10) with

$$\mathbb{Z} = \begin{bmatrix} 2.5L_z & 2.5L_z & 2.5L_z \\ 2.5L_z & 2.5L_z & 2.5L_z \\ 2.5L_z & 2.5L_z & 2.5L_z \end{bmatrix} \tag{23}$$

and the search ranges for the z coordinates of the control points were set to be $z_{min} = 0.1L_z$ and $z_{max} = 3.0L_z$.

In general, when using iterative methods, for solving ill-posed problems then a stopping criterion is required in order to regularize the problem. The iterative processes are not convergent with respect to the number of iterations due to the accumulation of noise effects. The real errors obtained by comparing the numerical solution with the exact solution decrease up to a specific point and then start increasing slowly, as the numerical solution loses its smoothness. Therefore regularizing stopping criteria are used in order to locate the optimum point in iterative methods for ill-posed problems. However, since we are using the function specification method, i.e. the solution is parametrized by B-splines, see section 3.2, the problem is regularized by looking for a solution in a parametric form depending on a reduced number of parameters. Therefore when only the parameters in eqn.(23) are to be retrieved, the iterative process is convergent with respect to the number of generations and simpler stopping criteria can be used, for example, one may stop the iterative process using a Cauchy type criterion, i.e. stop the iterative process when the solution does not change over a large number of iterations/generations.

Figure 2(a) presents the objective function obtained by the various hybrid algorithms in comparison with the objective functions obtained by the GA and by the SQP method starting with various initial guesses. It should be noted that for this test examples the SQP algorithms outperforms all the other algorithms since it converges to the real solution for all the initial guesses and it has a high convergence rate. GA has a much slower convergence rate but the hybrid algorithms have an improved convergence rate outperforming the standard GA. It should be noted that H1, H2 and H4 reach the same accuracy as SQP after 1500 generations even if they are outperformed by the SQP in the first stages. The rate of convergence of the H3 algorithm is lower than that of the other hybrid algorithms. This can be explained by the fact that in



(a) (b)

Figure 2: The evolution of the objective function as a function of the number of evaluations as obtained by various algorithm, namely H1 ($\circ$), H2 ($\times$), H3 ($+$), H4 (Δ), GA (- - -) and SQP ($\cdot\cdot\cdot\cdot$) for (a) the test example given by equation 23 and (b) the test example given by equation 24

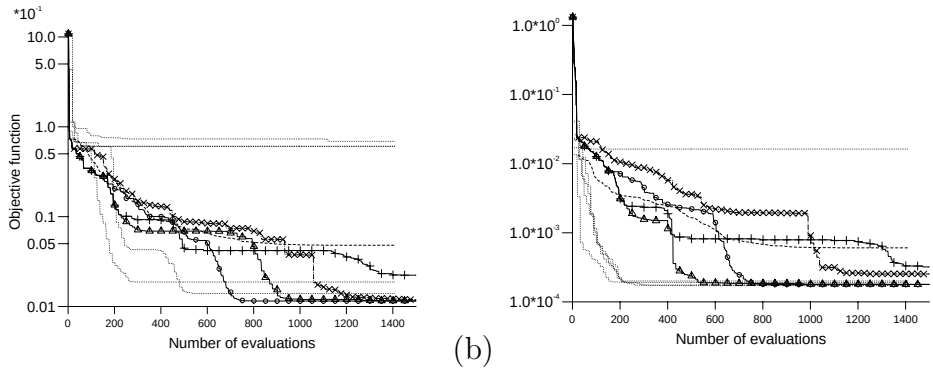
H3 the SQP operator is applied to every individual in the GA population and thus a large number of evaluations are wasted within SQP operators starting from neighboring initial guesses and from initial guesses which are not in the promising areas of the search space.

A non-symmetrical test example may be obtained by a B-spline surface given by eqn.(10) and the control points

$$\mathbb{Z} = \begin{bmatrix} 2.5L_z & 0.5L_z & 0.5L_z \\ 2.5L_z & 0.5L_z & 0.5L_z \\ 2.5L_z & 0.5L_z & 0.5L_z \end{bmatrix} \tag{24}$$

Figure 2(b) presents the objective function obtained by the various hybrid algorithms in comparison with the objective functions obtained by the GA and by the SQP method starting with various initial guesses. It can be seen that for this test examples SQP fails for some initial guesses which are not close to the exact solution. Again, the hybrid algorithms are found to outperform the standard GA, exhibiting rates of convergence which are approaching the rate of convergence of the SQP algorithm with a good initial guess. On the other hand, in every run the hybrid algorithms were found to converge to the real solution and escape local optima. Thus, the hybrid algorithms have an increased rate of convergence while they maintain the global search properties from the standard GA.

Two more complex boundary γ are considered by utilising the following



(a) (b)

Figure 3: The evolution of the objective function as a function of the number of evaluations as obtained by various algorithm, namely H1 ($\circ$), H2 ($\times$), H3 ($+$), H4 (Δ), GA (- - -) and SQP ($\cdot\cdot\cdot\cdot$) for (a) the test example given by equation 25 and (b) the test example given by equation 26

control points

$$\mathbb{Z} = \begin{bmatrix} 2.5L_z & 0.0 & 2.5L_z \\ 2.5L_z & 0.0 & 2.5L_z \\ 2.5L_z & 0.0 & 2.5L_z \end{bmatrix} \tag{25}$$

and

$$\mathbb{Z} = \begin{bmatrix} 1.4L_z & 0.5L_z & -0.4L_z \\ 1.4L_z & 0.5L_z & -0.4L_z \\ 1.4L_z & 0.5L_z & -0.4L_z \end{bmatrix} \tag{26}$$

and the objective functions obtained by various algorithms are presented in Figure 3. For both test examples the hybrid algorithm are converging to the global optimum with a convergence rate higher than that of the standard GA. For some test examples the H2 algorithm was found to present long periods of stagnation, see Figure 3(b). These are caused by the fact that if only one individual is accelerated with the SQP operator, it will have a much better objective function that the rest of the population and it will attract the whole population in the area around it, causing premature convergence toward a possible local optima. The local optima can still be escaped due to the mutation operator, but this operator might require a large number of evaluations before generating an objective function lower than the one obtained by the SQP operator. This effect might interfere with both the global search properties and

with the rate of convergence of the algorithm, and therefore the H2 algorithm may exhibit a poorer performance when compared to other hybrid algorithms. In the H3 algorithm by applying the SQP operator to all the individuals in the population, the dominance of only one individual is avoided. However, by applying SQP to all the individuals, a large number or evaluations are used for poor initial guesses and the rate of convergence of the algorithm is decreased.

The H1 algorithm was found to be fast and accurate for all the test

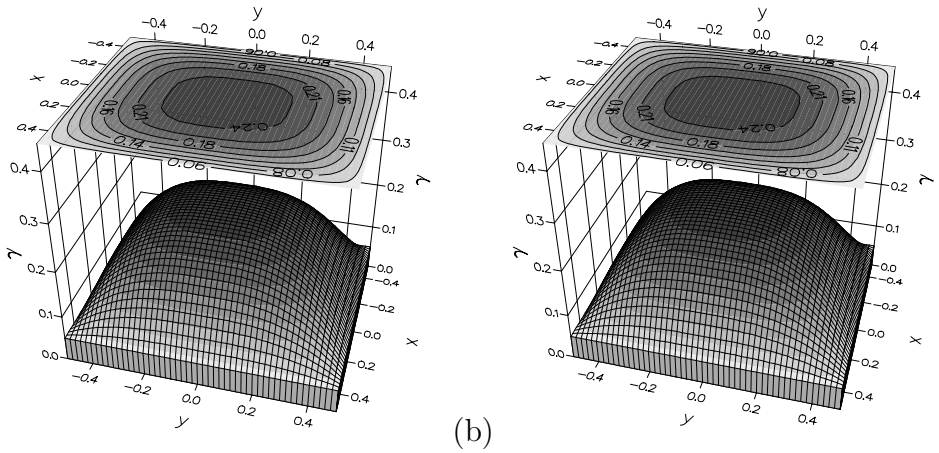
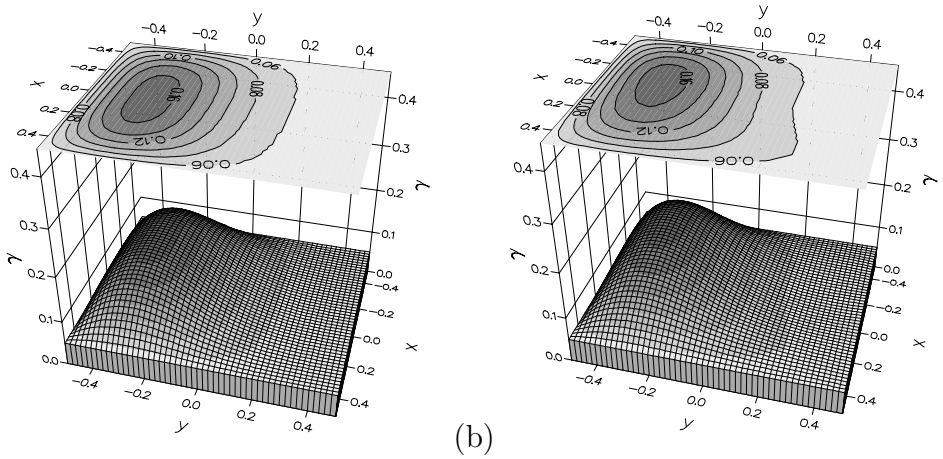


Figure 4: The exact (a) and the numerical (b) solution obtained by the H4 algorithm for the unknown boundary γ for the test example given by equation (23) for $s = 1\%$ noise after 1000 evaluations.

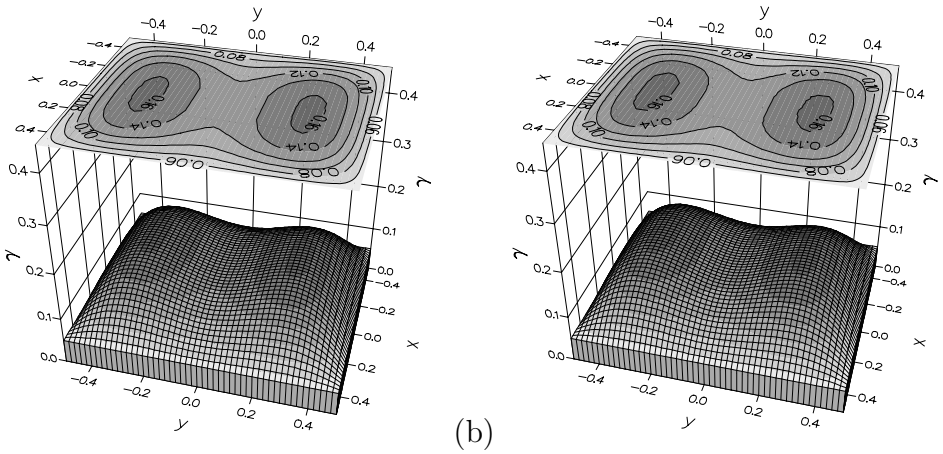
examples. However, also this algorithm presents a drawback in the fact that the number of generations to be performed by the GA before switching to SQP optimisation cannot be determined *a priori*. There are no estimates of how many evaluations are required in order to drive the GA within the radius of convergence of the SQP. If too many generations are performed or if the GA is allowed to converge, the hybrid algorithm will have a global rate of convergence not faster that that of a standard GA. On the other hand, if the GA if not run for long enough, and it has not entered the zone of convergence of the SQP, then the hybrid algorithm does not benefit on the global search propoerties of the GA and it will possible converge to a local optima.

The H4 hybrid algorithm was design to eliminate these drawbacks by combining the H1 and H2 algorithms. In the H4 algorithm the SQP operator of the H2 algorithm is applied to the best individual but there is no limit on the



(a) (b)

Figure 5: The exact (a) and the numerical (b) solution obtained by the H4 algorithm for the unknown boundary γ for the test example given by equation (24) for $s = 1\%$ noise after 1000 evaluations.



(a) (b)

Figure 6: The exact (a) and the numerical (b) solution obtained by the H4 algorithm for the unknown boundary γ for the test example given by equation (25) for $s = 1\%$ noise after 1000 evaluations.

number of SQP steps, the SQP algorithms is allow to converge as it is in the H1 algorithm. Thus the fast convergence rate of the SQP algorithm is brought in from the first generations, which causes this algorithm to outperform the

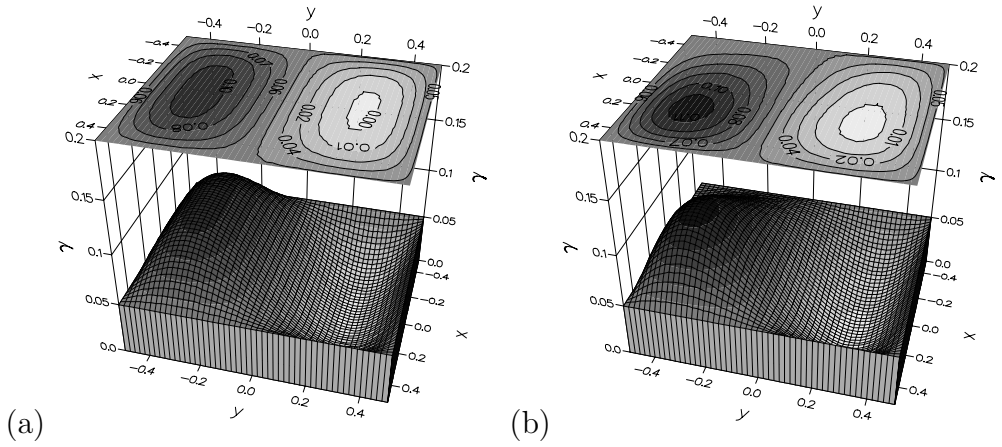


Figure 7: The exact (a) and the numerical (b) solution obtained by the H4 algorithm for the unknown boundary γ for the test example given by equation (26) for $s = 1\%$ noise after 1000 evaluations.

H1 algorithm. At the same time, the premature convergence which appears sometimes in the H2 algorithm is avoided by not introducing the SQP evolved individual back in the GA population. Thus, the GA is allowed to evolve and its global search properties are exploited without being affected by individuals with very good objective function generated by the SQP algorithm. In all the test examples investigated the H4 algorithm was found to converge to the real solution at a rate of convergence close to that of the SQP algorithm.

The numerical solutions obtained by the H4 using 1000 function evaluations for the four test examples considered are presented in Figures 4-7 in comparison with the exact solution. It can be seen that for all the test examples considered the numerical solution is a good approximations to the exact solution. Thus it can be concluded that using hybrid algorithms, complex boundaries can be retrieved within a small number of objective function evaluation.

5. CONCLUSIONS

In this study the problem of boundary detection has been reduced to an optimisation problem, namely to the minimisation of the objective functional (9). Once the problem has been reformulated as a minimisation problem, various

optimisation techniques have been employed in order to find the optimal solution and hybrid SQP and GA methods have been compared on various test examples. It has been found that by combining GAs and SQP optimisation algorithms it is possible to generate hybrid algorithms that preserve the global search properties of the GA and at the same time exhibit local rates of convergence much faster that that of the standard GA and close to the SQP rate of convergence. Overall, the hybrid GQ-SQP algorithms were found to be fast, robust and accurate methods for boundary identification.

References

1. M. Charton, H.J. Reinhardt and D.N. Hao, Numerical solution of a shape optimisation problem, Eurotherm seminar 68, Inverse Problems and Experimental Design in Thermal and Mechanical Engineering, Poitier, France, 2001, 101-105.

2. G. Fairweather and A. Karageorghis, The method of fundamental solutions for elliptic boundary value problems, Adv. Comput. Math., 9, 1998, 69-95.

3. P.C. Hansen, Analysis of discrete ill-posed problems by means of the *L*-curve, SIAM Review, 34, 1992, 561-580.

4. P.C. Hansen, *Rank Deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion*, SIAM, Philadelphia, 1998.

5. P.G. Kaup and F. Santosa , Nondestructive evaluation of corrosion damage using electrostatic boundary measurements, J. Nondestruct. Eval., 14, 1995, 127-136.

6. P.G. Kaup, F. Santosa and M. Vogelius, Method for imaging corrosion damage in thin plates from electrostatic data, Inverse Problems ,12, 1996, 279-293.

7. N.S. Mera, L. Elliott and D.B. Ingham, Boundary identification for a 3D Laplace equation using a genetic algorithm, Comput. Meth. App. Mech. Eng., 194, 2005, 42-44.

8. N.S. Mera and D. Lesnic, A three-dimensional boundary determination problem in potential corrosion damage, Comput. Mech., 36,2, 2005, 129-138.

9. N.S. Mera, L. Elliott, D.B. Ingham and D. Lesnic, A three-dimensional boundary determination problem in potential corrosion damage, 5th International Conference on Inverse Problems in Engineering, Theory and Practice, vol2, Leeds University Press, Leeds, M04, 2005.

10. Z. Michalewicz, Genetic Algorithms + Data Structures = Evolution Programs, Springer-Verlag, Berlin, 1996.

11. V.A. Morozov, On the solution of functional equations by the method of regularization, Math. Dokl., 7, 1966, 414-417.

12. NAG Fortran Library Manual, Mark 18, The Numerical Algorithms Ltd., Oxford, 1999.

13. L. Piegl and W. Tiller, The NURBS book , Springer, Berlin, 1997.

14. A.N. Tikhonov, A.V. Goncharky, V.V. Stepanov and A.G. Yagola, Numerical Methods for the Solution of Ill-Posed Problems, Kluwer Acad. Publ., Dordrecht, 1995.

Nicolae S Mera
Centre for Computational Fluid Dynamics
University of Leeds
Leeds, LS2 9JT, UK
email: *N.S.Mera@leeds.ac.uk*