MAKING A DECISION WHEN DEALING WITH UNCERTAIN CONDITIONS

by
Lucia Cabulea and Mihaela Aldea

1. Introduction

The decision theory offers the opportunity of choosing the best alternative from more available possibilities offering a logical process of making decisions. For making the best decision the following must be mentioned:

1. All the alternatives or possible variants must be identified, that is the ways in which the decision maker can act.

2. All possible natural states must be specified but defined in such a way that these events be mutually exclusive.

3. The results of the choice of any natural states must be evaluated. These evaluations or results represent benefits or costs and may be presented as tables or matrixes.

Such a table is also called the *payment matrix (table)*.

If the variants are V_i , i = $\overline{1,m}$ and the natural states N_j, j = $\overline{1,n}$, then the result of the variant V_i in the natural state N_j is noted R_{ij} and represents a benefit or a cost.

According to these remarks the way of making the best decision will be determined if the variants and the natural states will be a finite number.

The degree of knowing the natural states is of highest importance for the decision maker. According to the degree of knowing their appearance the following classification ca be made.

I. Making the decision under certain circumstances.

Under the circumstances, the decision maker is aware of the variants, he knows for sure what natural state will appear and knows the results of choosing variants in all natural states.

II. Making the decision under uncertain circumstances.

The decision maker knows the alternatives, has no information upon the probabilities of appearance of none of the natural states, but he can asses the results of choosing each alternative in all natural states.

III. Making decisions under risky conditions.

The decision maker knows the alternatives, has enough information to determine the probability of appearance of each of the natural states and can asses the results. The problems of making decisions under risky conditions are also called stochastic problems.

2. Decisions under uncertain conditions

When information regarding factors or events which can influence the results of choosing the variants is missing, a very important role belongs to the psychological factors.

The decision will depend mostly on subjective rationalizations of the decision maker, on the fact that he may be an optimistic or a pessimistic person.

As it has already been mentioned, the decision maker can establish all the variants or alternatives V_i, i = $\overline{1,m}$ and the results of choosing V_i in N_j state, noted R_{ij}. He has no information upon the probabilities of natural states appearance. The payment table looks like this:

Table 1

	N_1	N_2	…	N_n
V_1	R_{11}	R_{12}	…	R_{1n}
V_2	R_{21}	R_{22}	…	R_{2n}
…			…	
V_m	R_{m1}	R_{m2}	…	R_{mn}

For making the decision the decision criteria are being used and we mention here:

I. *The maxi-max criterion:*

This criterion corresponds to an optimistic decision maker. He thinks that whichever variant he may choose, the “nature” will act in such a way that he will get the best result.

He will choose the maximum value of the result of each variant (on line) and then the maximum value of these maximums:

$$\max_i(\max_j R_{ij}) = V \qquad \text{or} \qquad \begin{cases} V_i = \max_j R_{ij}, i = \overline{1,m} \\ V = \max_i V_i \end{cases}$$

where V represents the benefit (the result) that he hopes to obtain, and the line whose corresponds this V, determines the variant which have to be chose. This decision is risky, especially in long term.

II. *The maxi-min criterion*

This criterion corresponds to a pessimistic decision maker, who thinks that the nature acts against him: any variant he would choose, he would get the worst possible cashing, so the worst result.

The decision maker will choose, on line, the minimal result for each variant and then, he will select the maximal value of these minimal results:

$$\max_i(\min_j R_{ij}) = V \quad \text{or} \quad \begin{cases} V_i = \min_j R_{ij}, \quad i = \overline{1,m} \\ V = \max_i V_i \end{cases}$$

Applying this criterion obviously limits the obtaining of better results, wishing to obtain full security of the result that corresponds to the variant that was chosen.

III. *Hurwicz's criterion*

The criterion may be applied for optimistic, pessimistic or in-between decision makers.

Optimism is expressed by the so-called optimism index or coefficient, $\alpha \in [0,1]$, in such a way that (1- α) is the pessimism index.

Choosing the α coefficient depends on the decision maker, so it is subjective.

In this case, a shared value of the result of each alternative will be introduced.

$$\begin{cases} V_{pi} = \alpha \max_j R_{ij} + (1-\alpha) \min_j R_{ij} \\ V = \max_i V_{pi} \end{cases}$$

If α = 1, the optimism criterion is obtained and if α = 0, the pessimism one is obtained. For example, for α = 0,3 the decision maker is inclined to pessimism.

IV. *Laplace's criterion*

This criterion is also called "equal chance" criterion or the criterion of mathematical hope.

Equal chances of appearance are given to each natural state, so to n states, the appearance probability is 1/n. the natural states are equally probable.

The expected value of the result, which is the mathematical expectation, is:

$$\begin{cases} V_i = \dfrac{1}{n}\sum_{j=1}^{n} R_{ij} \\ V = \max_i V_i \end{cases}$$

so the arithmetic mean of all elements on each line is calculated and then we consider the maximum value of these results.

V. The mini-max criterion of regrets

This one is also called Savage's criterion, the one who introduced the notion of "regret", a measure of loss due to the miss choice of the best variant.

First a regrets table is drawn, r_{ij}, and then the mini-max criterion is applied.

The regret is measured through the difference between the best result that we would have got if we had known the natural state that was going to appear and the result obtained by making the decision.

$r_{ij} = \max_i R_{ij} - R_{ij}$, $j = \overline{1,n}$; $i = \overline{1,m}$.

Practically, for drawing the table of regrets, $r_{ij,}$ we must follow the following steps: from the highest element in each column in the payment table or the elements in the respective column are subtracted; thus the column of the regret matrix will result. The mini-max criterion, the table of regrets is applied, so for each line the maximum element is chosen and then we select the minimum regret.

The three stages necessary for the application of the criterion are:

- drawing up the regrets table;
- identifying the maximum regret for each alternative;
- choosing the alternative that minimizes the maximum values of regrets;

3. Case study

A firm owns a selling department for its products and wants to increase its sales. The choices it has are:

V_1: open a new selling department;

V_2: to extend the existing one;

V_3: to increase the number of ours the shop is open for buyers;

The owner's decision will depend on the buyer's request for his products. Noting the natural states N_i, these can be:

N_1: big request;

N_2: average request;

N_3: low request;

The firm manager evaluates the benefits R_{ij} that he can get with each variant, V_i, depending on the natural states, which are the market request, N_j, shown in table 2, where in variant 1, if the request is low, N_3, he loses 50 m.u. because he has investments. Using the decision criteria, let us find the variant that the manager should choose.

Solving:

I. The maxi-max criterion, of optimism:

Table 2

	N_1	N_2	N_3
V_1	600	300	-50
V_2	700	300	100
V_3	300	300	150

$$\begin{cases} V_1 = \max\limits_j R_{1j} = \max(600, 300, -50) = 600 \\ V_2 = \max\limits_j R_{2j} = \max(700, 300, 1000) = 700 \\ V_1 = \max\limits_j R_{3j} = \max(300, 300, 150) = 300 \end{cases}$$

V = $\max\limits_i V_i$ = max (600, 700, 300) = 700

The decision maker will choose the second variant, which is extending the existing shop, hoping that the request will be higher and the benefit obtained will be 7oom.u.

The best result, indicated by each of the five criteria, is given in Table 3.

Table 3

	N_1	N_2	N_3	I	II	III	IV	V
V_1	600	300	-50	600	-50	80	283,3	200
V_2	700	300	100	700*	100	220*	366,7*	50*
V_3	300	300	150	300	150*	180	250	400

II. The maxi-min criterion of pessimism

We choose the minimum value R_{ij} on line and then the maximum values of these limits.

The decision: the choice of V_3, expecting the request to be lower, with a benefit of 150m.u.

III. Hurwicz's criterion

The decision maker is inclined towards pessimism, thus $\alpha = 0,2$.

$V_{p1} = 0,2 \max_j R_{1j} + 0,8 \min_j R_{1j} = 0,2 \max (600, 300, -50) + 0,8 \min (600, 300, -50)$ $= 0,2 \cdot 600 + 0,8 \cdot (-50) = 80$

$V_{p2} = 0,2 \cdot 700 + 0,8 \cdot 100 = 220$

$V_{p3} = 0,2 \cdot 300 + 0,8 \cdot 150 = 180$

$V = \max (80, 220, 180) = 220$

The decision: he chooses V_2 expecting a benefit of 220 m.u.

IV. The criterion of equal chances, Laplace's criterion

$$V_1 = \frac{1}{3}\sum_{j=1}^{3} R_{1j} = \frac{1}{3}(600 + 300 - 50) = 283,3$$

$$V_2 = \frac{1}{3}(700 + 300 + 100) = 366,7$$

$$V_3 = \frac{1}{3}(300 + 300 + 150) = 250$$

$V = \max (283,3; 366,7; 250) = 366,7.$

The decision: he chooses V_2 expecting the benefit of 366,7m.u.

V. The mini-max criterion, the regrets criterion

We are going to build the regrets table

$j = 1 : r_{i1} = \max_i R_{i1} - R_{i1} = \max(R_{11}, R_{21}, R_{31}) - R_{i1}$ so:

$r_{i1} = \max(600, 700, 300) - R_{i1}$

which means that the first column is made of:

$$r_{i1} = 700 - r_{i1}, \begin{cases} r_{11} = 700 - 600 = 100 \\ r_{21} = 700 - 700 = 0 \\ r_{31} = 700 - 300 = 400 \end{cases}$$

From the maximum value of the results on the first column, the results on the first column were subtracted and the first column from the table of regrets has resulted. The rest of the columns are similarly obtained.

For the table of regrets, in table four the mini-max criterion is applied, the maximum on line value and then the minimum of these maximums.

Table 4

Regrets			Mini-max
100	0	200	200
0	0	50	50
400	0	0	400

The criterion indicates the 2nd variant, because it has the lowest regret.
The final decision is V_2: extending the existing space.

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Authors:

Lucia Cabulea and Mihaela Aldea, University "1 Decembrie 1918", Alba Iulia., lcabulea@uab.ro, maldea@uab.ro