AN APPLICATION OF CONSTRAINED NONLINEAR PROGRAMMING TO THE PHYSICAL PROBLEM: FE-MN BINARY ALLOYS

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Abstract. $Fe-Mn$ binary alloys having different Mn concentrations were subjected to the process of boriding at different times and different temperatures. Then, microhardnesses of the binary alloys were measured, [1].

The aim of this paper is to convert physical problem which is trying to reach maximum microhardness of $Fe-Mn$ binary alloys to constrained nonlinear problem (NLP) and find the common microhardness of these binary alloys at the same time and same temperature.

This study can be extended to many physical problems.

1.Introduction

In engineering and science one often has a number of data points, as obtained by sampling or experimentation, and tries to construct a function which closely fits those data points. This is called curve fitting or regression analysis. Interpolation is a specific case of curve fitting, in which the function must go exactly through the data points.

In mathematics, nonlinear programming (NLP) is the process of solving a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function are nonlinear , [2]. Mathematically, we can represent general form of a nonlinear problem as

$$\begin{array}{ll} \text{Maximize} & f(x_1, x_2, ..., x_n) \\ \text{Subject to:} & \\ & g_1(x_1, x_2, ..., x_n) \leq b_1, \\ & \vdots \\ & g_m(x_1, x_2, ..., x_n) \leq b_m, \end{array}$$

where the $x_i \in \mathbb{R}^n$ variables $x_1, x_2, ..., x_n$ represent the set of decision variables, $f(x_1, x_2, ..., x_n)$ is the objective function and each of the constraint functions g_1 through g_m is given.

Constrained nonlinear programming is using for modeling a series of real applications, emphasizing differences between linear and nonlinear constraints, convex and nonconvex feasible sets, separable and nonseparable objective functions, and so on.

A different problem which is closely related to interpolation is the approximation of a complicated function by a simple function. Suppose we know the function but it is too complex to evaluate efficiently. Then we could pick a few known data points from the complicated function, creating a lookup table, and try to interpolate those data points to construct a simpler function. Of course, when using the simple function to calculate new data points we usually do not receive the same result as when using the original function, but depending on the problem domain and the interpolation method used the gain in simplicity might offset the error.

In this article, we are especially interested in the question of what are the time and temperature at which these binary alloys have the common maximum microhardness.

2.Experimental Details

Fe-Mn binary alloys having different Mn concentrations at $2 - 8h$ and $1073 - 1373K$ temperature were subjected to the process of boriding. After boronizing process, three disctinct regions have been identified on the surface of binary alloys:

(i) Boride layer including borides,

(ii) Transition zone,

(iii) The matrix which is not affected by boron.

The microhardnesses of these regions were measured. For more information about this subject see, [1], [3].

2.1 Boride Layer Microhardness

In Fig.1, we observe that boride layer microhardness surface of Fe-Mn binary alloys have the same microhardness at 0.42, 0.76 and 0.94 wt.% Mn for different times ($2h$,$3h$,$4h$,$6h$ and $8h$) and temperatures ($1073K$,$1173K$,$1273K$ and $1373K$). The microhardness surfaces are intersects each other at many points.

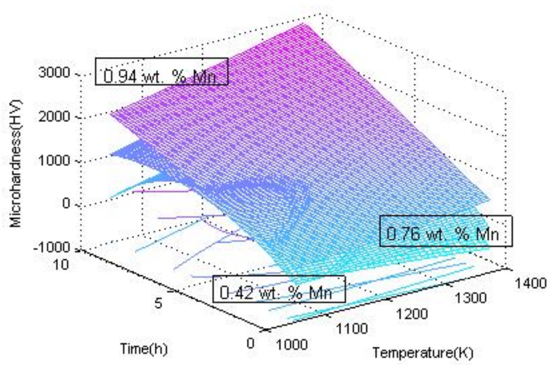

Figure 1.

In Fig.2, Boride layer surface of Fe-Mn binary alloys at 0.94 wt. % Mn and 0.76 wt. % Mn are never crossing each other at any point. That means the two binary alloys never will have the same microhardness at the same time and at the same temperature.

Boride layer surface of Fe-Mn binary alloys at 0.94 wt. % Mn and 0.42 wt. % Mn intersects at some points. These points constitute a common curve on the both surfaces. So, these two binary alloys have the same microhardness on this curve at common time and temperature. To find these common time and temperature we state the problem as the following:

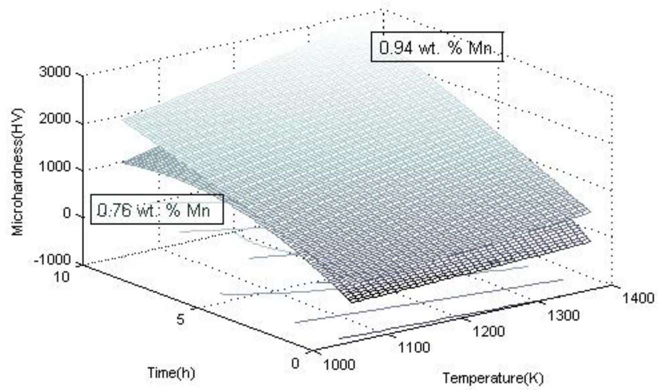

Figure 2.

$$\begin{aligned} \max_x f(x) \quad & \text{subject to: } c(x) \leq 0 \text{ (nonlinear constraints)} \\ & d(x) = 0 \text{ (nonlinear equality constraints)} \\ & Ax \leq b1 \text{ (linear inequality constraints)} \\ & Bx = b2 \text{ (linear equality constraints)} \\ & Ddown \leq x \leq Dup \text{ (bounds)} \end{aligned}$$

where

$$\begin{aligned} function[d] &= confunct1(x) \\ d(1) &= x(1)x(2) - 13730; \\ d(2) &= x(2) - x(1)^2 - 1273; \\ d(3) &= 0.7249 + (-0.6178)x(1) + 6.2899x(2) + 0.0008x(1)^2 \\ & \quad +0.1677x(1)x(2) + (-1.6009)x(2)^2 - 3.4160 - (0.7676)x(1) \\ & \quad -25.6942x(2) - (-0.0008)x(1)^2 - 0.4166x(1)x(2) - (-45.6621)x(2)^2 \end{aligned}$$

and we don't have other constraints. After solving this nonlinear problem with the following matlab code

```
x0=[1150;2.4];
A=[];
b1=[];B=[];b2=[];Dalt=[];Dust=[];
options=optimset('display','iter','largescale','off');
[x,fonkdegeri]=fmincon('f',x0,A,b1,B,b2,Dalt,Dust,'confunct1',options)
```

where f is objective function. We deduce that these two binary alloys can have maximum microhardness at $1116.8K$ temperature and at $4.5h$.

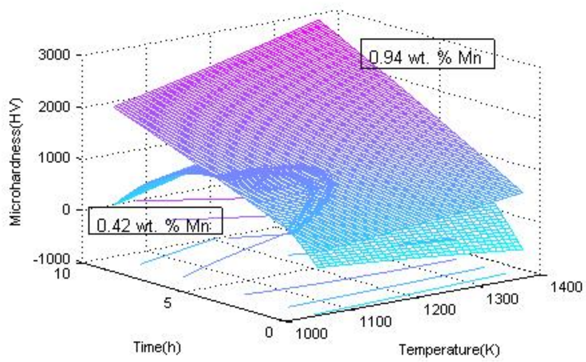

Figure 3.

In the following figure we have boride layer surface of Fe-Mn binary alloys at 0.76 wt. % Mn and 0.42 wt. % Mn intersecting each other. Again intersection points gives us another curve. So, these two binary alloys have the same microhardness on this curve at common time and temperature. These two binary alloys can have maximum microhardness at $1155K$ and $0.5h$ on this curve.

Now we will make similar observations for transition zone microhardness.

2.2 Transition Zone Microhardness

Fe-Mn binary alloys at 0.94 wt. % Mn and 0.76 wt. % Mn are never crossing each other at any point in this layer. So, these two binary alloys never have the same microhardness.

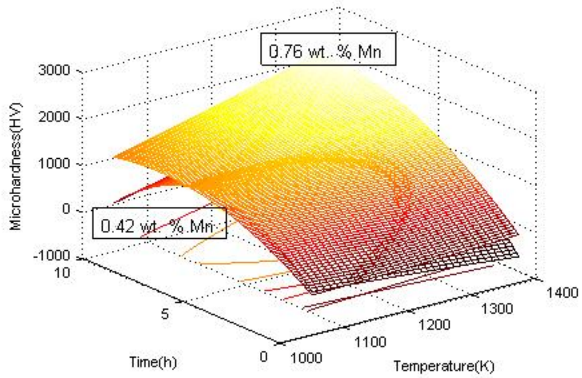

Figure 4.

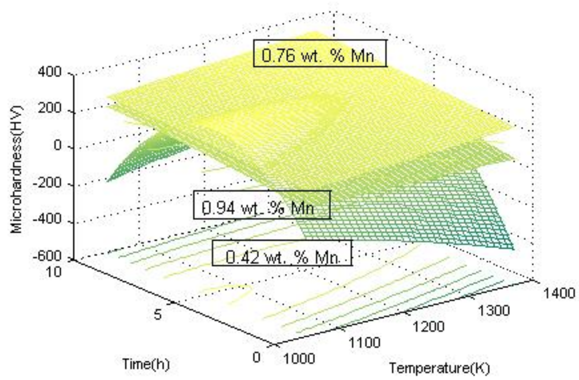

Figure 5.

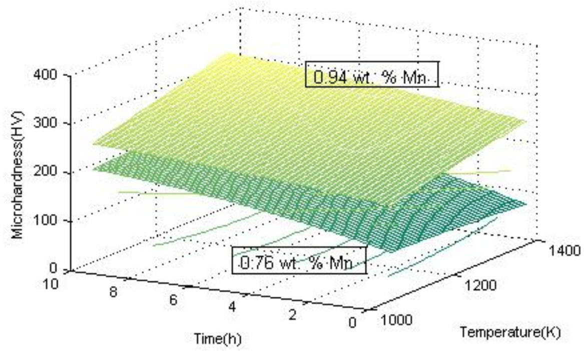

Figure 6.

Transition zone layer surface of Fe-Mn binary alloys at 0.94 wt. % Mn and 0.42 wt. %Mn intersects at some points. Again these points constitute a curve on the both surfaces. So, these two binary alloys have the same microhardness on this curve. The same method used as in boride layer we find maximum common microhardness is achieved at the temperature $1245K$ and the time $5.6h$.

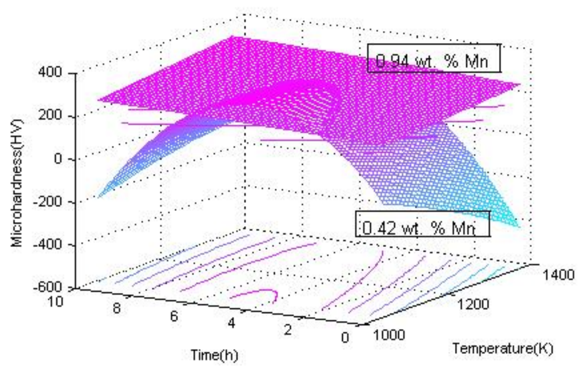

Figure 7.

Using the same arguments for Fe-Mn binary alloys at 0.76 wt. % Mn and 0.42 wt. % Mn we see that the maximum microhardness for mentioned binary alloys is attained at the temperature $1150.2K$ and the time $2h$.

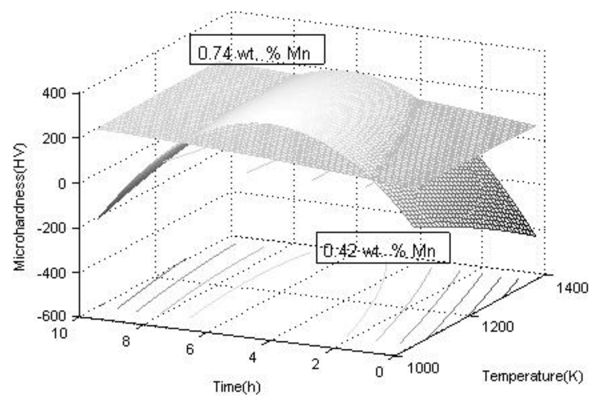

Figure 8.

2.3 Matrix Microhardness

In this region just one pair, Fe-Mn binary alloys at 0.94 wt. % Mn and 0.76 wt. % Mn intersects. These two binary alloys can have maximum microhardness at $886.7K$ and $2.1h$.

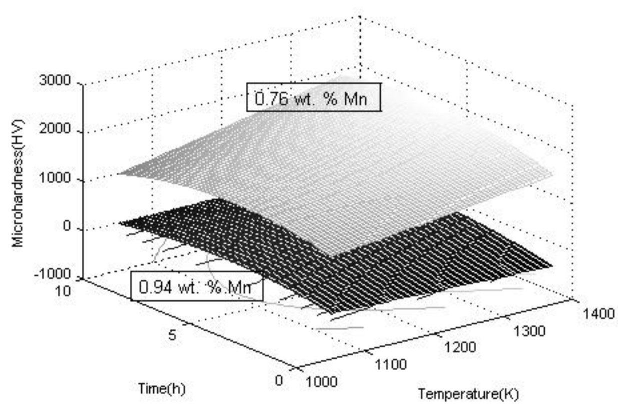

Figure 9.

3.Conclusion

Constrained nonlinear optimization methods can be used in many branches of science such as physics, chemistry, geology, astronomy, as well as many others. A constrained nonlinear mathematical programming used for determination of common maximum microhardness that different binary alloys can attain at the same time and the same temperature.

In this study, two different samples were boronized in an electrical resistance furnace at the same time and the same temperature to reach maximum microhardness.

This observation may be more useful when investigating under which common time and temperature any binary alloys can have the maximum common microhardness and this may help us save time as well as reducing the cost.

References

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[2] Bertsekas, Dimitri P. Nonlinear Programming (Second ed.), Cambridge, MA.: Athena Scientific, ISBN 1999; $1-886529-00-0$.

[3] Calik A., Sahin O., Ekinci A., E., Ucar N. Mechanical Properties of Boronized Fe-0.94% Mn Binary Alloy, Z. Naturforsch, 62, (2007), $545-548$.

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