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ON A CLASS OF SINGULAR ELLIPTIC SYSTEM WITH COMBINED NONLINEAR EFFECTS

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Abstract. We study the existence of positive solution for the following nonlinear system

$$\begin{cases} -div(|x|^{-ap}\,|\nabla u|^{p-2}\,\nabla u) = \lambda\,|x|^{-(a+1)p+c_1}\,f(u,v), & x \in \Omega, \\ -div(|x|^{-bq}\,|\nabla v|^{q-2}\,\nabla v) = \lambda\,|x|^{-(b+1)q+c_2}\,g(u,v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain of R^N with $0 \in \Omega$, $1 < p, q < N$, $0 \leq a < \frac{N-p}{p}$, $0 \leq b < \frac{N-q}{q}$ and c_1, c_2, λ are positive parameters. Here $f, g : [0,\infty) \times [0,\infty) \to [0,\infty)$ are nondecresing continuous functions and

$$\lim_{x \to \infty} \frac{f(x, A[g(x,x)]^{\frac{1}{q-1}})}{x^{p-1}} = 0.$$

for every $A > 0$.

We discuss the existence of positive solution when f and g satisfy certain additional conditions. We use the method of sub-super solutions to establish our results.

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1. Introduction

The paper deal with the existence of positive solution for the nonlinear system

$$\begin{cases} -div(|x|^{-ap}\,|\nabla u|^{p-2}\,\nabla u) = \lambda\,|x|^{-(a+1)p+c_1}\,f(u,v), & x \in \Omega, \\ -div(|x|^{-bq}\,|\nabla v|^{q-2}\,\nabla v) = \lambda\,|x|^{-(b+1)q+c_2}\,g(u,v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded smooth domain of R^N with $0 \in \Omega$, $1 < p, q < N$, $0 \leq a < \frac{N-p}{p}$, $0 \leq b < \frac{N-q}{q}$ and c_1, c_2, λ are positive parameters. Here $f, g : [0, \infty) \times [0, \infty) \to [0, \infty)$ are nondecresing continuous functions.

Elliptic problems involving more general operator, such as the degenerate quasilinear elliptic operator given by $-div(|x|^{-ap}\,|\nabla u|^{p-2}\,\nabla u)$, were motivated by the following Caaffarelli, Kohn and Nirenberg's inequality (see [1], [2]). The study of this type of problem is motivated by its various applications, for example, in fluid mechanics, in newtonian fluids, in flow through porous media and in glaciology (see [3, 4]). On the other hand, quasilinear elliptic systems has an extensive practical background. It can be used to describe the multiplicate chemical reaction catalyzed by the catalyst grains under constant or variant temperature, it can be used in the theory of quasiregular and quasiconformal mappings in Riemannian manifolds with boundary (see [5, 6]) and can be a simple model of tubular chemical reaction, more naturally, it can be a correspondence of the stable station of dynamical system determined by the reaction-diffusion system, see Ladde and Lakshmikantham et al. [7]. More naturally, it can be the populations of two competing species [8]. So, the study of positive solutions of elliptic systems has more practical meanings. We refer to [9], [10], [11], [12] for additional results on elliptic problems.

For the regular case, that is, when $a = b = 0$, $c_1 = p$ and $c_2 = q$, the quasilinear elliptic equation has been studied by several authors (see [13, 14]). See [14, 15] where the authors discussed the system (1) when $a = b = 0$, $c_1 = c_2 = p = q = 2$, $f(u, v) = \tilde{f}(u)$, $g(u, v) = \tilde{g}(u)$, $\tilde{f}, \tilde{g}$ are increasing and $\tilde{f}, \tilde{g} \geq 0$. In [16], the authors extended the study of [15], to the case when no sign conditions on $f(0)$ or $g(0)$ were required and in [17] they extend this study to the case when $p = q > 1$. Here we focus on further extending the study in [13] for the quasilinear elliptic systems involving singularity. Due to this singularity in the weights, the extensions are challenging and nontrivial. Our approach is based on the method of sub-super solutions, see [18, 19].

2. Preliminaries

In this paper, we denote $W_0^{1,p}(\Omega, |x|^{-ap})$, the completion of $C_0^\infty(\Omega)$, with respect to the norm $\|u\| = (\int_\Omega |x|^{-ap}|\nabla u|^p dx)^{\frac{1}{p}}$. To precisely state our existence result we consider the eigenvalue problem

$$\begin{cases} -div(|x|^{-sr}\,|\nabla \phi|^{r-2}\,\nabla \phi) = \lambda |x|^{-(s+1)p+t}\,|\phi|^{r-2}\phi, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega. \end{cases} \tag{2}$$

For $r = p$, $s = a$ and $t = c_1$, let $\phi_{1,p}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,p}$ of (2) such that $\phi_{1,p}(x) > 0$ in Ω, and $||\phi_{1,p}||_\infty = 1$ and for $r = q$,

$s = b$ and $t = c_2$, let $\phi_{1,q}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,q}$ of (2) such that $\phi_{1,q}(x) > 0$ in Ω, and $||\phi_{1,q}||_\infty = 1$ (see [20, 21]). It can be shown that $\frac{\partial \phi_{1,r}}{\partial n} < 0$ on $\partial\Omega$ for $r = p, q$. Here n is the outward normal. This result is well known and hence, depending on Ω, there exist positive constants $\epsilon, \delta, \sigma_p, \sigma_q$ such that

$$\lambda_{1,r}\, |x|^{-(s+1)r+t}\, \phi_{1,r}^r - |x|^{-sr}\, |\nabla \phi_{1,r}|^r \, \leq -\epsilon, \qquad x \in \bar{\Omega}_\delta, \tag{3}$$

$$\phi_{1,r} \geq \sigma_r, \qquad x \in \Omega_0 = \Omega \setminus \bar{\Omega}_\delta, \tag{4}$$

with $r = p, q$; $s = a, b$; $t = c_1, c_2$ and $\bar{\Omega}_\delta = \{x \in \Omega \mid d(x, \partial\Omega) \leq \delta\}$ (see [20]). We will also consider the unique solution $(\zeta_p(x), \zeta_q(x)) \in W_0(\Omega, \|x\|^{-ap}) \times W_0(\Omega, \|x\|^{-bq})$ for the system

$$\begin{cases} -div(|x|^{-ap}\, |\nabla \zeta_p|^{p-2}\, \nabla \zeta_p) = |x|^{-(a+1)p+c_1}, & x \in \Omega, \\ -div(|x|^{-bq}\, |\nabla \zeta_q|^{q-2}\, \nabla \zeta_p) = |x|^{-(b+1)q+c_2}, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

to discuss our existence result. It is known that $\zeta_r(x) > 0$ in Ω and $\frac{\partial \zeta_r(x)}{\partial n} < 0$ on $\partial\Omega$, for $r = p, q$ (see [20]).

3. Existence results

In this section, we shall establish our existence result via the method of sub and supersolutions. A pair of nonnegative functions (ψ_1, ψ_2), (z_1, z_2) are called a subsolution and supersolution of (1) if they satisfy $(\psi_1, \psi_2) = (0, 0) = (z_1, z_2)$ on $\partial\Omega$ and

$$\int_\Omega |x|^{-ap}\, |\nabla \psi_1|^{p-2}\, \nabla \psi_1 \cdot \nabla w\, dx \leq \lambda \int_\Omega |x|^{-(a+1)p+c_1}\, f(\psi_1, \psi_2)\, w\, dx,$$

$$\int_\Omega |x|^{-bq}\, |\nabla \psi_2|^{q-2}\, \nabla \psi_2 \cdot \nabla w\, dx \leq \lambda \int_\Omega |x|^{-(b+1)q+c_2}\, g(\psi_1, \psi_2)\, w\, dx,$$

$$\int_\Omega |x|^{-ap}\, |\nabla z_1|^{p-2}\, \nabla z_1 \cdot \nabla w\, dx \geq \lambda \int_\Omega |x|^{-(a+1)p+c_1}\, f(z_1, z_2)\, w\, dx,$$

$$\int_\Omega |x|^{-bq}\, |\nabla z_2|^{q-2}\, \nabla z_2 \cdot \nabla w\, dx \geq \lambda \int_\Omega |x|^{-(b+1)q+c_2}\, g(z_1, z_2)\, w\, dx,$$

for all $w \in W = \{w \in C_0^\infty(\Omega) | w \geq 0 \in \Omega\}$. Then the following result holds:

Lemma 1. *(See [20]) Suppose there exist sub and super- solutions* (ψ_1, ψ_2) *and* (z_1, z_2) *respectively of* (1) *such that* $(\psi_1, \psi_2) \leq (z_1, z_2)$*. Then* (1) *has a solution* (u, v) *such that* $(u, v) \in [(\psi_1, \psi_2), (z_1, z_2)]$.

We make the following assumptions:

(H1) $f, g : [0, \infty) \times [0, \infty) \to [0, \infty)$ *are* C^1 *functions such that* $f_u, f_v, g_u, g_v \geq 0$ *and*

$$\lim_{u,v\to\infty} f(u, v) = \lim_{u,v\to\infty} g(u, v) = \infty.$$

(H2) *For all* $A > 0$,

$$\lim_{x\to\infty} \frac{f(x, A[g(x, x)]^{\frac{1}{q-1}})}{x^{p-1}} = 0.$$

(H3)

$$\lim_{x\to\infty} \frac{g(x, x)}{x^{q-1}} = 0.$$

Now we are ready to state our existence result.

Theorem 2. *Assume (H1), (H2) and (H3) hold. Then the system* (1) *admits a positive large solution when* λ *is large.*

Proof. Since f, g are continuous and nondecreasing, we have $f(x), g(x) \geq -a_0$ for all $x \geq 0$ and for some $a_0 > 0$. Choose $\eta > 0$ such that

$$\eta \leq \min\{|x|^{-(a+1)p+c_1}, |x|^{-(b+1)q+c_2}\},$$

in $\bar{\Omega}_\delta$. We shall verify that

$$(\psi_{1,\lambda}, \psi_{2,\lambda}) = \Big([\frac{\lambda a_0 \eta}{\epsilon}]^{\frac{1}{p-1}} (\frac{p-1}{p})\, \phi_{1,p}^{\frac{p}{p-1}}\ ,\ [\frac{\lambda a_0 \eta}{\epsilon}]^{\frac{1}{q-1}} (\frac{q-1}{q})\, \phi_{1,q}^{\frac{q}{q-1}}\Big),$$

is a sub-solution of (1). Let $w \in W$. Then a calculation shows that

$$\begin{aligned}
&\int_\Omega |x|^{-ap}\, |\nabla \psi_{1,\lambda}|^{p-2}\, \nabla \psi_{1,\lambda}\, \nabla w\, dx \\
= &\Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_\Omega |x|^{-ap}\, \phi_{1,p}\, |\nabla \phi_{1,p}|^{p-2}\, \nabla \phi_{1,p}\, \nabla w\, dx \\
= &\Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_\Omega |x|^{-ap}\, |\nabla \phi_{1,p}|^{p-2}\, \nabla \phi_{1,p}\, [\nabla(\phi_{1,p} w) - |\nabla \phi_{1,p}|^p\, w]\, dx \\
= &\Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_\Omega [\lambda_{1,p}\, |x|^{-(a+1)p+c_1}\, \phi_{1,p}^p - |x|^{-ap}\, |\nabla \phi_{1,p}|^p] w\, dx.
\end{aligned}$$

Similarly

$$\int_\Omega |x|^{-bq}\,|\nabla\psi_{2,\lambda}|^{q-2}\nabla\psi_{2,\lambda}\nabla w\,dx = \Big(\frac{\lambda a_0\eta}{\epsilon}\Big)\int_\Omega [\lambda_{1,q}\,|x|^{-(b+1)q+c_2}\,\phi_{1,q}^q - |x|^{-bq}\,|\nabla\phi_{1,q}|^q]w\,dx.$$

First we consider the case when $x\in\bar{\Omega}_\delta$. We have $\lambda_{1,p}\,|x|^{-(a+1)p+c_1}\,\phi_{1,p}^p - |x|^{-ap}\,|\nabla\phi_{1,p}|^p \le -\epsilon$ on $\bar{\Omega}_\delta$. Since $\psi_{1,\lambda}(x),\psi_{2,\lambda}(x)\ge 0$ in Ω, it follows that

$$-a_0\eta \le \min\{|x|^{-(a+1)p+c_1}f(\psi_{1,\lambda},\psi_{2,\lambda}),|x|^{-(a+1)p+c_1}g(\psi_{1,\lambda},\psi_{2,\lambda})\},$$

in $\bar{\Omega}_\delta$. Hence, we have

$$\begin{aligned}
&\Big(\frac{\lambda a_0\eta}{\epsilon}\Big)\int_{\bar{\Omega}_\delta}[\lambda_{1,p}\,|x|^{-(a+1)p+c_1}\,\phi_{1,p}^p - |x|^{-ap}\,|\nabla\phi_{1,p}|^p]w\,dx\\
\le\;& -\lambda a_0\epsilon\int_{\bar{\Omega}_\delta} w\,dx\\
\le\;& \lambda\int_{\bar{\Omega}_\delta}|x|^{-(a+1)p+c_1}\,f(\psi_{1,\lambda},\psi_{2,\lambda})\,w\,dx.
\end{aligned}$$

A similar argument shows that

$$\begin{aligned}
\Big(\frac{\lambda a_0\eta}{\epsilon}\Big)&\int_{\bar{\Omega}_\delta}[\lambda_{1,q}|x|^{-(b+1)q+c_2}\phi_{1,q}^q - |x|^{-bq}|\nabla\phi_{1,q}|^q]wdx\\
&\le \lambda\int_{\bar{\Omega}_\delta}|x|^{-(b+1)q+c_2}\,g(\psi_{1,\lambda},\psi_{2,\lambda})w\,dx.
\end{aligned}$$

On the other hand, on $\Omega\setminus\bar{\Omega}_\delta$, we have $\phi_{1,p}\ge\sigma_p,\phi_{1,q}\ge\sigma_q$ for some $0<\sigma_p,\sigma_q<1$. Therefore

$$\psi_{1,\lambda}\ge \Big(\frac{\lambda a_0\eta}{\epsilon}\Big)^{\frac{1}{p-1}}\Big(\frac{p-1}{p}\Big)\,\sigma_p^{\frac{p}{p-1}}\to\infty, \tag{5}$$

$$\psi_{2,\lambda}\ge \Big(\frac{\lambda a_0\eta}{\epsilon}\Big)^{\frac{1}{q-1}}\Big(\frac{q-1}{q}\Big)\,\sigma_q^{\frac{q}{q-1}}\to\infty, \tag{6}$$

as $\lambda\to\infty$, uniformly in $\Omega\setminus\bar{\Omega}_\delta$. By (5), (6) and (**H1**) we can find λ_* sufficiently large such that

$$f(\psi_{1,\lambda},\psi_{2,\lambda}),g(\psi_{1,\lambda},\psi_{2,\lambda})\ge\frac{a_0\eta}{\epsilon}\,\max\{\lambda_{1,p},\lambda_{1,q}\}.$$

for all $x \in \Omega \setminus \bar{\Omega}_\delta$ and for all $\lambda \geq \lambda_*$. Hence

$$\begin{aligned}
&\Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_{\Omega\setminus\bar{\Omega}_\delta} [\lambda_{1,p}\, |x|^{-(a+1)p+c_1}\, \phi_{1,p}^{p} - |x|^{-ap}\, |\nabla \phi_{1,p}|^{p}] w\, dx \\
\leq\ & \Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_{\Omega\setminus\bar{\Omega}_\delta} |x|^{-(a+1)p+c_1}\, \lambda_{1,p}\, w\, dx \\
\leq\ & \lambda \int_{\Omega\setminus\bar{\Omega}_\delta} |x|^{-(a+1)p+c_1}\, f(\psi_{1,\lambda}, \psi_{2,\lambda})\, w\, dx.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Big(\frac{\lambda a_0 \eta}{\epsilon}\Big) \int_{\Omega\setminus\bar{\Omega}_\delta} [\lambda_{1,q}\, |x|^{-(b+1)q+c_2}\, \phi_{1,q}^{q} - |x|^{-bq}\, |\nabla \phi_{1,q}|^{q}] w\, dx \\
\leq \int_{\Omega\setminus\bar{\Omega}_\delta} |x|^{-(b+1)q+c_2}\, g(\psi_{1,\lambda}, \psi_{2,\lambda})\, w\, dx.
\end{aligned}$$

Hence

$$\int_\Omega |x|^{-ap}\, |\nabla \psi_1|^{p-2}\, |\nabla \psi_1| \cdot \nabla w\, dx \leq \int_\Omega |x|^{-(a+1)p+c_1}\, f(\psi_{1,\lambda}, \psi_{2,\lambda})\, w\, dx,$$

$$\int_\Omega |x|^{-bq}\, |\nabla \psi_2|^{q-2}\, |\nabla \psi_2| \cdot \nabla w\, dx \leq \int_\Omega |x|^{-(b+1)q+c_2}\, g(\psi_{1,\lambda}, \psi_{2,\lambda})\, w\, dx.$$

i.e., (ψ_1, ψ_2) is a sub-solution of (1).

Now, we will prove there exists a M large enough so that

$$(z_1, z_2) = \Big(M\, \theta_p^{-1} \lambda^{\frac{1}{p-1}}\, \zeta_p(x), [g(M\, \lambda^{\frac{1}{p-1}}, M\, \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}\, \lambda^{\frac{1}{q-1}}\, \zeta_q(x)\Big),$$

is a super-solution of (1), where $\theta_r = \|\zeta_r\|_\infty; r = p, q$. A calculation shows that:

$$\begin{aligned}
\int_\Omega |x|^{-ap}\, |\nabla z_1|^{p-2}\, \nabla z_1\, \nabla w\, dx &= \lambda (M \theta_p^{-1})^{p-1} \int_\Omega |x|^{-ap}\, |\nabla \zeta_p|^{p-2}\, \nabla \zeta_p\, \nabla w\, dx \\
&= \theta_p^{1-p} (M \lambda^{\frac{1}{p-1}})^{p-1} \int_\Omega |x|^{-(a+1)p+c_1}\, w\, dx.
\end{aligned}$$

By monotonicity condition on f and **(H2)** we can choose M large enough so that

$$\begin{aligned}
\theta_p^{1-p} (M \lambda^{\frac{1}{p-1}})^{p-1} &\geq \lambda f\Big(M\, \lambda^{\frac{1}{p-1}}, [g(M\, \lambda^{\frac{1}{p-1}}, M\, \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}\, \lambda^{\frac{1}{q-1}}\, \theta_q(x)\Big) \\
&\geq \lambda f\Big(M\, \theta_p^{-1} \lambda^{\frac{1}{p-1}}\, \zeta_p(x), [g(M\, \lambda^{\frac{1}{p-1}}, M\, \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}\, \lambda^{\frac{1}{q-1}}\, \zeta_q(x)\Big) \\
&= \lambda f(z_1, z_2).
\end{aligned}$$

Hence

$$\int_\Omega |x|^{-ap}\,|\nabla z_1|^{p-2}\,|\nabla z_1|\cdot\nabla w\,dx \geq \lambda\int_\Omega |x|^{-(a+1)p+c_1}\,f(z_1,z_2)\,w\,dx.$$

Next, by **(H3)** for M large enough we have

$$\frac{[g(M\,\lambda^{\frac{1}{p-1}},M\,\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}}{M\lambda^{\frac{1}{p-1}}} \leq \lambda^{\frac{1}{1-q}}\theta_q(x).$$

Hence

$$\begin{aligned}
&\int_\Omega |x|^{-bq}|\nabla z_2|^{q-2}\nabla z_2\nabla w\,dx\\
=\;& \lambda[g(M\,\lambda^{\frac{1}{p-1}},M\,\lambda^{\frac{1}{p-1}})]\int_\Omega |x|^{-bq}\,|\nabla\zeta_q|^{q-2}\,\nabla\zeta_q\,\nabla w\,dx\\
=\;& \lambda[g(M\,\lambda^{\frac{1}{p-1}},M\,\lambda^{\frac{1}{p-1}})]\int_\Omega |x|^{-(b+1)q+c_2}\,w\,dx\\
\geq\;& \lambda\int_\Omega |x|^{-(b+1)q+c_2}g\Big(M\,\lambda^{\frac{1}{p-1}},[g(M\,\lambda^{\frac{1}{p-1}},M\,\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}\,\lambda^{\frac{1}{q-1}}\,\theta_q(x)\Big)w\,dx\\
\geq\;& \lambda\int_\Omega |x|^{-(b+1)q+c_2}g\Big(M\,\theta_p^{-1}\lambda^{\frac{1}{p-1}}\,\zeta_p(x),[g(M\,\lambda^{\frac{1}{p-1}},M\,\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}\,\lambda^{\frac{1}{q-1}}\zeta_q(x)\Big)w\,dx
\end{aligned}$$

i.e. (z_1,z_2) is a super-solution of (1) with $z_i\geq\psi_i$ for M large, $i=1,2$. Thus, by [20] there exists a positive solution (u,v) of (1) such that $(\psi,\psi)\leq(u,v)\leq(z_1,z_2)$. This completes the proof of Theorem 1.

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