Surface family with a common Mannheim B-geodesic curve

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Abstract. In this paper, we construct a surface family possessing a Mannheim B pair of a given curve as a geodesic curve. Using the Bishop frame of the given Mannheim B curves, we present the surface as a linear combination of this frame and analyse the necessary and sufficient condition for a given curve such that its Mannheim B pairs is both isoparametric and geodesic on a parametric surface. Also we analyze the conditions when the resulting surface is a ruled surface. In addition, developability along the common Mannheim B -geodesic of the members of surface family are discussed. Finally, we present some interesting examples to show the validity of this study.

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Key words: Bishop frame; Geodesic curve; Mannheim B-pair; Mannheim B-curves; ruled surface.

1 Introduction

In differential geometry, there are many important consequences and properties of curves [20, 10, 21]. Researches follow labours about the curves. In the light of the existing studies, authors always introduce new curves. One of the most significant curve on a surface is geodesic curve. Geodesics are important in the relativistic description of gravity. Einstein's principle of equivalence tells us that geodesics represent the paths of freely falling particles in a given space. In architecture, some special curves have nice properties in terms of structural functionality and manufacturing cost. One example is planar curves in vertical planes, which can be used as support elements. Another example is the one of geodesic curves [11] described methods to create patterns of special curves on surfaces, which find applications in design and realization of freeform architecture. At the corresponding points of associated curves, one of the Frenet vectors of a curve coincides with one of the Frenet vectors of other curve. This has attracted the attention of many mathematicians. One of the well-known curves is the Mannheim curve, where the principal normal line of a curve coincides with the

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binormal line of another curve at the corresponding points of these curves. The first study of Mannheim curves has been presented by Mannheim in 1878 and has a special position in the theory of curves [7]. Other studies have been revealed, which introduce some characterized properties in the Euclidean and Minkowski space [17, 18, 22, 25]. Liu and Wang called these new curves as Mannheim partner curves (see [18] for details). Later, Mannheim offset the ruled surfaces and dual Mannheim curves have been defined in [23, 24, 13]. Recently, in [19] Masal and Azak, defined of Mannheim B-pair according to Bishop frame in the Euclidean 3-space was provided which followed by the calculitions of the relations between Bishop and Frenet vectors of Mannheim B-pair. Besides, some theorems and results about the curvatures of Mannheim B-pair were stated. However, recent researchers focused on the reverse problem: given a 3D curve, find surfaces interpolating the given curve as a special curve, rather than finding and classifying curves on analytical curved surfaces. The first study related with this problem was proposed by Wang et al. [28] in Euclidean 3-space. They constructed parametric surfaces possessing a given curve as a common geodesic. In this construction, they obtained the condition on marching-scale functions, coefficients of the Frenet vectors. Kasap et al. [16] generalized the marching-scale functions of Wang and obtained a larger family of surfaces. Saffak and Kasap [27, 1] constructed surfaces with a common null geodesic and null asymptotic. Atalay and Kasap [2, 3] studied the problem: given a curve (with Bishop frame), how to characterize those surfaces that posess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space. Also they studied the problem: given a curve (with Frenet frame), how to characterize those surfaces that possess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space. Recently, in [5, 4], Atalay studied the necessary and sufficient condition for a given curve (with Frenet frame) such that its Mannheim pair is both isoparametric and geodesic (asymptotic) on a parametric surfaces. Bishop frame, which is also called alternative or parallel frame of the curves, was introduced by L. R. Bishop in 1975 by means of parallel vector fields, [6]. Recently, many research papers related to this concept have been treated in the Euclidean space, see [8, 9]. And, recently, this special frame is extended to study of canal and tubular surfaces, we refer to [14, 15]. Bishop and Frenet-Serret frames have a common vector field, namely the tangent vector field of the Frenet-Serret frame. A practical application of Bishop frames is that they are used in the area of Biology and Computer Graphics. For example, it may be possible to compute information about the shope of sequences of DNA using a curve defined by the Bishop frame. The Bishop frame may also provide a new way to control virtual cameras in computer animatons, [26]. In this paper, we obtain the necessary and sufficient condition for a given curve (with Bishop frame) such that its Mannheim B-pair is both isoparametric and geodesic on a parametric surfaces. Furthermore, we present important results for ruled surfaces. Finally, we illustrate the method with some examples.

2 Preliminaries

Let be a 3-dimensional Euclidean space provided with the metric given by $\langle\ ,\ \rangle = dx_1^2 + dx_2^2 + dx_3^2$ where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. Recall that, the norm of a arbitrary vector $X \epsilon E^3$ is given by $\| X \| = \sqrt{\langle X, X \rangle}$. Let $\alpha = \alpha(s) : I \subset \mathbb{R} \longrightarrow E^3$ is an arbitrary curve of arc-length parameter s. The curve

α is called a unit speed curve if velocity $\alpha^{'}$ vector of a satisfies $\parallel \alpha^{'} \parallel = 1$. Let $\{T(s), N(s), B(s)\}$be the moving Frenet frame along α, Frenet formulas is given by

$$\frac{d}{ds}\left(\begin{array}{c} T(s) \\ N(s) \\ B(s) \end{array}\right) = \left(\begin{array}{ccc} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{array}\right)\left(\begin{array}{c} T(s) \\ N(s) \\ B(s) \end{array}\right) \tag{2.1}$$

where the function $\kappa(s) = \left\|\alpha^{''}(s)\right\|$ and $\tau(s) = \frac{\det(\alpha'(s),\alpha''(s),\alpha'''(s))}{\|\alpha'(s)\times\ \alpha''(s)\|}$ are called the curvature and torsion of the curve $\alpha(s)$, respectively [10].

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative. One can express parallel transport of an orthonormal frame along a curve simply by parallel transporting each component of the frame, [5]. The tangent vector and any convenient arbitrary basis for the remainder of the frame are used. The Bishop frame is expressed as

$$\frac{d}{ds}\left(\begin{array}{c} T(s) \\ N_1(s) \\ N_2(s) \end{array}\right) = \left(\begin{array}{ccc} 0 & k_1(s) & k_2(s) \\ -k_1(s) & 0 & 0 \\ -k_2(s) & 0 & 0 \end{array}\right)\left(\begin{array}{c} T(s) \\ N_1(s) \\ N_2(s) \end{array}\right) \tag{2.2}$$

Here, we shall call the set $\{T(s), N_1(s), N_2(s)\}$ as Bishop Frame and $k_1(s)$ and $k_2(s)$ as Bishop curvatures.

Let $\{T, N, B, \kappa, \tau\}$ and $\{T, N_1, N_2, k_1, k_2\}$ be the Frenet and Bishop apparatus of regular curve α with the arc-length parameter s respectively. The relations between Frenet and Bishop frames are given as follows:

$$\left\{\begin{array}{l} T = \alpha', \\ N = \cos\theta N_1 + \sin\theta N_2, \\ B = -\sin\theta N_1 + \cos\theta N_2 \end{array}\right. \tag{2.3}$$

and

$$\tau(s) = -\theta'(s),\ \ \kappa(s) = \sqrt{k_1^2 + k_2^2}, \tag{2.4}$$

where $\theta(s) = \arctan\left(\frac{k_2}{k_1}\right)$, $N_2 = T \times N_1$. Furthermore, the relations

$$\left\{\begin{array}{l} k_1(s) = \kappa(s)\cos\theta(s), \\ k_2(s) = \kappa(s)\sin\theta(s) \end{array}\right. \tag{2.5}$$

can be written for the Bishop curvatures of the curve α [5].

A curve on a surface is geodesic if and only if the normal vector to the curve is everywhere parallel to the local normal vector of the surface. Another criterion for a curve in a surface M to be geodesic is that its geodesic curvature vanishes [10]. An isoparametric curve $\alpha(s)$ is a curve on a surface $\varphi = \varphi(s, v)$ is that has a constant s or v-parameter value. In other words, there exist a parameter s_0 or v_0, such that

$\alpha(s) = \varphi(s, v_0)$ or $\alpha(v) = \varphi(s_0, v)$. Given a parametric curve $\alpha(s)$, we call $\alpha(s)$ an isogeodesic of a surface φ if it is both an geodesic and an isoparametric curve on φ.

Now, the Mannheim B-curves and some characterizations of these curves will be introduced.

Definition 2.1. Let C and C^* be unit speed curves with the arc-length parameters of s and s^* respectively. Denote the Bishop apparatus of C and C^* by $\{T, N_1, N_2, k_1, k_2\}$ and $\{T^*, N_1^*, N_2^*, k_1^*, k_2^*\}$ respectively. If the Bishop vector N_1 coincides with the Bishop vector N_2^* at the corresponding points of the curves C and C^* then the curve is said to be a Mannheim partner B-curve of C^* or a (C, C^*) curve couple is called Mannheim B-pair, see Figure 1 [19].

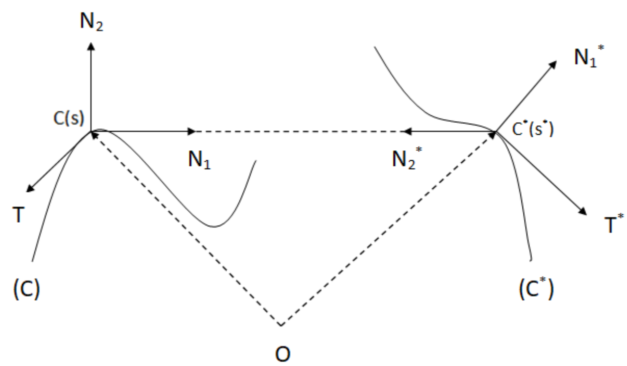

Figure 1: Mannheim B-curves.

Let γ be the angle between the tangents T and T^* of $(C\ ,\ C^*)$ Mannheim B-pair. Thus from the definition of Mannheim B-pair the following matrix representation can be written [19]

$$\begin{pmatrix} T \\ N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ 0 & 0 & 1 \\ -\sin\gamma & \cos\gamma & 0 \end{pmatrix} \begin{pmatrix} T^* \\ N_1^* \\ N_2^* \end{pmatrix} \tag{2.6}$$

Theorem 2.1. *The distance between the corresponding points of the Mannheim B-curves is constant E^3, (see for details, [19]).*

In other words, the Mannheim B pairs of a curve $C(s)$ with arc length s are given by

$$C^*(s) = C(s) + \lambda N_1(s), \tag{2.7}$$

where λ is a reel constant and $\lambda \neq 0$.

Theorem 2.2. *Let $(C,\ C^*)$ be a Mannheim B-pair in E^3. Then the relationships between the Bishop vectors of C and C^* is given by*

$$T = \mu T^*, \quad N_1 = \mu N_2^*, \quad N_2 = \mu N_1^*,$$

such that

$$\mu = \begin{cases} 1, & \textit{for } \gamma = 0 \\ -1, & \textit{for } \gamma = \pi, \end{cases}$$

where γ is the angle between the tangent vectors of C and C^*, (see for details, [19]).

3 Surfaces family with a common Mannheim B-geodesic curve

Suppose we are given a 3-dimensional parametric curve $\alpha(s), L_1 \leq s \leq L_2$, in which s is the arc length and $\|\alpha''(s)\| \neq 0$. Let $\overline{\alpha}(s), L_1 \leq s \leq L_2$, be the Mannheim partner B-curve of the given curve $\alpha(s)$.

The surface pencil that interpolates $\overline{\alpha}(s)$ as a common curve is given in the parametric form as

$$\varphi(s,v) = \overline{\alpha}(s) + x(s,v)\overline{T}(s) + y(s,v)\overline{N_1}(s) + z(s,v)\overline{N_2}(s), L_1 \leq s \leq L_2, K_1 \leq v \leq K_2, \tag{3.1}$$

where $x(s,v)$, $y(s,v)$ and $z(s,v)$ are C^1 functions. The values of the marching-scale functions $x(s,v)$, $y(s,v)$ and $z(s,v)$ indicate, respectively; the extension-like, flexion-like and retortion-like effects, by the point unit through the time v, starting from $\overline{\alpha}(s)$ and $\{\overline{T}(s), \overline{N_1}(s), \overline{N_2}(s)\}$ is the Bishop frame associated with the curve $\overline{\alpha}(s)$.

Let $\overline{\alpha}(s)$ be the Mannheim partner B-curve of the given curve $\alpha(s)$. Then $\overline{\alpha}(s)$ is given by

$$\overline{\alpha}(s) = \alpha(s) + \lambda N_1(s) \tag{3.2}$$

where λ is a non-zero constant.

Now, let us first examine the problem of finding the surface family with a common Mannheim B-geodesic curve in the case of $\gamma = 0$ from Theorem 2.2.

Using (3.2) and Theorem 2.2, we obtain

$$\varphi(s,v) = \alpha(s) + x(s,v)T(s) + (\lambda + z(s,v))\, N_1(s) + y(s,v)N_2(s) \tag{3.3}$$

Remark 3.1. Observe that choosing different marching-scale functions yields different surfaces possessing $\overline{\alpha}(s)$ as a common curve.

Our goal is to find the necessary and sufficient conditions for which the Mannheim partner B-curve of the given curve $\alpha(s)$ is isoparametric and geodesic on the surface $\varphi(s,v)$. Firstly, since $\overline{\alpha}(s)$ is an isoparametric curve on the surface $\varphi(s,v)$, then there exists a parameter $v_0 \in [K_1, K_2]$ such that

$$x(s,v_0) = y(s,v_0) \equiv 0,\ z(s,v_0) = -\lambda,\ L_1 \leq s \leq L_2\ ,\ K_1 \leq v_0 \leq K_2\ . \tag{3.4}$$

Secondly, since the Mannheim partner B-curve of $\alpha(s)$ is a geodesic curve on the surface $\varphi(s,v)$, then there exists a parameter $v_0 \in [K_1, K_2]$ such that

$$n(s,v_0) \parallel N(s) \tag{3.5}$$

where $n(s,v_0)$ is a normal vector $\varphi=\varphi(s,v)$ and $N(s)$ is principal normal vector of $\alpha(s)$. The normal vector of $\varphi=\varphi(s,v)$ can be written as

$$n(s,v)=\frac{\partial\varphi(s,v)}{\partial s}\times\frac{\partial\varphi(s,v)}{\partial v}.$$

By using the equations (2.2) and (3.3), we infer that along the curve α the normal vector can be expressed as:

$$\begin{aligned} n(s,v) \quad = \quad & \left[\begin{array}{c} \frac{\partial y(s,v)}{\partial v}\left(x(s,v)k_1+\frac{\partial z(s,v)}{ds}\right) \\ -\frac{\partial z(s,v)}{\partial v}\left(-x(s,v)k_2+\frac{\partial y(s,v)}{ds}\right) \end{array}\right]T(s) \\ & +\left[\begin{array}{c} \frac{\partial x(s,v)}{\partial v}\left(x(s,v)k_2+\frac{\partial y(s,v)}{ds}\right) \\ -\frac{\partial y(s,v)}{\partial v}\left(1-k_1\left(z(s,v)+\lambda\right)-k_2y(s,v)+\frac{\partial x(s,v)}{ds}\right) \end{array}\right]N_1(s) \\ & +\left[\begin{array}{c} \frac{\partial z(s,v)}{\partial v}\left(1-k_1\left(z(s,v)+\lambda\right)-k_2y(s,v)+\frac{\partial x(s,v)}{ds}\right) \\ -\frac{\partial x(s,v)}{\partial v}\left(x(s,v)k_1+\frac{\partial z(s,v)}{ds}\right) \end{array}\right]N_2(s). \end{aligned}$$

Thus,

$$n(s,v_0)=\varphi_1(s,v_0)T(s)+\varphi_2(s,v_0)N_1(s)+\varphi_3(s,v_0)N_2(s) \tag{3.6}$$

where

$$\begin{cases} \varphi_1(s,v_0)=0 \\ \varphi_2(s,v_0)=-\frac{\partial y}{\partial v}(s,v_0) \\ \varphi_3(s,v_0)=\frac{\partial z}{\partial v}(s,v_0) \end{cases} \tag{3.7}$$

Also, from equation (2.3), we obtain

$$\begin{aligned} n(s,v_0)&=\varphi_1(s,v_0)T(s)+\left(\cos\theta\varphi_2(s,v_0)+\sin\theta\varphi_3(s,v_0)\right)N(s) \\ &+\left(-\sin\theta\varphi_2(s,v_0)+\cos\theta\varphi_3(s,v_0)\right)B(s). \end{aligned}$$

From equation (3.5), we have

$$\begin{cases} -\frac{\partial y}{\partial v}(s,v_0)\cos\theta(s)+\frac{\partial z}{\partial v}(s,v_0)\sin\theta(s)\neq 0 \\ -\frac{\partial y}{\partial v}(s,v_0)\sin\theta(s)+\frac{\partial z}{\partial v}(s,v_0)\cos\theta(s)=0 \end{cases}$$

From $\beta(s)\neq 0$, we infer

$$\begin{cases} \frac{\partial z}{\partial v}(s,v_0)=\beta(s)\sin\theta(s) \\ \frac{\partial y}{\partial v}(s,v_0)=-\beta(s)\cos\theta(s) \end{cases} \tag{3.8}$$

So, we can present the following theorem:

Theorem 3.1. *Let $\alpha(s)$,$L_1\le s\le L_2$, be a unit speed curve with nonvanishing curvature and let $\overline{\alpha}(s)$, $L_1\le s\le L_2$, be a Mannheim partner B-curve. $\overline{\alpha}$ is a* *isogeodesic curve on the surface (3.1) if and only if*

$$\begin{cases} x(s,v_0) = y(s,v_0) \equiv 0, \ z(s,v_0) = -\lambda, \\ \frac{\partial z}{\partial v}(s,v_0) = \beta(s)\sin\theta(s) \\ \frac{\partial y}{\partial v}(s,v_0) = -\beta(s)\cos\theta(s) \quad , \quad \beta(s) \neq 0 \\ \theta'(s) = -\tau(s). \end{cases} \tag{3.9}$$

where $L_1 \leq s \leq L_2$, $K_1 \leq v, v_0 \leq K_2$ *(*v_0 *fixed)* θ *is the angle between the* N_1 *and the* N *vector of the curve* α*.*

Corollary 3.2. *Combining the conditions (3.4) and (3.8), we have found the necessary and sufficient conditions for the surface* φ *to have the Mannheim partner B-curve of the given curve* α *an isogeodesic. We call the set of surfaces defined by (3.4) and (3.8)* the family of surfaces with common geodesic. *Any surface* $\varphi(s, v)$ *defined by (3.3) and satisfying (3.9) is a member of this family.*

Secondly, from Theorem 2.2 for $\gamma = \pi$, similar calculations lead to

$$\begin{cases} x(s,v_0) = y(s,v_0) \equiv 0, \ z(s,v_0) = -\lambda, \\ \frac{\partial z}{\partial v}(s,v_0) = \beta(s)\sin\theta(s) \\ \frac{\partial y}{\partial v}(s,v_0) = \beta(s)\cos\theta(s) \quad , \quad \beta(s) \neq 0 \\ \theta'(s) = -\tau(s). \end{cases}$$

4 Ruled surfaces with a common Mannheim B-geodesic curve

As is well-known, a surface is said to be "ruled" if it is generated by moving a straight line continuously in Euclidean space E^3. Ruled surfaces are one of the simplest objects in geometric modelling as they are generated basically by moving a line in space. One important fact about ruled surfaces is that they can be generated by straight lines. A practical application of this type surfaces is that they are used in civil engineering and physics, [12]. A surface φ is called a ruled surface in Euclidean space, if it is a surface swept out by a straight line l moving alone a curve α. The generating line l and the curve α are called the rulings and the base curve of the surface, respectively.

We show how to derive the formulations of a ruled surfaces family such that the common Mannheim B-geodesic is also the base curve of ruled surfaces.

Theorem 4.1. *Given an arc-length curve* $\alpha(s)$*, there exists a ruled surface family possessing* $\alpha(s)$ *as a common Mannheim B-geodesic.*

Proof. Choosing marching-scale functions as

$$\begin{cases} x(s,v) = (v - v_0)x(s) \\ y(s,v) = (v - v_0)\beta(s)\sin\theta(s) \\ z(s,v) = (v - v_0)\beta(s)\cos\theta(s) - \lambda, \end{cases} \tag{4.1}$$

and $\beta(s) \neq 0$ and $\theta'(s) = -\tau(s)$, equation (3.1) takes the following form of a ruled surface

$$\varphi(s,v) = \alpha(s) + (v - v_0)\left[x(s)T(s) + \beta(s)\sin\theta(s)N_1(s) - \beta(s)\cos\theta(s)N_2(s)\right], \tag{4.2}$$

which satisfies equation (3.9) interpolating $\alpha(s)$ as a common Mannheim B-geodesic curve. □

Remark 4.1. Observe that, changing $x(s)$ and $\beta(s)$ in equation (4.2) yields different ruled surfaces interpolating $\alpha(s)$ as a common Mannheim B-geodesic.

Corollary 4.2. *Ruled surface (4.2) is developable if and only if*

$$x(s) = \frac{\tau(s)}{\kappa(s)}\beta(s), \beta(s) \neq 0,$$

for some real valued function $x(s)$*.*

Proof. The surface

$$\varphi(s,v) = \alpha(s) + (v - v_0)\left[x(s)T(s) + \beta(s)\sin\theta(s)N_1(s) - \beta(s)\cos\theta(s)N_2(s)\right]$$

is developable if and only if $\det(\alpha', R, R') = 0$, where

$$R(s) = x(s)T(s) + \beta(s)\sin\theta(s)N_1(s) - \beta(s)\cos\theta(s)N_2(s).$$

If necessary calculations are made and determinants are used we get

$$x(s)\beta(s)\left[k_2(s)\sin\theta(s) + k_1(s)\cos\theta(s)\right] + \beta^2(s)\theta'(s) = 0,$$

where

$$\begin{cases} k_1(s) = \kappa(s)\cos\theta(s), \\ k_2(s) = \kappa(s)\sin\theta(s) \end{cases} \text{and } \theta'(s) = -\tau(s),\ \beta(s) \neq 0$$

are used, we obtain $x(s) = \frac{\tau(s)}{\kappa(s)}\beta(s), \beta(s) \neq 0$, which completes the proof. □

5 Examples of generating simple surfaces and ruled surface with common Mannheim B-geodesic curve

Example 5.1. Let $\alpha(s) = (\cos s, \sin s, 0)$ be a unit speed curve. Then it is easy to show that $\kappa(s) = 1$, $\tau(s) = 0$.

From equation (2.4), $\theta'(s) = -\tau(s) \Rightarrow \theta = c,\ c =$constant. Here $c = 0$ can be taken.
From equation (2.5), $k_1(s) = \cos 0 = 1,\ k_2(s) = \sin 0 = 0$.
From equation (2.3),

$$\begin{cases} T(s) = (-\sin(s), \cos(s), 0), \\ N_1(s) = (-\cos s, -\sin s, 0), \\ N_2(s) = (0, 0, 1). \end{cases}$$

a) If we take $x(s,v)=0$, $y(s,v)=-\sin(v)$, $z(s,v)=-\lambda+v\sin(v)$ and $\lambda=1$, $v_0=0$, $\beta(s)=1$, then the equation (3.9) is satisfied. Thus, we obtain a member of the surface with common Mannheim B-geodesic curve as

$$\varphi_1(s,v)=(\cos s(1-v\sin v),\sin s(1-v\sin v),-\sin v)\,,$$

where $0\leq s\leq 2\pi$, $0\leq v\leq 2\pi$ (Fig.2).

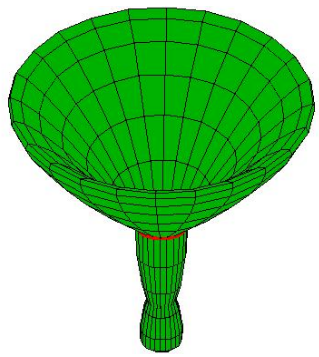

Fig.2. $\varphi_1(s,v)$ **as a member of the surface and its common Mannheim B-geodesic curve.**

b) If we take $x(s,v)=0$, $y(s,v)=-v$, $z(s,v)=-\lambda-v^2$ and $\lambda=1$, $v_0=0,\beta(s)=1$ then the equation (3.9) is satisfied. Thus, we obtain another member of the surface with common Mannheim B-geodesic curve as

$$\varphi_2(s,v)=\left(\cos s+v^2\cos s,\sin s+v^2\sin s,-v\right),$$

where $0\leq s\leq 2\pi$, $0\leq v\leq 2\pi$ (Fig.3).

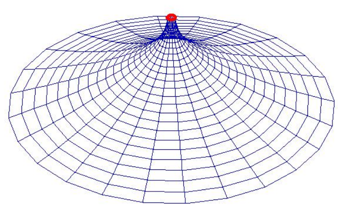

Fig.3. $\varphi_2(s,v)$ **as a member of the surface and its common Mannheim B-geodesic curve.**

Example 5.2. Let $\alpha(s)=\left(\frac{4}{5}\cos s,-\frac{3}{5}\cos s,1-\sin s\right)$ be a unit speed curve. Then it is easy to show that

$$\begin{cases} T(s)=\left(-\frac{4}{5}\sin s,\frac{3}{5}\sin s,-\cos s\right) \\ N(s)=\left(-\frac{4}{5}\cos s,\frac{3}{5}\cos s,\sin s\right) \\ B(s)=(\frac{3}{5},\frac{4}{5},0) \end{cases} ,\kappa(s)=1 \ \ ,\ \tau(s)=0.$$

From equation (2.4), $\theta'(s)=-\tau(s)\Rightarrow\theta=c$, $c=$constant. Here $c=0$ can be taken.

From equation (2.5), $k_1(s)=\cos 0=1$, $k_2(s)=\sin 0=0$.

From equation (2.3),

$$N_1(s) = \left(\tfrac{2}{5}\cos s, -\tfrac{3}{5}\cos s, \tfrac{1}{2}\sin s\right)$$
$$N_2(s) = \left(\tfrac{2\sqrt{3}}{5}\cos s, \tfrac{3\sqrt{3}}{10}\cos s, \tfrac{\sqrt{3}}{2}\sin s\right).$$

If we take $x(s,v) = 0$, $y(s,v) = -\sin(v)$, $z(s,v) = -\lambda + v\sin(v)$ and $\lambda = 1$, $v_0 = 0, \beta(s) = 1$ then the equation (3.9) is satisfied. Thus, we obtain a member of the surface with common Mannheim B-geodesic curve as

$$\varphi_3(s,v) = \begin{pmatrix} \tfrac{4}{5}\cos s + \tfrac{2}{5}v\sin v\cos s - \tfrac{2\sqrt{3}}{5}\sin v\cos s, \\ -\tfrac{3}{5}\cos s - \tfrac{3}{5}v\sin v\cos s - \tfrac{3\sqrt{3}}{10}\sin v\cos s, \\ 1 - \sin s + \tfrac{1}{2}v\sin v\sin s - \tfrac{\sqrt{3}}{2}\sin v\sin s \end{pmatrix}$$

where $0 \leq s \leq 2\pi$, $0 \leq v \leq \pi$ (Fig.4).

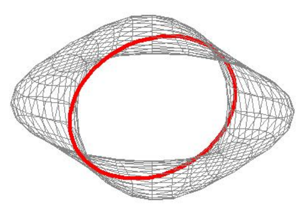

Fig.4. $\varphi_3(s,v)$ **as a member of the surface and its common Mannheim B-geodesic curve.**

In equation (4.2), if we take $x(s) = 0$, then we obtain the following developable ruled surface with a common Mannheim B-geodesic curve as

$$\varphi_4(s,v) = \left(\frac{1}{5}\cos s\left(4 - 2\sqrt{3}v\right), -\frac{3}{5}\cos s\left(1 + \frac{\sqrt{3}}{2}v\right), 1 - \sin s\left(1 + \frac{\sqrt{3}}{2}v\right)\right),$$

where $0 \leq s \leq 2\pi$, $0 \leq v \leq 2\pi$ (Fig.5).

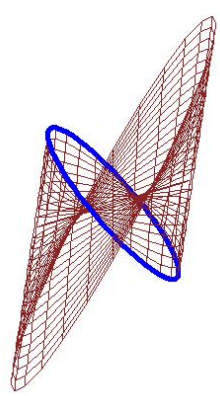

Fig.5. $\varphi_4(s,v)$ **as a member of the developable ruled surface and its common Mannheim B-geodesic curve.**

In equation (4.2), if we take $x(s) = s$, then we obtain the following nondevelopable ruled surface with a common Mannheim B-geodesic curve as

$$\varphi_5(s,v) = \begin{pmatrix} \frac{4}{5}\cos s - \frac{v}{5}\left(4s\sin s + 2\sqrt{3}\cos s\right), -\frac{3}{5}\cos s + \frac{3v}{5}\left(s\sin s - \frac{\sqrt{3}}{2}\cos s\right), \\ 1 - \sin s - v\left(s\cos s + \frac{\sqrt{3}}{2}\sin s\right) \end{pmatrix},$$

where -$2\pi \leq s \leq 2\pi$, $0 \leq v \leq 2\pi$ (Fig.6).

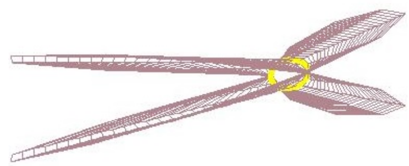

Fig.6. $\varphi_5(s,v)$ **as a member of the nondevelopable ruled surface and its common Mannheim B-geodesic curve.**

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