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CERTAIN CLASSES OF k-UNIFORMLY CLOSE-TO-CONVEX FUNCTIONS AND OTHER RELATED FUNCTIONS DEFINED BY USING THE DZIOK-SRIVASTAVA OPERATOR

(DEDICATED IN OCCASION OF THE 65-YEARS OF PROFESSOR R.K. RAINA)

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Abstract. Several interesting classes of k-uniformly close-to-convex functions and k-uniformly quasi-convex functions are defined here by using the Dziok-Srivastava operator. We provide necessary and sufficient coefficient conditions, extreme points, integral representations, and distortion bounds for functions belonging to each of these classes of k-uniformly close-to-convex functions and k-uniformly quasi-convex functions.

1. Introduction, Definitions and Preliminaries

Let $\mathcal{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Also let $\mathcal{A}^-$ denote a subclass of $\mathcal{A}$ consisting of functions of the form:

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \qquad (a_n \geqq 0)\,, \tag{1.2}$$

which are analytic in $\mathbb{U}$.

A function $f(z) \in \mathcal{A}$ is said to be in the class of k-uniformly convex functions of order β $(0 \leqq \beta < 1)$, denoted by $\mathcal{UK}(k, \beta)$ (*cf.* [10]; see also [6] and [8]) if

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > k\left|\frac{zf''(z)}{f'(z)}\right| + \beta \qquad (k \geqq 0;\ 0 \leqq \beta < 1;\ z \in \mathbb{U}). \tag{1.3}$$

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A corresponding class of k-uniformly starlike functions, denoted by $\mathcal{US}(k,\beta)$ consists of functions $f(z)\in\mathcal{A}$ such that

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > k\left|\frac{zf'(z)}{f(z)}-1\right|+\beta \qquad (k\geqq 0;\ 0\leqq\beta<1;\ z\in\mathbb{U}). \tag{1.4}$$

It is obvious from the inequalities in (1.3) and (1.4) that (see [10])

$$f(z)\in\mathcal{UK}(k,\beta) \quad\Longleftrightarrow\quad zf'(z)\in\mathcal{US}(k,\beta). \tag{1.5}$$

Each of the function classes $\mathcal{UK}(k,\beta)$ and $\mathcal{US}(k,\beta)$ provides unifications and generalizations various other (known or new) subclasses of $\mathcal{A}$. Several properties of some of the subclasses of the function classes $\mathcal{UK}(k,\beta)$ and $\mathcal{US}(k,\beta)$ were studied recently in [9] (see also [6] and [8]).

Definition 1 (see [1]). Define $\mathcal{UC}(k,\gamma,\beta)$ to be the family of functions $f(z)\in\mathcal{A}$ such that

$$\Re\left(\frac{zf'(z)}{g(z)}\right) > k\left|\frac{zf'(z)}{g(z)}-1\right|+\gamma \qquad \big(k\geqq 0;\ \gamma\in[0,1);\ z\in\mathbb{U}\big) \tag{1.6}$$

for some function $g(z)\in\mathcal{US}(k,\beta)$.

Definition 2 (see [1]). Define $\mathcal{UQ}(k,\gamma,\beta)$ to be the family of functions $f(z)\in\mathcal{A}$ such that

$$\Re\left(\frac{(zf'(z))'}{g'(z)}\right) > k\left|\frac{(zf'(z))'}{g'(z)}-1\right|+\gamma \qquad \big(k\geqq 0;\ \gamma\in[0,1);\ z\in\mathbb{U}\big) \tag{1.7}$$

for some function $g(z)\in\mathcal{UK}(k,\beta)$.

It readily follows from Definitions 1 and 2 that

$$f(z)\in\mathcal{UQ}(k,\gamma,\beta) \quad\Longleftrightarrow\quad zf'(z)\in\mathcal{UC}(k,\gamma,\beta). \tag{1.8}$$

We say that $\mathcal{UC}(0,\gamma,\beta)$ is the class of *close-to-convex functions of order* γ *and type* β in $\mathbb{U}$ and that $\mathcal{UQ}(0,\gamma,\beta)$ is the class of *quasi-convex functions of order* γ *and type* β in $\mathbb{U}$.

Definition 3. For functions $f(z)\in\mathcal{A}$ given by (1.1), and $g(z)\in\mathcal{A}$ given by

$$g(z) = z+\sum_{n=2}^{\infty} b_n z^n, \tag{1.9}$$

we define the Hadamard product (or convolution) of $f(z)$ and $g(z)$ by

$$(f*g)(z) := z+\sum_{n=2}^{\infty} a_n\, b_n z^n =: (g*f)(z) \qquad (z\in\mathbb{U}). \tag{1.10}$$

For complex parameters

$$\alpha_j\in\mathbb{C}\ \ (j=1,\cdots,l) \quad\text{and}\quad \beta_j\in\mathbb{C}\setminus\mathbb{Z}_0^-\ \ (j=1,\cdots,m;\ \mathbb{Z}_0^- := \{0,-1,-2,\cdots\}),$$

the generalized hypergeometric function ${}_lF_m$ (with l numerator and m denominator parameters) is defined by

$${}_lF_m(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m) = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_l)_n}{(\beta_1)_n\cdots(\beta_m)_n}\cdot\frac{z^n}{n!} \tag{1.11}$$

$$(l \leqq m+l;\ l,m \in \mathbb{N}_0 := \{0,1,2,\cdots\} = \mathbb{N} \cup \{0\}),$$

where $(\lambda)_\nu$ denotes the Pochhammer symbol (or the *shifted* factorial, since $(1)_n = n!$ for $n \in \mathbb{N}$) defined, in terms of the familiar Gamma functions, by

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0;\ \lambda \in \mathbb{C} \setminus \{0\}) \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (\nu = n \in \mathbb{N};\ \lambda \in \mathbb{C}). \end{cases}$$

Now, corresponding to the function

$$h(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m;z) = z\ {}_lF_m(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m),$$

the *Dziok-Srivastava linear operator* (see [3], [4], [5] and [11]; see also [7], [14] and [15])

$$H_m^l(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)$$

is defined as follows by using the Hadamard product (or convolution):

$$\begin{aligned} &H_m^l(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)f(z) \\ &\qquad = h(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m;z) * f(z) \\ &\qquad = z + \sum_{n=2}^{\infty} \varphi_n(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m) a_n z^n, \end{aligned} \tag{1.12}$$

where, for convenience,

$$\varphi_n(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)$$

is given by

$$\varphi_n(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m) := \frac{(\alpha_1)_{n-1}\cdots(\alpha_l)_{n-1}}{(\beta_1)_{n-1}\cdots(\beta_m)_{n-1}} \cdot \frac{1}{(n-1)!}. \tag{1.13}$$

It is well known (see, for example, [5]) that

$$\begin{aligned} &\alpha_1 H_m^l(\alpha_1+1,\alpha_2,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)f(z) \\ &\qquad = z\big(H_m^l(\alpha_1+1,\alpha_2,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)f(z)\big)' \\ &\qquad\quad + (\alpha_1-1)H_m^l(\alpha_1,\alpha_2,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)f(z). \end{aligned} \tag{1.14}$$

For notational simplification in our investigation, we write

$$H_m^l[\alpha_1]f(z) = H_m^l(\alpha_1,\cdots,\alpha_l;\beta_1,\cdots,\beta_m)f(z). \tag{1.15}$$

We now define the linear operator $L_{\lambda,j,m}^{\tau,\alpha_1}$ as follows:

$$L_{\lambda,\alpha_1}^0 f(z) = f(z), \tag{1.16}$$

$$\begin{aligned} L_{\lambda,j,m}^{1,\alpha_1} f(z) &= (1-\lambda)H_m^l[\alpha_1]f(z) + \lambda z\big(H_m^l[\alpha_1]f(z)\big)' \\ &= L_{\lambda,j,m}^{\alpha_1} f(z) \qquad (\lambda \geqq 0), \end{aligned} \tag{1.17}$$

$$L_{\lambda,j,m}^{2,\alpha_1} f(z) = L_{\lambda,j,m}^{\alpha_1}\big(L_{\lambda,j,m}^{1,\alpha_1} f(z)\big) \tag{1.18}$$

and, in general,

$$L_{\lambda,j,m}^{\tau,\alpha_1} f(z) = L_{\lambda,j,m}^{\alpha_1}\big(L_{\lambda,j,m}^{\tau-1,\alpha_1} f(z)\big) \qquad (l \leqq m+1;\ l,m \in \mathbb{N}_0;\ \tau \in \mathbb{N}). \tag{1.19}$$

If the function $f(z)$ is given by (1.1), then we see from (1.12), (1.13), (1.17) and (1.19) that

$$L_{\lambda,j,m}^{\tau,\alpha_1} f(z) = z + \sum_{n=2}^{\infty} \phi_n^\tau(\alpha_1,\lambda,l,m) a_n z^n \qquad (\tau \in \mathbb{N}_0), \tag{1.20}$$

where

$$\phi_n^\tau(\alpha_1,\lambda,l,m)=\left(\frac{(\alpha_1)_{n-1}\cdots(\alpha_l)_{n-1}}{(\beta_1)_{n-1}\cdots(\beta_m)_{n-1}}\cdot\frac{[1+\lambda(n-1)]}{(n-1)!}\right)^\tau \tag{1.21}$$
$$(n\in\mathbb{N}\setminus\{1\};\ \tau\in\mathbb{N}_0).$$

When

$$\tau=1 \qquad \text{and} \qquad \lambda=0,$$

the linear operator $L_{\lambda,j,m}^{\tau,\alpha_1}$ would reduce to the familiar Dziok-Srivastava linear operator given by (1.12) above (see, for example, [3]). For a linear operator which is essentially analogous to the Dziok-Srivastava operator in (1.12), but uses instead the Fox-Wright generalization of the hypergeometric function ${}_lF_m$ defined here by (1.11), the interested reader may be referred to the recent works [2] and [12] as well as to the closely-related works cited in each of these recent works.

By applying the general operator $L_{\lambda,j,m}^{\tau,\alpha_1}$, we define the following subclasses of the function class $\mathcal{A}$.

I. Let $\mathcal{US}_m^l(\tau,\lambda,k,\beta)$ be the class of functions $f(z)\in\mathcal{A}$ satisfying the following inequality:

$$\Re\left(\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}{L_{\lambda,j,m}^{\tau,\alpha_1}f(z)}\right)>k\left|\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}{L_{\lambda,j,m}^{\tau,\alpha_1}f(z)}-1\right|+\beta \qquad \big(k\geqq 0;\ \beta\in[0,1)\big). \tag{1.22}$$

Observe that

$$L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\in\mathcal{US}(k,\beta).$$

II. Let $\mathcal{UK}_m^l(\tau,\lambda,k,\beta)$ be the class of functions $f(z)\in\mathcal{A}$ satisfying the following inequality:

$$\Re\left(1+\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)''}{\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}\right)>k\left|\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)''}{\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}\right|+\beta \qquad \big(k\geqq 0;\ \beta\in[0,1)\big). \tag{1.23}$$

Observe that

$$L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\in\mathcal{UK}(k,\beta).$$

III. Let $\mathcal{UC}_m^l(\tau,\lambda,k,\gamma,\beta)$ be the class of functions $f\in\mathcal{A}$ such that

$$\Re\left(\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}{L_{\lambda,j,m}^{\tau,\alpha_1}g(z)}\right)>k\left|\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)'}{L_{\lambda,j,m}^{\tau,\alpha_1}g(z)}-1\right|+\gamma \qquad \big(k\geqq 0;\ \gamma\in[0,1)\big) \tag{1.24}$$

for some function $g(z)\in\mathcal{US}_m^l(\tau,k,\beta)$. Observe that

$$L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\in\mathcal{UC}(k,\gamma,\beta).$$

IV. Let $\mathcal{UQ}_m^l(\tau,\lambda,k,\gamma,\beta)$ be the class of functions $f\in\mathcal{A}$ such that

$$\Re\left(1+\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)''}{\big(L_{\lambda,j,m}^{\tau,\alpha_1}g(z)\big)'}\right)>k\left|\frac{z\big(L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\big)''}{\big(L_{\lambda,j,m}^{\tau,\alpha_1}g(z)\big)'}\right|+\gamma \qquad \big(k\geqq 0;\ \gamma\in[0,1)\big) \tag{1.25}$$

for some function $g(z)\in\mathcal{UK}_m^l(\tau,\lambda,k,\beta)$. Observe that

$$L_{\lambda,j,m}^{\tau,\alpha_1}f(z)\in\mathcal{UK}(k,\gamma,\beta).$$

It is clear from two of the above definitions that

$$f(z) \in \mathcal{UK}_m^l(\tau,\lambda,k,\beta) \Longleftrightarrow zf'(z) \in \mathcal{UC}_m^l(\tau,\lambda,k,\beta). \tag{1.26}$$

Finally, in terms of the above-defined function classes, we write

$$\mathcal{US}_{l,m}^-(\tau,\lambda,k,\beta) = \mathcal{A}^- \cap \mathcal{US}_m^l(\tau,\lambda,k,\beta),$$
$$\mathcal{UK}_{l,m}^-(\tau,\lambda,k,\beta) = \mathcal{A}^- \cap \mathcal{UK}_m^l(\tau,\lambda,k,\beta),$$
$$\mathcal{UC}_{l,m}^-(\tau,\lambda,k,\gamma,\beta) = \mathcal{A}^- \cap \mathcal{UC}_m^l(\tau,\lambda,k,\gamma,\beta)$$

and

$$\mathcal{UQ}_{l,m}^-(\tau,\lambda,k,\gamma,\beta) = \mathcal{A}^- \cap \mathcal{UQ}_m^l(\tau,\lambda,k,\gamma,\beta).$$

The various properties and characteristics of functions in the class $\mathcal{US}_m^l(1,0,k,\beta)$ were investigated by Dziok and Srivastava [3]. In this paper, we obtain several relationships and properties of the convolution operators considered here. Our paper mainly studies the functions in the class $\mathcal{UC}_m^l(\tau,\lambda,k,\beta)$. We first prove a sufficient condition for a function $f \in \mathcal{A}$ to be in the class $\mathcal{UC}_m^l(\tau,\lambda,k,\beta)$. We then provide necessary and sufficient coefficient conditions, extreme points, integral representations, distortion bounds, radii of starlikeness and convexity for functions in the class $\mathcal{UC}_m^l(\tau,\lambda,k,\beta)$.

2. First Set of Main Results

First of all, we obtain a sufficient condition for a function $f \in \mathcal{A}$ to be in the class $\mathcal{UC}_m^l(\tau,\lambda,k,\gamma,\beta)$.

Theorem 1. *Let $f(z) \in \mathcal{A}$ be given by* (1.1)*. Suppose also that $\phi_n^\tau(\alpha_1,\lambda,l,m)$ is given by* (1.21)*. If*

$$k \geqq 0, \quad \beta \in [0,1), \quad \gamma \in [0,1), \quad \lambda \geqq 0, \quad \tau \in \mathbb{N}_0$$

and

$$\sum_{n=2}^{\infty} \left[2k|na_n - b_n| + (1-\gamma)|b_n|\right] \phi_n^\tau(\alpha_1,\lambda,l,m) < 1-\gamma,$$

then $f(z) \in \mathcal{UC}_m^l(\tau,\lambda,k,\gamma,\beta)$.

Proof. By the definition of the function class $\mathcal{UC}_m^l(\tau,\lambda,k,\gamma,\beta)$, it suffices to show for a function $f(z) \in \mathcal{A}$ given by (1.1) that

$$\begin{aligned} k\left|\frac{z\left(L_{\lambda,j,m}^{\tau,\alpha_1} f(z)\right)'}{L_{\lambda,j,m}^{\tau,\alpha_1} g(z)} - 1\right| - \Re\left(\frac{z\left(L_{\lambda,j,m}^{\tau,\alpha_1} f(z)\right)'}{L_{\lambda,j,m}^{\tau,\alpha_1} g(z)} - \gamma\right) &\leqq 2k\left|\frac{z\left(L_{\lambda,j,m}^{\tau,\alpha_1} f(z)\right)'}{L_{\lambda,j,m}^{\tau,\alpha_1} g(z)} - 1\right| \\ &\leqq 2k\,\frac{\sum\limits_{n=2}^{\infty} \phi_n^\tau(\alpha_1,\lambda,l,m)|na_n - b_n| \cdot |z|^{n-1}}{1 - \sum\limits_{n=2}^{\infty} \phi_n^\tau(\alpha_1,\lambda,l,m)\,|b_n| \cdot |z|^{n-1}}. \end{aligned} \tag{2.1}$$

Now the last expression in (2.1) is bounded above by $1-\gamma$ if and only if

$$\sum_{n=2}^{\infty} \left[2k|na_n - b_n| + (1-\gamma)|b_n|\right] \phi_n^\tau(\alpha_1,\lambda,l,m) < 1-\gamma,$$

which evidently completes the proof of Theorem 1. □

We next provide a necessary and sufficient coefficient bound for a given function $f(z) \in \mathcal{A}^-$ to belong to the class $\mathcal{UC}^-_{l,m}(\tau, \lambda, k, \gamma, \beta)$.

Theorem 2. *Let $f(z) \in \mathcal{A}^-$ be given by (1.2). Also let $\phi^\tau_n(\alpha_1, \lambda, l, m)$ be given by (1.21). Then $f \in \mathcal{UC}^-_{l,m}(\tau, \lambda, k, \gamma, \beta)$ if and only if*

$$\sum_{n=2}^{\infty} \big[n(1+k)a_n - (k+\gamma)b_n\big]\phi^\tau_n(\alpha_1, \lambda, l, m) < 1-\gamma. \tag{2.2}$$

Proof. Suppose that $f(z) \in \mathcal{UC}^-_{l,m}(\tau, \lambda, k, \gamma, \beta)$. Then, making use of the fact that

$$\Re(\omega) > k|\omega - 1| + \gamma \Longleftrightarrow \Re\big(\omega(1+ke^{i\phi}) - ke^{i\phi}\big) > \gamma \qquad (\gamma \in \mathbb{R})$$

and letting

$$\omega = \frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m} f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)}$$

in (1.3), we obtain

$$\Re\left(\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m} f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)}(1+ke^{i\phi}) - ke^{i\phi}\right) > \gamma$$

or, equivalently,

$$\Re\left(\frac{(1+ke^{i\phi})z\left(z - \sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)a_n z^n\right)' - (ke^{i\phi}+\gamma)\left(z - \sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)b_n z^n\right)}{z - \sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)b_n z^n}\right) > 0,$$

which holds true for all $z \in \mathbb{U}$. By letting $z \to 1-$ through real values, we thus find that

$$\Re\left(\frac{(1-\gamma) - (1+ke^{i\phi})\sum_{n=2}^{\infty} n\phi^\tau_n(\alpha_1,\lambda,l,m)a_n + (\gamma+ke^{i\phi})\sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)b_n}{1 - \sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)b_n}\right) > 0,$$

and so (by the mean value theorem) we have

$$\Re\left((1-\beta) - (1+ke^{i\gamma})\sum_{n=2}^{\infty} n\phi^\tau_n(\alpha_1,\lambda,l,m)a_n + (\beta+ke^{i\phi})\sum_{n=2}^{\infty}\phi^\tau_n(\alpha_1,\lambda,l,m)b_n\right) > 0.$$

Therefore, we get

$$\sum_{n=2}^{\infty} \big[n(1+k)a_n - (k+\gamma)b_n\big]\phi^\tau_n(\alpha_1, \lambda, l, m) < 1-\gamma,$$

which proves the first part of Theorem 2.

Conversely, we let the inequality (2.2) hold true.
Then, in light of the fact that

$$\Re(\omega) > \gamma \Longleftrightarrow |\omega - (1+\gamma)| < |\omega + (1-\gamma)| \qquad (\gamma \in \mathbb{R}),$$

we need only to show that

$$\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-\left(1+k\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-1\right|\right)+\gamma\right|$$
$$<\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}+\left(1-k\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-1\right|\right)-\gamma\right|$$

By setting

$$\frac{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}=e^{i\vartheta},$$

we may write

$$\begin{aligned}
\mathfrak{E}&=\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}+\left(1-k\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-1\right|-\gamma\right)\right|\\
&=\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\left|\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'+(1-\gamma)\frac{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}{z}-k\left|\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'-\frac{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}{z}\right|\right|\\
&=\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\Bigg|(2-\gamma)-\sum_{n=2}^{\infty}[na_n+(1-\gamma)b_n]\phi^{\tau}_n(\alpha_1,\lambda,l,m)z^{n-1}\\
&\qquad -e^{i\vartheta}\left|-\sum_{n=2}^{\infty}(kna_n-kb_n)\phi^{\tau}_n(\alpha_1,\lambda,l,m)z^{n-1}\right|\Bigg|\\
&>\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\left((2-\gamma)-\sum_{n=2}^{\infty}(n(1+k)a_n+(1-k-\gamma)b_n)\phi^{\tau}_n(\alpha_1,\lambda,l,m)\right)
\end{aligned}$$

and

$$\begin{aligned}
\mathfrak{F}&=\left|\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-\left(1+k\left|\frac{z(L^{\tau,\alpha_1}_{\lambda,j,m}f(z))'}{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}-1\right|+\gamma\right)\right|\\
&=\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\left|\big(L^{\tau,\alpha_1}_{\lambda,j,m}f(z)\big)'-(1+\gamma)\frac{L^{\tau,\alpha_1}_{\lambda,l,m}g(z)}{z}-k\left|\big(H^{l}_{m}[\alpha_1]f(z)\big)'-\frac{L^{\tau,\alpha_1}_{\lambda,j,m}g(z)}{z}\right|\right|\\
&=\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\Bigg|-\gamma-\sum_{n=2}^{\infty}[na_n-(1+\gamma)b_n]\phi^{\tau}_n(\alpha_1,\lambda,l,m)z^{n-1}\\
&\qquad -e^{i\vartheta}\left|-\sum_{n=2}^{\infty}(kna_n-kb_n)\phi^{\tau}_n(\alpha_1,\lambda,l,m)z^{n-1}\right|\Bigg|\\
&<\frac{|z|}{\left|L^{\tau,\alpha_1}_{\lambda,j,m}g(z)\right|}\left(\gamma+\sum_{n=2}^{\infty}[n(1+k)a_n-(1+k+\gamma)b_n]\phi^{\tau}_n(\alpha_1,\lambda,l,m)\right).
\end{aligned}$$

It is easy to verify that

$$\mathfrak{E}-\mathfrak{F}>0$$

in case the inequality (2.2) holds true. The proof of Theorem 2 is thus completed. □

When

$$f(z) = g(z) \qquad (z \in \mathbb{U}),$$

Theorem 2 would yield the following corollary.

Corollary 1. *Let $g(z) \in \mathcal{A}^-$ be given by*

$$g(z) = z - \sum_{n=2}^{\infty} b_n z^n \qquad (b_n \geqq 0)\,, \tag{2.3}$$

Then $g(z) \in \mathcal{US}^-_{l,m}(\tau, \lambda, k, \beta)$ if and only if

$$\sum_{n=2}^{\infty} \frac{[(n-1)k + n - \beta] b_n \phi_n^\tau(\alpha_1, \lambda, l, m)}{1 - \beta} < 1.$$

Corollary 2. *If $g(z) \in \mathcal{US}^-_{l,m}(\tau, \lambda, k, \beta)$ is given by (2.3), then*

$$\sum_{n=2}^{\infty} b_n < \frac{1-\beta}{(2+k-\beta)\phi_2^\tau(\alpha_1, \lambda, l, m)}.$$

Proof. Since $g(z) \in \mathcal{US}^-_{l,m}(\tau, \lambda, k, \beta)$ is given by (2.3), we can apply Corollary 1 to obtain

$$\begin{aligned}(k+2-\beta)\phi_2^\tau(\alpha_1, \lambda, l, m) \sum_{n=2}^{\infty} b_n \\ &\leqq \sum_{n=2}^{\infty} b_n [(n-1)k + n - \beta] \phi_n^\tau(\alpha_1, \lambda, l, m) \\ &< 1 - \beta.\end{aligned}$$

We thus find that

$$\sum_{n=2}^{\infty} b_n < \frac{1-\beta}{(2+k-\beta)\phi_2^\tau(\alpha_1, \lambda, l, m)},$$

which proves Corollary 2. □

Corollary 3. *If $g(z) \in \mathcal{US}^-_{l,m}(\tau, \lambda, k, \beta)$ is given by (2.3), then*

$$b_n < \frac{1-\beta}{(2+k-\beta) a_n \phi_2^\tau(\alpha_1, \lambda, l, m)}.$$

3. Further Results and Consequences

In this section, several further results involving the various function classes which were introduced in Section 1.

Theorem 3. *If $g(z) \in \mathcal{US}^-_{l,m}(\tau, \lambda, k, \beta)$, then*

$$L^{\tau;\alpha_1}_{\lambda,j,m} g(z) = \exp\left(\int_0^z \frac{k - \beta Q(t)}{t[k - Q(t)]} dt\right) \qquad \big(|Q(z)| < 1;\ z \in \mathbb{U}\big) \tag{3.1}$$

and

$$L^{\tau;\alpha_1}_{\lambda,j,m} g(z) = \exp\left(\int_{|x|=1} \log\left[(k - xz)^{-1-\beta}\right] d\mu(x)\right), \tag{3.2}$$

where $\mu(x)$ is a probability measure on the set:

$$X = \{x : |x| = 1\}.$$

Proof. The case $k = 0$ of the assertion (3.1) if Theorem 3 is obvious. Let $k \neq 0$. Then, for

$$g(z) \in US^{-}_{l,m}(k, \beta) \quad \text{and} \quad \omega = \frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m} g(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)},$$

we have

$$\Re(\omega) > k|\omega - 1| + \beta.$$

We thus find that

$$\left|\frac{\omega - 1}{\omega - \beta}\right| < \frac{1}{k} \quad \text{and} \quad \frac{\omega - 1}{\omega - \beta} = \frac{Q(z)}{k} \qquad \big(|Q(z)| < 1;\ z \in \mathbb{U}\big),$$

which readily yields

$$\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m} g(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)} = \frac{k - \beta Q(z)}{z[k - Q(z)]}$$

and, therefore,

$$L^{\tau,\alpha_1}_{\lambda,j,m} g(z) = \exp\left(\int_0^z \frac{k - \beta Q(t)}{t[k - Q(t)]} dt\right).$$

In order to derive the second representation (3.2), corresponding to the set:

$$X = \{x : |x| = 1\},$$

we observe that

$$\frac{\omega - 1}{\omega - \beta} < \frac{1}{k} xz$$

or, equivalently, that

$$\frac{z\big(L^{\tau,\alpha_1}_{\lambda,j,m} g(z)\big)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)} = \frac{k - \beta Q(z)}{z[k - Q(z)]}$$

$$\Longrightarrow \log\left(\frac{H^l_m[\alpha_1] g(z)}{z}\right) = -(1 + \beta) \log(k - xz).$$

Thus, if $\mu(x)$ is the probability measure on X, then

$$L^{\tau,\alpha_1}_{\lambda,j,m} g(z) = \exp\left(\int_{|x|=1} \log\left[(k - xz)^{-1-\beta}\right] d\mu(x)\right).$$

□

Theorem 4. *If $f(z) \in \mathcal{UC}^{-}_{l,m}(\tau, \lambda, k, \gamma, \beta)$, then*

$$L^{\tau,\alpha_1}_{\lambda,j,m} f(z) = \int_0^z \left[\frac{k - \gamma Q(t)}{k - Q(t)} \ \exp\left(\int_{|x|=1} \log\left[(k - xt)^{-1-\beta}\right] d\mu(x)\right)\right] dt, \quad (3.3)$$

where $\mu(x)$ is a probability measure on the following set:

$$X = \{x : |x| = 1\}.$$

Proof. The case $k = 0$ of the assertion (3.3) of Theorem 4 is obvious. Let $k \neq 0$. Then, for

$$f \in \mathcal{UC}^{-}_{l,m}(\tau, \lambda, k, \beta) \quad \text{and} \quad \omega = \frac{z\left(L^{\tau,\alpha_1}_{\lambda,j,m} f(z)\right)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)},$$

we have

$$\Re(\omega) > k|\omega - 1| + \gamma.$$

We thus find that

$$\left|\frac{\omega - 1}{\omega - \gamma}\right| < \frac{1}{k} \quad \text{and} \quad \frac{\omega - 1}{\omega - \gamma} = \frac{Q(z)}{k} \qquad \big(|Q(z)| < 1;\ z \in \mathbb{U}\big),$$

which easily yields

$$\frac{z\left(L^{\tau,\alpha_1}_{\lambda,j,m} f(z)\right)'}{L^{\tau,\alpha_1}_{\lambda,j,m} g(z)} = \frac{k - \gamma Q(z)}{z[k - Q(z)]}. \tag{3.4}$$

Moreover, from Theorem 3, we have

$$L^{\tau,\alpha_1}_{\lambda,j,m} g(z) = \exp\left(\int_{|x|=1} \log\left[(k - xz)^{-1-\beta}\right] d\mu(x)\right), \tag{3.5}$$

where $\mu(x)$ is a probability measure on the set:

$$X = \{x : |x| = 1\}.$$

The assertion (3.3) of Theorem 4 would now follow from (3.4) and (3.5). □

Next we obtain a distortion bounds for the functions $f(z)$ and $g(z)$.

Theorem 5. *If* $g(z) \in \mathcal{US}^{-}_{l,m}(\tau, \lambda, k, \beta)$, *then*

$$|z| - \frac{1-\beta}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z|^2 < |g(z)| < |z| + \frac{1-\beta}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z|^2 \qquad (z \in \mathbb{U}) \tag{3.6}$$

and

$$1 - \frac{2(1-\beta)}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z| < |g'(z)| < 1 + \frac{2(1-\beta)}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z| \qquad (z \in \mathbb{U}). \tag{3.7}$$

Proof. For $g(z) \in \mathcal{US}^{-}_{l,m}(\tau, \lambda, k, \beta)$ given by (2.3), we find from Corollary 2 that

$$\sum_{n=2}^{\infty} b_n < \frac{1-\beta}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}, \tag{3.8}$$

which implies that

$$|g(z)| \leqq |z| + |z|^2 \sum_{n=2}^{\infty} b_n < |z| + \frac{1-\beta}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z|^2 \qquad (z \in \mathbb{U})$$

and

$$|g(z)| \geqq |z| - |z|^2 \sum_{n=2}^{\infty} b_n > |z| - \frac{1-\beta}{(2+k-\beta)\phi^{\tau}_2(\alpha_1, \lambda, l, m)}\ |z|^2 \qquad (z \in \mathbb{U}).$$

Thus the assertion (3.6) of Theorem 5 follows at once.

In a similar manner, for the derivative $g'(z)$, the following inequalities:

$$|g'(z)| \leqq 1 + \sum_{n=2}^{\infty} n b_n |z|^{n-1} < 1 + |z| \sum_{n=2}^{\infty} n b_n \qquad (z \in \mathbb{U})$$

and

$$\sum_{n=2}^{\infty} n b_n < \frac{2(1-\beta)}{(2+k-\beta)\phi_2^{\tau}(\alpha_1, \lambda, l, m)}$$

lead us immediately to the assertion (3.7) of Theorem 5. This completes the proof of Theorem 5. □

Theorem 6. *If* $f \in \mathcal{UC}_{l,m}^{-}(\tau, \lambda, k, \gamma, \beta)$, *then*

$$|z| - \frac{1-\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right) |z|^2 < |f(z)|$$
$$< |z| + \frac{1-\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right) |z|^2 \qquad (z \in \mathbb{U}) \quad (3.9)$$

and

$$1 - \frac{1-\gamma}{(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right) |z| < |f'(z)|$$
$$< 1 + \frac{1-\gamma}{(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \left[1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right] |z| \qquad (z \in \mathbb{U}). \quad (3.10)$$

Proof. For $f \in \mathcal{UC}_{l,m}^{-}(\tau, \lambda, k, \gamma, \beta)$ given by (1.2), by using Theorem 1, we obtain

$$2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m) \sum_{n=2}^{\infty} a_n < \sum_{n=2}^{\infty} n(1+k) a_n \phi_n^{\tau}(\alpha_1, \lambda, l, m)$$
$$< 1 - \gamma + \sum_{n=2}^{\infty} (k+\gamma) b_n \phi_n^{\tau}(\alpha_1, \lambda, l, m), \qquad (3.11)$$

which immediately yields

$$\sum_{n=2}^{\infty} a_n < \frac{1-\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)}$$
$$+ \frac{k+\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \sum_{n=2}^{\infty} b_n \phi_n^{\tau}(\alpha_1, \lambda, l, m). \qquad (3.12)$$

Also, by applying Corollary 1, we have

$$\sum_{n=2}^{\infty} b_n \phi_n^{\tau}(\alpha_1, \lambda, l, m) < \frac{1-\beta}{2+k-\beta},$$

so that

$$|f(z)| \leqq |z| + |z|^2 \sum_{n=2}^{\infty} a_n$$
$$< |z| + \frac{1-\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1, \lambda, l, m)} \left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right) |z|^2 \qquad (z \in \mathbb{U}).$$

Similarly, we can show that

$$\begin{aligned}|f(z)| &\geqq |z| - |z|^2 \sum_{n=2}^{\infty} a_n \\ &> |z| - \frac{1-\gamma}{2(1+k)\phi_2^{\tau}(\alpha_1,\lambda,l,m)} \left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right)|z|^2 \qquad (z \in \mathbb{U}).\end{aligned}$$

We thus have proved the assertion (3.9) of Theorem 6.

In a similar manner, for the derivative $f'(z)$, the following inequalities:

$$|f'(z)| \leqq 1 + \sum_{n=2}^{\infty} n a_n |z|^{n-1} < 1 + |z| \sum_{n=2}^{\infty} n a_n \qquad (z \in \mathbb{U})$$

and

$$\sum_{n=2}^{\infty} n a_n < \frac{1-\gamma}{(1+k)\phi_2^{\tau}(\alpha_1,\lambda,l,m)} \left[1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right]$$

lead us to the assertion (3.12) of Theorem 6. This evidently completes the proof of Theorem 6. □

It is not difficult to deduce Corollary 4 below.

Corollary 4. *Let* $f \in \mathcal{UC}_{l,m}^{-}(\tau,\lambda,k,\gamma,\beta)$. *Then*

$$\begin{aligned}&\left\{\omega : |\omega| < 1 - \frac{1-\gamma}{(1+k)\phi_2^{\tau}(\alpha_1,\lambda,l,m)}\left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right)\right\} \subset f(\mathbb{U}) \\ &\quad \subset \left\{\omega : |\omega| < 1 + \frac{1-\gamma}{(1+k)\phi_2^{\tau}(\alpha_1,\lambda,l,m)}\left(1 + \frac{(k+\gamma)(1-\beta)}{(1-\gamma)(2+k-\beta)}\right)\right\}. \qquad (3.13)\end{aligned}$$

Theorem 7 below follows easily from Corollary 1. In fact, the proof of Theorem 7 is essentially analogous to that of Theorem 8, which we have chosen to present here in detail.

Theorem 7. *Let*

$$g_m(z) = z - \sum_{n=2}^{\infty} b_{j,m} z^j \in \mathcal{US}_{l,m}^{-}(\tau,\lambda,k,\gamma,\beta) \qquad (m = 1, 2).$$

Then

$$\begin{aligned}g(z) &= (1-\xi)g_1(z) + \xi g_2(z) = z - \sum_{j=2}^{\infty} b_j z^j \\ &\in \mathcal{US}_{l,m}^{-}(\tau,\lambda,k,\gamma,\beta) \qquad (0 \leqq \xi \leqq 1). \qquad (3.14)\end{aligned}$$

Theorem 8. *Let*

$$f_m(z) = z - \sum_{n=2}^{\infty} a_{j,m} z^j \in \mathcal{UC}_{l,m}^{-}(\tau,\lambda,k,\gamma,\beta) \qquad (m = 1, 2).$$

Then

$$f(z) = (1-\xi)f_1(z) + \xi f_2(z) = z - \sum_{j=2}^{\infty} a_j z^j$$
$$\in \mathcal{UC}_{l,m}^{-}(\tau, \lambda, k, \gamma, \beta) \qquad (0 \leqq \xi \leqq 1). \tag{3.15}$$

Proof. Since

$$f_m(z) \in \mathcal{UC}_{l,m}^{-}(\tau, \lambda, k, \gamma, \beta) \qquad (m = 1, 2),$$

by using Theorem 2, we get the following coefficient inequalities:

$$\sum_{j=2}^{\infty}[(1+k)ja_{j,1}\phi_j^{\tau}(\alpha_1, \lambda, l, m) - (k+\gamma)b_{j,1}\phi_j^{\tau}(\alpha_1, \lambda, l, m)] < 1-\gamma$$

and

$$\sum_{j=2}^{\infty}[(1+k)ja_{j,2}\phi_j^{\tau}(\alpha_1, \lambda, l, m) - (k+\gamma)b_{j,2}\phi_j^{\tau}(\alpha_1, \lambda, l, m)] < 1-\gamma.$$

Furthermore, in view of the following obvious relationships:

$$a_j = (1-\xi)a_{j,1} + \xi a_{j,2} \quad \text{and} \quad b_j = (1-\xi)b_{j,1} + \xi b_{j,2}$$
$$(j \in \mathbb{N} \setminus \{1\};\ 0 \leqq xi \leqq 1),$$

we thus find that

$$\begin{aligned}
&\sum_{j=2}^{\infty} \left[(1+k)ja_j\phi_j^{\tau}(\alpha_1, \lambda, l, m) - (k+\gamma)b_j\phi_j^{\tau}(\alpha_1, \lambda, l, m)\right] \\
&= \sum_{j=2}^{\infty}(1+k)j\phi_j^{\tau}(\alpha_1, \lambda, l, m)\left[(1-\xi)a_{j,1}(z) + \xi a_{j,2}(z)\right] \\
&\quad - \sum_{j=2}^{\infty}(k+\gamma)b_j\phi_j^{\tau}(\alpha_1, \lambda, l, m)\left[(1-\xi)b_{j,1}(z) + \xi b_{j,2}(z)\right] \\
&= \sum_{j=2}^{\infty}(1-\xi)\left[(1+k)ja_{j,1}\phi_j^{\tau}(\alpha_1, \lambda, l, m) - (k+\beta)b_{j,1}\phi_j^{\tau}(\alpha_1, \lambda, l, m)\right] \\
&\quad + \sum_{j=2}^{\infty}\xi[(1+k)ja_{j,2}\phi_j^{\tau}(\alpha_1, \lambda, l, m) - (k+\gamma)b_{j,2}\phi_j^{\tau}(\alpha_1, \lambda, l, m)] \\
&\leqq (1-\xi)(1-\gamma) + \xi(1-\gamma) = 1-\gamma.
\end{aligned}$$

Thus, by using Theorem 2 again, we finally obtain

$$f(z) \in \mathcal{UC}_{l,m}^{-}(\tau, \lambda, k, \gamma, \beta),$$

which completes the proof of Theorem 8. □

We remark in conclusion that, by suitably specializing the parameters involved in the results presented in this paper, we can deduce numerous *further* corollaries and consequences of each of these results.

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