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ON DIFFERENCE FUZZY ANTI λ-IDEAL CONVERGENT DOUBLE SEQUENCE SPACES

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Abstract. The concept of fuzzy sets was introduced by Zadeh as a means of representing data that was not precise but rather fuzzy. Recently, Kočinac [24] studied some topological properties of fuzzy antinormed linear spaces. This has motivated us to introduce and study the fuzzy antinormed double sequence spaces with respect to ideal by using a difference operator Δ^n and prove some theorems, in particular convergence and completeness theorems on these new double sequence spaces.

1. Introduction

Fuzzy set theory was formalised by Professor Lofti Zadeh [34] at the University of California in 1965. Thereafter, fuzzy set theory found applications in different areas of mathematics and in other fields. The concept of fuzzy norm was introduced by Katsaras [13] in 1984. In 1992, by using fuzzy numbers, Felbin [11] introduced the fuzzy norm on a linear space. Cheng and Mordeson [3] introduced another idea of fuzzy norm on a linear space, and in 2003 Bag and Samanta [1] modified the definition of fuzzy norm of Cheng-Mordeson [3]. In [2] a comparative study of the fuzzy norms defined by Katsaras [13], Felbin [11] and Bag and Samanta [1] was given.

Later on, Jebril and Samanta [12] introduced the concept of fuzzy anti-norm on a linear space depending on the idea of fuzzy anti norm, introduced by Bag and Samanta [2]. The motivation of introducing fuzzy anti-norm is to study fuzzy set theory with respect to the non-membership function. It is useful in the process of decision making. Moreover, in 1981, the idea of difference sequence spaces was introduced by Kizmaz (see[9]). Malkowsky et al.[31] introduced the difference sequence spaces of order m. The generalized difference ideal convergence of real sequences was introduced and studied by Hazarika[8] and Gumus and Nuray[7] independently. Recently[10], Hazarika introduced the concept of generalized difference ideal convergence in random 2-normed spaces.

The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast[6] and Schoenberg[33]. There has

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been an effort to introduce several generalizations and variants of statistical convergence in different spaces (see, for example, [5],[16],[17],[21],[22],[23] and references therein). One such very important generalization of this notion was introduced by Kostyrko et al. [26] by using an ideal I of subsets of the set of natural numbers, which they called I-convergence. After that the idea of I-convergence for double sequence was introduced by Das et al. [4] in 2008 (see also [30], [25],[20],[18],[19] for ideal convergence in fuzzy context).

Now, we recall some terms and definitions which will be used throughout the article.

Let X be a non empty set. A family $I \subset 2^X$ is said to be an **ideal** in X if $\emptyset \in I$, I is additive i.e for all $A, B \in I \Rightarrow A \cup B \in I$ and I is hereditary i.e for all $A \in I, B \subseteq A \Rightarrow B \in I$ [14, 15]. A non empty family of sets $\mathcal{F} \subset 2^X$ is said to be a **filter** on X if for all $A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$ and for all $A \in \mathcal{F}$ with $A \subseteq B$ implies $B \in \mathcal{F}$. An ideal $I \subset 2^X$ is said to be **non trivial** if $I \neq 2^X$; a non trivial ideal is said to be admissible if $I \supseteq \{\{x\} : x \in X\}$ and is said to be **maximal** if there cannot exist any non trivial ideal $J \neq I$ containing I as a subset. For each ideal I there is a filter $\mathcal{F}(I)$ called the filter associate with ideal I, that is

$$\mathcal{F}(I) = \{K \subseteq X : K^c \in I\}, \text{ where } K^c = X \backslash K.$$

Throughout the article, I is an admissible ideal on $\mathbb{N} \times \mathbb{N}$, and ${}_2\omega$ denotes the class of all double real sequences. The spaces ${}_2l_\infty$, ${}_2c$ and ${}_2c_0$ are the Banach spaces of bounded, convergent, and null double sequences of reals respectively with the norm

$$\|x\| = \sup_{i,j \in \mathbb{N}} |x_{ij}|. \tag{1.1}$$

Definition: 1.1 [28, 29] A double sequence $x = (x_{ij}) \in {}_2\omega$ is said to be I-convergent to a number L, if for every $\epsilon > 0$

$$\{(i,j) : |x_{ij} - L| \geq \epsilon\} \in I. \tag{1.2}$$

In this case, we write $I - \lim x_{ij} = L$.

Definition: 1.2 [28, 29] A double sequence $(x_{ij}) \in {}_2\omega$ is said to be $I-$Cauchy if for every $\epsilon > 0$, there exist $(m, n) \in \mathbb{N} \times \mathbb{N}$ such that

$$\{(i,j) : |x_{ij} - x_{mn}| \geq \epsilon\} \in I. \tag{1.3}$$

Definition: 1.3 [28, 29] A double sequence $(x_{ij}) \in {}_2\omega$ is said to be I-bounded if there exists $M > 0$ such that

$$\{(i,j) : |x_{ij}| > M\} \in I. \tag{1.4}$$

Definition: 1.4 [27, 32] A binary operation $\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:
(a) $\diamond$ is associative and commutative,
(b) $\diamond$ is continuous,
(c) $a \diamond 0 = a$ for all $a \in [0,1]$,
(d) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.
Some examples of continuous t-conorm are:
(i) $a \diamond b$= a+b-ab (ii) $a \diamond b$= max {a,b} (iii) $a \diamond b$= min {a+b, 1}.

Remark. *(a) For any* $r_1, r_2 \in (0,1)$ *with* $r_1 > r_2$ *, there exist* $r_3 \in (0,1)$ *such that* $r_1 > r_4 \diamond r_2$.
(b) For any $r_4 \in (0,1)$*, there exist* $r_5 \in (0,1)$ *such that and* $r_5 \diamond r_5 \leq r4$.

Recall now the notion of fuzzy antinorm in a linear space with respect to a continuous t-conorm following.

Definition: 1.5[24] Let X be a real linear space and $\diamond$ a t-conorm. A fuzzy subset $\nu : X \times \mathbb{R} \to \mathbb{R}$ of $X \times \mathbb{R}$ is called a fuzzy antinorm on X with respect to the t-conorm if, for all $x, y \in X$

(FaN1) for each $t \in (-\infty, 0]$, $\nu(x,t) = 1$;
(FaN2) for each $t \in (0,\infty), \nu(x,t) = 0$ if and only if $x = \theta$;
(FaN3) for each $t \in (0,\infty), \nu(\alpha x) = \nu(x, |\alpha|)$ if $\alpha \neq 0$;
(FaN4) for all $s, t \in \mathbb{R}, \nu(x+y, s+t) \leq \nu(x,s) \diamond \nu(y,t)$;
(FaN5) $\lim_{t\to\infty} \nu(x,t) = 0$.

Note that if ν is the antinorm in the definition above, then $\nu(x,t)$ is nonincreasing with respect to t for each $x \in X$. The followings are examples of fuzzy antinorms with respect to a corresponding t-conorm and show how a fuzzy antinorm can be obtained from a norm.

Example: 1.1 Let $(X, \|.\|)$ be a normed linear space and let the t-conorm $\diamond$ be given by $a \diamond b = a + b - ab$. Define $\nu : X \times \mathbb{R} \to [0,1]$ by

$$\nu(x,t) = \begin{cases} 0, & \text{if } t > \|x\| \\ 1, & \text{if } t \leq \|x\|. \end{cases}$$

Then ν is a fuzzy antinorm on X with respect to the t-conorm $\diamond$. This antinorm ν satisfies also the following:

(FaN6) For each $t > 0, \nu(x,t) < 1$ implies $x = \theta$.

Example: 1.2 Let $(X, \|.\|)$ be a normed linear space and consider the t-conorm $\diamond$ defined by $a \diamond b = \min\{a+b, 1\}$. Define $\nu : X \times \mathbb{R} \to [0,1]$ by

$$\nu(x,t) = \begin{cases} \frac{\|x\|}{2t - \|x\|}, & \text{if } t > \|x\| \\ 1, & \text{if } t \leq \|x\|. \end{cases}$$

Then ν is a fuzzy antinorm on X with respect to the t-norm $\diamond$. Note that this ν satisfies the condition (FaN6) and also the following:

(FaN7) $\nu(x,.)$ is a continuous function on $\mathbb{R}$ and strictly decreasing on the subset $\{t : 0 < \nu(x,t) < 1\}$ of $\mathbb{R}$.

Definition: 1.6 [24] A sequence $(x_n)_{n\in\mathbb{N}}$ in a fuzzy antinormed linear space $(X, \nu, \diamond)$ is said to be ν-convergent to a point $x \in X$ if for each $\epsilon > 0$ and each $t > 0$ there is $n_0 \in \mathbb{N}$ such that

$$\nu(x_n - x, t) < \epsilon \text{ for each } n \geq n_0 \tag{1.5}$$

Let $(X, \nu, \diamond)$ be a fuzzy antinormed linear space with respect to an idempotent t-conorm $\diamond$, and let ν satisfy (FaN6). Then for each $\lambda \in (0,1)$ the function $\|x\|_\lambda : X \to [0,\infty)$ defined by

$$\|x\|_\lambda = \{t > 0 : \nu(x,t) \leq 1 - \lambda\} \tag{1.6}$$

is a norm on X and $\varphi = \{\|x\|_\lambda : \lambda \in (0,1)\}$ is an asscending family of norms on X. In this paper we generalize the definition of fuzzy anti-norm on a linear space. Later on we study some relations and results on them.

2. Fuzzy(anti) $\Delta^n I_\lambda$- convergence

Now, in this section we define fuzzy $\Delta^n I_\lambda$-convergence, fuzzy $\Delta^n I_\lambda$- anti- convergence, fuzzy $\Delta^n I_\lambda$- anti-Cauchy and fuzzy $\Delta^n I_\lambda$- completeness for double sequences with respect to an ideal I on $\mathbb{N} \times \mathbb{N}$.

Definition: 2.1 Let X be a fuzzy antinormed double sequence space. A sequence (x_{ij}) is said to be fuzzy $\Delta^n I_\nu$-convergent to a point $x \in X$ if for each $\epsilon > 0$ and each $t > 0$ the set

$$\{(i,j) : \nu(\Delta^n x_{ij} - x, t) < \epsilon\} \in I. \tag{2.1}$$

where,
$\Delta^n x_{i,j} = (\Delta^{n-1} x_{i,j} - \Delta^{n-1} x_{i,j+1} - \Delta^{n-1} x_{i+1,j} + \Delta^{n-1} x_{i+1,j+1})$
$(\Delta^1 x_{i,j}) = (\Delta x_{i,j}) = (x_{i,j} - x_{i,j+1} - x_{i+1,j} + x_{i+1,j+1}), \Delta^0 x = (x_{i,j})$
and this generalized difference double notion has the following binomial representation:

$$\Delta^n x_{i,j} = \sum_{k=0}^{n} \sum_{l=0}^{n} (-1)^{k+l} \binom{n}{k} \binom{n}{l} x_{i+k,j+l}.$$

In this case, we write fuzzy $I_\nu - \lim \Delta^n x_{ij} = x$ and x is called a fuzzy $\Delta^n I_\nu$-limit of (x_{ij}).

Definition: 2.2 Let X be a fuzzy antinormed double sequence space and $\lambda \in (0,1)$. A sequence $(x_{ij}) \in X$ is said to be fuzzy $\Delta^n I_\lambda$-convergent to $x \in X$ if for all $t > 0$, the set

$$\{(i,j) : \nu(\Delta^n x_{ij} - x, t) < 1 - \lambda\} \in I. \tag{2.2}$$

In this case we write fuzzy $I_\lambda - \lim \nu(\Delta^n x_{ij} - x, t) = 0$ and x is called a fuzzy I_λ-limit of $(\Delta^n x_{ij})$.

Definition: 2.3 Let X be a fuzzy antinormed double sequence space and $\lambda \in (0,1)$. A sequence $(x_{ij}) \in X$ is said to be fuzzy $\Delta^n I_\lambda$-anti-convergent in X if there exist $x \in X$ and $M \in \mathcal{F}(I)$ such that for all $t > 0$,

$$M = \{(i,j) : \nu(\Delta^n x_{ij} - x, t) < 1 - \lambda\}. \tag{2.3}$$

In this case, we write $(\Delta^n x_{ij})$ anti-convergent to x and x is called a fuzzy I_λ-anti-limit of $(\Delta^n x_{ij})$.

Definition: 2.4 Let $\lambda \in (0,1)$. A sequence (x_{ij}) in a fuzzy antinormed double sequence space X is said to be fuzzy $\Delta^n I_\lambda$-anti-Cauchy if there exist numbers $m, n \in \mathbb{N}$ and $\mathcal{S} \in \mathcal{F}(I)$ such that for all $t > 0$,

$$\mathcal{S} = \{(i,j) : \nu(\Delta^n x_{ij} - \Delta^n x_{pq}, t) < 1 - \lambda\}. \tag{2.4}$$

Definition: 2.5 A fuzzy antinormed double sequence space X is said to be fuzzy $\Delta^n I_\lambda$-anti-complete, $\lambda \in (0,1)$, if for every fuzzy $\Delta^n I_\lambda$-anti-Cauchy sequence in X

is fuzzy $\Delta^n I_\lambda$- anti-convergent in X.

Now, here we define two fuzzy antinormed double difference sequence spaces by using operator Δ^n as follows:

$$_2\mathcal{F}_\nu^I(\Delta^n) = \{(x_{ij}) \in {}_2\ell_\infty : (i,j) : \nu(\Delta^n x_{ij} - x, t) < \epsilon\}; \tag{2.5}$$

$$_2\mathcal{F}_{0\nu}^I(\Delta^n) = \{(x_{ij}) \in {}_2\ell_\infty : (i,j) : \nu(\Delta^n x_{ij}, t) < \epsilon\}. \tag{2.6}$$

It is easy to check that these are really fuzzy antinormed double difference sequence spaces defined by Δ^n as difference operator. We also define an open ball with centre x and radius r with respect to t as follows:

$$_2\mathcal{B}_x^{\Delta^n}(r,t) = \{(y_{ij}) \in {}_2\ell_\infty : (i,j) : \nu(\Delta^n x_{ij} - \Delta^n y_{ij}, t) < r\}. \tag{2.7}$$

Theorem 2.1. *In the fuzzy antinormed double difference sequence space $_2\mathcal{F}_\nu^I(\Delta^n)$ with respect to an idempotent t-conorm $\diamond$ satisfying (FaN6) and (FaN7) a sequence is $\Delta^n I_\nu$-convergent if and only if it is $\Delta^n I_\lambda$-convergent for each $\lambda \in (0,1)$.*

Proof. Let (x_{ij}) be a sequence in $_2\mathcal{F}_\nu^I(\Delta^n)$ such that (x_{ij}) is $\Delta^n I_\nu$- convergent to x, i.e., for each $t > 0$

$$I_\nu - \lim_{i,j\to\infty} \nu(\Delta^n x_{ij} - x, t) = 0. \tag{2.8}$$

Fix $\lambda \in (0,1)$. So, $I_\nu - \lim_{i,j\to\infty} \nu(\Delta^n x_{ij} - x, t) = 0 < 1 - \lambda$. There exists a set $P \in I$ such that for each $(m,n) \in P$,

$$\nu(\Delta^n x_{mn} - x, t) < 1 - \lambda \tag{2.9}$$

Since $\|\Delta^n x_{mn} - x\|_\lambda = \varphi\{t > 0 : \nu(\Delta^n x_{mn} - x, t) \le 1 - \lambda\}$, we have $\|\Delta^n x_{mn} - x\|_\lambda \le t$ for all $(m,n) \in P$. As $t > 0$, for each $\lambda \in (0,1)$, by (FaN6), we have $\|\Delta^n x_{mn} - x\|_\lambda$ I- converges to 0.

Conversely, suppose now that for each $\lambda \in (0,1)$, $\|\Delta^n x_{ij} - x\|_\lambda$ I-converges to 0. This means that for each $\lambda \in (0,1)$ and each $\epsilon > 0$ there is a set $P_\lambda \in I$ such that, for each $(i,j) \in P$

$$\|\Delta^n x_{ij} - x\|_\lambda \le \epsilon. \tag{2.10}$$

Therefore,

$$\nu(\Delta^n x_{ij} - x, \epsilon) = \varphi\{1 - \lambda : \|\Delta^n x_{ij} - x\|_\lambda \le \epsilon\} \tag{2.11}$$

implies $\nu(\Delta^n x_{ij} - x, \epsilon) \le 1 - \lambda$ for each $\lambda \in (0,1)$ and each $(i,j) \in P$, which means

$$I_\nu - \lim \nu(\Delta^n x_{ij} - x, \epsilon) = 0 \tag{2.12}$$

that is, (x_{ij}) is $\Delta^n I_\nu$-convergent to x as $i,j \to \infty$. □

Theorem 2.2. *Let $_2\mathcal{F}_\nu^I(\Delta^n)$ be a fuzzy antinormed double difference sequence space with respect to an idempotent t-conorm $\diamond$ satisfying (FaN6). Then fuzzy $\Delta^n I_\lambda$-anti-limit of a fuzzy $\Delta^n I_\lambda$-anti-convergent sequence is unique.*

Proof. Let $(x_{ij}) \in {}_2\mathcal{F}_\nu^I(\Delta^n)$ be fuzzy $\Delta^n I_\lambda$- anti convergent double sequence anti-converging two distinct points x and y in $_2\mathcal{F}_\nu^I(\Delta^n)$. This means that for each $t > 0$, there exist $x, y \in X$ and $A_1, A_2 \in \mathcal{F}(I)$ such that

$$A_1 = \{(i,j) : \nu(\Delta^n x_{ij} - x, t) < 1 - \lambda\}; \tag{2.13}$$

$$A_2 = \{(i,j) : \nu(\Delta^n x_{ij} - y, t) < 1 - \lambda\}. \tag{2.14}$$

The set $A = A_1 \cap A_2 \in \mathcal{F}(I)$ and by the assumption on $\diamond$ for each $(i,j) \in A$, we have

$$\begin{aligned}\nu(x-y,t) &\le \nu(\Delta^n x_{ij} - x, t) \diamond \nu(\Delta^n x_{ij} - y, t)\\ &< (1-\lambda) \diamond (1-\lambda) = 1-\lambda.\end{aligned}$$

So we have

$$\begin{aligned}\{(i,j) : \nu(\Delta^n(x-y),t) < 1-\lambda\} \supseteq \{(i,j) : \nu(\Delta^n(x_{ij}-x),t) < 1-\lambda\}\\ \cap \{(i,j) : \nu(\Delta^n(x_{ij}-y),t) < 1-\lambda\} \qquad (2.15)\end{aligned}$$

Thus, the sets on right hand side of the above equation (21) belong to $\mathcal{F}(I)$. Therefore,
$\nu(\Delta^n(x-y),t) < 1-\lambda$ for each $t > 0$ by $(FaN6)$ one obtains $x - y = \theta$ i.e., $x = y$. □

Theorem 2.3. *Let ${}_2\mathcal{F}^I_\nu(\Delta^n)$ and ${}_2\mathcal{F}^I_{0\nu}(\Delta^n)$ be a fuzzy antinormed double difference sequence spaces with respect to an idempotent t-conorm $\diamond$ satisfying (FaN6). Then*

1. *if $I_\lambda - anti - \lim \Delta^n x_{ij} = x$ and $I_\lambda - anti - \lim \Delta^n y_{ij} = y$, then $I_\lambda - anti - \lim \Delta^n(x_{ij} + y_{ij}) = x + y$*
2. *if $I_\lambda - anti - \lim \Delta^n x_{ij} = x$ and $r \in \mathbb{R}$, then $I_\lambda - anti - \lim r(\Delta^n x_{ij}) = rx$.*

Proof. Since $I_\lambda - anti - \lim \Delta^n x_{ij} = x$ and $I_\lambda - anti - \lim \Delta^n y_{ij} = y$, there exist $M_1, M_2 \in \mathcal{F}(I)$ such that for all $t > 0$,

$$M_1 = \{(i,j) : \nu(\Delta^n x_{ij} - x, \frac{t}{2}) < 1-\lambda\}; \qquad (2.16)$$

$$M_2 = \{(i,j) : \nu(\Delta^n y_{ij} - y, \frac{t}{2}) < 1-\lambda\}. \qquad (2.17)$$

The set $M = M_1 \cap M_2 \in \mathcal{F}(I)$ and by the assumption on $\diamond$ for each $(i,j) \in M$, we have

$$\begin{aligned}\nu(\Delta^n(x_{ij}+y_{ij}) - \Delta^n(x+y),t) &\le \nu(\Delta^n(x_{ij}-x),\frac{t}{2}) \diamond \nu(\Delta^n(y_{ij}-y),\frac{t}{2})\\ &< (1-\lambda) \diamond (1-\lambda) = 1-\lambda.\end{aligned}$$

So we have

$$\begin{aligned}\{(i,j) : \nu(\Delta^n(x_{ij}+y_{ij}) - (x+y),t) < 1-\lambda\} \supseteq \{(i,j) : \nu(\Delta^n(x_{ij}-x),\tfrac{t}{2}) < 1-\lambda\}\\ \cap \{(i,j) : \nu(\Delta^n(x_{ij}-y),\frac{t}{2}) < 1-\lambda\} \qquad (2.18)\end{aligned}$$

Thus, the sets on right hand side of the above equation (24) belong $\mathcal{F}(I)$. So we have $M = \{(i,j) : \nu(\Delta^n(x_{ij}+y_{ij}) - (x+y),t) < 1-\lambda\} \notin I$ which means that $I_\lambda - anti - \lim \Delta^n(x_{ij}+y_{ij}) = x + y$.

(2) The fact $I_\lambda - anti - \lim \Delta^n x_{ij} = x$ implies that there exists $M \in \mathcal{F}(I)$ such that for all $t > 0$ we have

$$M = \{(i,j) : \nu(\Delta^n x_{ij} - x, t) < 1-\lambda\} \in \mathcal{F}(I). \qquad (2.19)$$

Therefore, for each $(i,j) \in M$, we have

$$\nu(r(\Delta^n x_{ij}) - r(\Delta^n x),t) = \nu(\Delta^n x_{ij} - x, \frac{t}{|r|}) < 1-\lambda.$$

We have

$$\{(i,j) : \nu(r\Delta^n x_{ij} - r\Delta^n x, t) < 1-\lambda\} \supseteq \{(i,j) : \nu(\Delta^n x_{ij} - \Delta^n x, t) < 1-\lambda\} \quad (2.20)$$

From this we get,$\{(i,j) : \nu(r\Delta^n x_{ij} - r\Delta^n x, t) \geq 1-\lambda\} \notin I$ which shows that $I_\lambda - anti - \lim\ r(\Delta^n x_{ij}) = rx$.

□

Theorem 2.4. *Let ${}_2\mathcal{F}_\nu^I(\Delta^n)$ be a fuzzy antinormed double difference sequence space with respect to an idempotent t-conorm $\diamond$. If $(x_{ij}) \in {}_2\mathcal{F}_\nu^I(\Delta^n)$ is I_λ-anti-convergent to $x \in {}_2\mathcal{F}_\nu^I(\Delta^n)$, then $\|\Delta^n x_{ij} - x\|_\lambda$ is I-convergent to 0.*

Theorem 2.5. *Let ${}_2\mathcal{F}_\nu^I(\Delta^n)$ be a fuzzy antinormed double difference sequence space with respect to an idempotent t-conorm $\diamond$ satisfying (FaN6) and $\lambda \in (0,1)$. Then every fuzzy $\Delta^n I_\lambda$-anti-convergent double sequence $(x_{ij}) \in {}_2\mathcal{F}_\nu^I(\Delta^n)$ is fuzzy $\Delta^n I_\lambda$-anti-cauchy.*

Proof. Let $(x_{ij}) \in {}_2\mathcal{F}_\nu^I(\Delta^n)$ be fuzzy $\Delta^n I_\lambda$- anti-convergent double sequence. This shows that there exists $\mathcal{S} \in \mathcal{F}(I)$ such that for all $t > 0$ we have

$$\{(i,j) : \nu(\Delta^n x_{ij} - x, \frac{t}{2}) < 1-\lambda\} \in \mathcal{F}(I). \tag{2.21}$$

Therefore for each $(i,j),(m,n) \in M$, we have

$$\nu(\Delta^n x_{ij} - \Delta^n x_{mn}, t) \leq \nu(\Delta^n x_{ij} - x, \frac{t}{2}) \diamond \nu(\Delta^n x_{ij} - x, \frac{t}{2}).$$

$$< (1-\lambda) \diamond (1-\lambda) = 1-\lambda$$

which means that (x_{ij}) is fuzzy $\Delta^n I_\lambda$-anti- Cauchy in ${}_2\mathcal{F}_\nu^I(\Delta^n)$.

□

Theorem 2.6. *Let ${}_2\mathcal{F}_\nu^I(\Delta^n)$ be a fuzzy antinormed double difference sequence space with respect to an idempotent t-conorm $\diamond$. If ${}_2\mathcal{F}_\nu^I(\Delta^n)$ is fuzzy $\Delta^n I_\lambda$-anti-complete, then ${}_2\mathcal{F}_\nu^I(\Delta^n)$ is $\Delta^n I$- complete with respect to $\|.\|_\lambda, \lambda \in (0,1)$.*

Proof. Let (x_{ij}) be fuzzy $\Delta^n I_\lambda$- anti-Cauchy sequence in ${}_2\mathcal{F}_\nu^I(\Delta^n)$. As ${}_2\mathcal{F}_\nu^I(\Delta^n)$ is fuzzy $\Delta^n I_\lambda$-anti-complete then fuzzy $\Delta^n I_\lambda$- anti-Cauchy sequence (x_{ij}) is fuzzy $\Delta^n I_\lambda$- anti-convergent to x. By Theorem (2.4), this means that $\|\Delta^n(x_{ij} - x)\|_\lambda$ is convergent to 0; i.e. (x_{ij}) is $\Delta^n I_\lambda$-convergent to 0. Hence ${}_2\mathcal{F}_\nu^I(\Delta^n)$ is I_λ-complete with respect to $\|.\|_\lambda,\ \lambda \in (0,1)$. Therefore $({}_2\mathcal{F}_\nu^I(\Delta^n), \|.\|_\lambda)$ is I- complete.

□

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