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RIESZ PROJECTION AND WEYL'S THEOREM FOR HEREDITARILY ABSOLUTE-(p,r)-PARANORMAL OPERATORS

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Abstract. A bounded linear operator $T \in B(H)$, H a Hilbert space is hereditarily absolute-(p,r)-paranormal (HAP) , if when ever $M \subseteq H$ is a closed invariant subspace of T , the restriction $T|M$ of T to M is absolute-(p,r)-paranormal. We study the necessary and sufficient condition for the self-adjoinness of Riesz Projection P_λ associated with $\lambda \in \sigma(T)$, T is hereditarily absolute-(p,r)-paranormal and show that Weyl's theorem holds for hereditarily absolute-(p,r)-paranormal operators.

1. Introduction and Priliminaries

Let $B(H)$ denote the algebra of all bounded linear operators on infinite dimensional separable Hilbert space H. An operator $T \in B(H)$ is said to be p-paranormal if $\||T|^pU|T|^px\| \geq \||T|^px\|^2$ for every unit vector x and $p > 0$, where the polar decomposition of T is defined by $T = U|T|$. The class of p-paranormal operators was introduced in [11], and have since been studied in [26] and [15]. An operator $T \in B(H)$ is said to be absolute (p,r)-paranormal for $p > 0$ and $r > 0$ if $\||T|^p|T^*|^rx\|^r \geq \||T^*|^rx\|^{p+r}$ for every unit vector x and normaloid if $r(T) = \|T\|$, where $r(T)$ denotes the spectral radius of T . The class of absolute (p,r)-paranormal operators was defined and studied by Yamazaki and Yanagida [27]. It is well known that every absolute (p,p)-paranormal is p-paranormal and every absolute (k,1)-paranormal is absolute-k-paranormal, see [27].

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The quasinilpotent part $H_0(T)$ and the analytic core $K(T)$ of a Hilbert space operator T are defined by

$$H_0(T) = \{x \in H : \lim_{n\to\infty} \|T^n x\|^{\frac{1}{n}} = 0\}$$

and

$K(T)$=$\{x \in X$: there exists a sequence $\{x_n\} \subset X$ and $\delta > 0$ for which x=x_0, $T(x_{n+1})$=x_n and $\|x_n\| \leq \delta^n \|x\|$ for all n=$1, 2, ...\}$.

It is well known that $H_0(T)$ and $K(T)$ are non - closed hyperinvariant subspaces of T such that $T^{-q}(0) \subseteq H_0(T)$ for all q=$0, 1, 2, ...$ and $TK(T)$=$K(T)$ [17]. An operator $T \in B(H)$ is said to be semi regular if $T(H)$ is closed and $T^{-1}(0) \subset T^{\infty}(H)$=$\bigcap_{n\in N} T^n(H)$. An operator T admits a generalized Kato decomposition , GKD for short , if there exists a pair of T - invarient closed subspaces (M, N) such that H=$M \oplus N$, the restriction $T|M$ is quasinilpotent and $T|N$ is semi regular. For more information, see [1] and [18].

If the range $T(H)$ of $T \in B(H)$ is closed and $\alpha(T)$=dim$(T^{-1}(0)) < \infty$ (resp., $\beta(T)$=dim$(H/T(H)) < \infty$) then T is upper semi Fredholm (resp., lower semi Fredholm) operator. Let $\Phi_+(H)$ (resp.,$\Phi_-(H)$) denote the semigroup of upper semi Fredholm (resp., lower semi Fredholm) operator on H. An operator $T \in B(H)$ is said to be semi Fredhom if $T \in \Phi_+(H) \cup \Phi_-(H)$ and Fredholm if $T \in \Phi_+(H) \cap \Phi_-(H)$. If T is semi Fredholm then the index of T is defined by

$$\text{ind}(T) = \alpha(T) - \beta(T).$$

The ascent of T, asc(T), is the least non negative integer n such that $T^{-n}(0) = T^{-(n+1)}(0)$. If such n does not exist, then asc$(T) = \infty$. The descent of T, des(T), is the least non negative integer n such that $T^n(H)$=$T^{n+1}(H)$. If such n does not exist, then des$(T) = \infty$. We say that T is of finite ascent (resp., finite descent) if asc$(T-\lambda) < \infty$ (resp., des$(T-\lambda) < \infty$) for all complex numbers λ. It is well known that if asc(T) and des(T) are both finite then they are equal [14, Proposition 38.6].

An operator $T \in B(H)$ is Weyl if it is Fredholm of index zero and Browder if T is Fredholm and asc(T)= des (T)< ∞ . Let $\mathbb{C}$ denote the set of complex numbers and let $\sigma(T)$ denote the spectrum of T . The Weyl spectrum $\sigma_w(T)$ and Browder spectrum $\sigma_b(T)$ of T are defined by

$$\sigma_w(T) = \{\lambda \in \mathbb{C} : T - \lambda \text{ is not Weyl}\},$$

$$\sigma_b(T) = \{\lambda \in \mathbb{C} : T - \lambda \text{ is not Browder }\}\cdot$$

Let $\pi_{00}(T)$ denote the set of eigenvalues of T of finite geometric multiplicity and let $\pi_0(T) := \sigma(T) \setminus \sigma_b(T)$ denote set of all Riesz points of T . According to Coburn [6], Weyl's theorem holds for T if

$$\sigma(T) \setminus \sigma_w(T) = \pi_{00}(T),$$

and that Browder's theorm holds for T if

$$\sigma(T) \setminus \sigma_w(T) = \pi_0(T).$$

Note that Weyl's theorem $\Longrightarrow$ Browder's theorem , see [13].

Hermann Weyl [25] examined the spectra of all compact perturbations $T + K$ of a single hermitian operator T and discovered that $\lambda \in \sigma(T + K)$ for every compact operator K if and only if λ is not an isolated eigenvalue of finite multiplicity in $\sigma(T)$. This remarkable result is known as Weyl's theorem. Many mathematicians extended Weyl's theorem to several classes of operators including p-hyponormal [5], paranormal [4] , w-hyponormal and Class A, see [12] and [22].

Let $K(H)$ denote the ideal of all compact operators on H and let $\sigma_a(T)$ be the approximate point spectrum of $T \in B(H)$. The essential approximate point spectrum $\sigma_{ea}(T)$ is defined by

$$\sigma_{ea}(T) = \cap\{\sigma_a(T + K) : K \in K(H)\}.$$

In [19], Rakočevič introduced the concept of a-Weyl's theorem. An operator $T \in B(H)$ holds a-Weyl's theorem if

$$\sigma_{ea}(T) = \sigma_a(T) \setminus \pi_{a0}(T),$$

where $\pi_{a0}(T) = \{\lambda \in \mathbb{C} : \lambda \in iso\sigma_a(T) \text{ and } 0 < \alpha(T - \lambda) < \infty\}$. This approximate point spectrum version of Weyl's theorem have been much investigated in [7] and [8].

An operator $T \in B(H)$ has the single valued extension property (SVEP) at $\lambda_0 \in \mathbb{C}$, if for every open disc D_{λ_0} centered at λ_0 the only analytic function $f : D_{\lambda_0} \to H$ which satisfies $(T - \lambda)f(\lambda)$=0 for all $\lambda \in D_{\lambda_0}$ is the function $f \equiv 0$. We say that T has SVEP if it has SVEP at every $\lambda \in \mathbb{C}$.

Let $T \in B(H)$ and let λ be an isolated point of $\sigma(T)$. If there exist a closed disc D_λ centered at λ that satisfies $D_\lambda \cap \sigma(T) = \{\lambda\}$, then the operator

$$P_\lambda = \frac{1}{2\pi i}\int_{\partial D_\lambda}(\lambda - T)^{-1}d\lambda$$

associated with λ is defined by familiar Cauchy integral[14] is called Riesz projection with respect to λ , which has the properties that $P_\lambda^2 = P_\lambda$, $P_\lambda T = TP_\lambda$, and $\sigma(T|P_\lambda H) = \{\lambda\}$.

Self-adjointness of Riesz Projection P_λ associated with $\lambda \in \sigma(T)$ for hyponormal operators has been proved by Stampfli [20] and this result was extended for w-hyponormal by Han, Lee and Wang [12], for class A and paranormal operators by Uchiyama ([23], [24]). Duggal [9] investigated the necessary and sufficient condition for the self-adjointness of Riesz Projection for operators of class CHN. In this paper, we show the necessary and sufficient condition for the self-adjointness of Riesz Projection associated with $\lambda \in iso\sigma(T)$ and Weyl's theorem holds for $T \in HAP$.

2. Riesz projection and Weyl's theorem

The class of p-paranormal operators inherit some of the properties of paranormal operators , as in the case , if T is invertible and p-paranormal then T^{-1} is p-paranormal [15]. If T is invertible and absolute (p,r)-paranormal, then T^{-1} is absolute-(r,p)-paranormal [27]. A part of an operator is a restriction of it to an invariant subspace. If T is paranormal then we see that every part of it is paranormal. Now we define class of hereditarily absolute-(p,r)-paranormal operators (HAP) as follows .

Definition 2.1. *The class HAP of hereditarily absolute-(p,r)-paranormal operators between Hilbert Spaces consists of those operators $T \in B(H)$ for which , when ever $M \subseteq H$ is a closed invariant subspace of T , the restriction $T|M$ of T to M is absolute-(p,r)-paranormal.*

The class HAP is large; it contains , among others, the classes of hyponormal ($T \in B(H)$:$T^*T \geq TT^*$) , p-hyponormal ($T \in B(H)$:$(T^*T)^p \geq (TT^*)^p$, $0 < p < 1$) and class A ($T \in B(H)$:$|T^2| \geq |T|^2$) operators. Every class HAP operator is normaloid.

For hereditarily absolute-(p,r)-paranormal operators, isolated points of spectrum are simple poles of resolvent set.

Theorem 2.2. *If $T \in HAP$, then every isolated point of $\sigma(T)$ is simple pole of the resolvent of T .*

Proof. If $T \in HAP$ and λ is an isolated point of $\sigma(T)$, then

$$H{=}H_0(T-\lambda) \oplus K(T-\lambda)$$

where $H_0(T-\lambda) \neq \{0\}$ and $(T-\lambda)K(T-\lambda){=}K(T-\lambda)$ [17]. If $\lambda = 0$, consider the hereditarily absolute (p,r)-paranormal operator $T|H_0(T)$. Since $T|H_0(T)$ is absolute (p,r)-paranormal, $T|H_0(T)$ is normaloid by [27, Theorem 8]. Therefore, $\sigma(T|H_0(T)) = \{0\}$ implies $T|H_0(T) = 0$. If $\lambda \neq 0$, we may assume $\lambda = 1$. Since $T|H_0(T-1)$ is hereditarily absolute (p,r) paranormal and $\sup \|(T|H_0(T-1))^n\| \leq 1$, where supremum is taken over all integers n, it follows that $T|H_0(T-1) = I|H_0(T-1)$ by [16, Theorem 1.5.14] which implies that $H_0(T-1) = (T-1)^{-1}(0)$ and so $H_0(T-\lambda) = (T-\lambda)^{-1}(0)$. Hence $(T-\lambda)H{=}0 \oplus (T-\lambda)K(T-\lambda){=}0 \oplus K(T-\lambda)$. Thus $H = (T-\lambda)^{-1}(0) \oplus (T-\lambda)H$. Hence λ is a simple pole of the resolvent of T. □

An operator $T \in B(H)$ is said to be reguloid if λ is an isolated point of $\sigma(T)$ implies $(T-\lambda)^{-1}(0)$ and $(T-\lambda)H$ are complimented in H. Evidently, T is reguloid implies T is isoloid (ie., every isolated point of $\sigma(T)$ is an eigenvalue of T). The following corollaries are immediate consequences of Theorem 2.2.

Corollary 2.3. *If $T \in HAP$ then T is reguloid.*

Corollary 2.4. *If $T \in HAP$ then $\pi_0(T){=}\pi_{00}(T)$.*

Theorem 2.5. *If $T \in HAP$ and $\lambda \in iso\sigma(T)$, then P_λ is self-adjoint if and only if $(T-\lambda)^{-1}(0) \subseteq (T^*-\overline{\lambda})^{-1}(0)$.*

Proof. If $T \in HAP$ and $\lambda \in iso\sigma(T)$, then

$$H{=}H_0(T-\lambda) \oplus K(T-\lambda) = (T-\lambda)^{-1}(0) \oplus (T-\lambda)H$$

as a topological direct sum and T has matrix decomposition

$$T{=}\begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix} \begin{pmatrix} (T-\lambda)^{-1}(0) \\ (T-\lambda)H \end{pmatrix}$$

where $\sigma(T_1){=}\{\lambda\}$ and $\sigma(T_3){=}\sigma(T) \setminus \{\lambda\}$.

Suppose that $(T-\lambda)^{-1}(0) \subseteq (T^*-\overline{\lambda})^{-1}(0)$. If $x{=}x_1 \oplus x_2 \in (T-\lambda)^{-1}(0)$, then $x_1 \in (T_1-\lambda)^{-1}(0)$ and $x_2 = 0$. Since $(T-\lambda)^{-1}(0) \subseteq (T^*-\overline{\lambda})^{-1}(0)$, $(T^*-\overline{\lambda})(x_1 \oplus 0) = 0 \oplus T_2^* x_1 = 0$ and so T_2 is the zero operator. Thus $(T-\lambda)^{-1}(0)$ reduces T. Consequently, $P_\lambda^{-1}(0)^\perp = P_\lambda H$. Thus, P_λ is self-adjoint.

Conversely, suppose the Riesz Projection P_λ is self-adjoint. From [14, Theorem 49.1], $P_\lambda H = H_0(T-\lambda) = (T-\lambda)^{-1}(0)$ and $P_\lambda^{-1}(0) = K(T-\lambda) = (T-\lambda)H$. Since $P_\lambda H^\perp = P_\lambda^{-1}(0)$, and since $P_\lambda H^\perp = (T-\lambda)H^\perp = (T^*-\overline{\lambda})^{-1}(0)$, the condition $(T-\lambda)^{-1}(0) \subseteq (T^*-\overline{\lambda})^{-1}(0)$ is necessary.

□

Let M and N be linear subspaces of a Banach space X. Then M is said to be orthogonal to N, $M \perp N$, in the sense of G.Brikhoff, if $\|x\| \leq \|x+y\|$ for every $x \in M$ and $y \in N$. Note that in general this is not symmetric relation. when X is Hilbert Space it reduces to the usual (symmetric) orthogonality. If $T \in B(H)$ we shall write $R(T)$ and $N(T)$ for the range and null space of T and $\nu(T)$ denote the numerical radius of T .

Proposition 2.6. *If $T \in HAP$, then N($T-\alpha$)$\perp$N($T-\beta$) for distinct complex numbers $\alpha(\neq 0)$ and β.*

Proof. Suppose $|\alpha| \geq |\beta|$. Let M denote the subspace generated by x and y such that $(T-\alpha)x = 0 = (T-\beta)y$ and $T_1 = T|M$. Then $\sigma(T_1) = \{\alpha, \beta\}$ and $r(T_1) = \nu(T_1) = |\alpha|$. Thus $\alpha \in \partial v(B(M), T_1)$, where $\partial v(B(M), T_1)$ denotes the boundary of the numerical range of $T_1 \in B(M)$. By [21, Proposition 1] it follows that $\|(T_1-\alpha)w + x\| \geq \|x\|$ for $x \in N(T-\alpha)$ and $w \in M$. Let $P_\alpha(T_1)$ denotes the Riesz projection of T_1 associated with α , then $R(T_1-\alpha) = R(I - P_\alpha(T_1)) = R(P_\beta(T_1)) = N(T_1-\beta)$ implies $\|x\| \leq \|x+y\|$. If $|\alpha| < |\beta|$ then T_1 is invertible with $\sigma(T_1^{-1}) = \{\alpha^{-1}, \beta^{-1}\}$. Being hereditarily absolute-(p,r)-paranormal, T_1^{-1} also normaloid. Thus $r(T_1^{-1}) = |\alpha^{-1}|$ and $(T_1^{-1}-\alpha^{-1})x = 0 = (T_1^{-1}-\beta^{-1})y$. This completes the proof.

□

For an operator $T \in B(H)$,the reduced minimum modulus is defined by

$$\gamma(T) = inf\{\frac{\|Tx\|}{dist(x,(T)^{-1}(0))} : x \in H \setminus T^{-1}(0)\}.$$

Obviously $\gamma(T^*) = \gamma(T)$, and $T(H)$ is closed if and only if $\gamma(T) > 0$[10].

Theorem 2.7. *If $T \in HAP$, then T and T^* have SVEP at points $\lambda \in \sigma(T)\backslash\sigma_w(T)$.*

Proof. Let $\lambda \in \sigma(T) \setminus \sigma_w(T)$ then $(T - \lambda)$ is Fredhlom of index zero . Suppose that the point spectrum of T clusters at λ, then there exists a sequence $\{\lambda_n\}$ of non zero eigenvalues of T converging to λ. Choose $\lambda_m \in \{\lambda_n\}$. By Proposition 2.6 eigenspaces corresponding to non zero eigenvalues of T are mutually orthogonal, and if $\lambda = 0$ then eigenspaces corresponding to the eigenvalue λ_m is orthogonal to the eigenspaces corresponding to the eigenvalue 0. Then $dist(x, (T - \lambda)^{-1}(0)) \geq 1$ for every unit vector $x \in (T - \lambda_m)^{-1}(0)$. We have

$\delta(\lambda_m, \lambda)$ =sup$\{dist(x, (T - \lambda)^{-1}(0)) : x \in (T - \lambda_m)^{-1}(0), \| x \|= 1\} \geq 1$for all m.

Thus

$$\gamma(T - \lambda)=\frac{|\lambda_m-\lambda|}{\delta(\lambda_m,\lambda)} \to 0 \text{ as } m \to \infty$$

which contradicts the fact that $(T - \lambda)H$ is closed. Hence the point spectrum of T does not clusters at λ . Applying [3, Corollary 2.10], it follows that T and T^* has SVEP at λ. □

Theorem 2.8. *If $T \in HAP$, then T and T^* holds Weyl's theorem.*

Proof. By Theorem 2.7, T have SVEP at points $\lambda \in \sigma(T) \setminus \sigma_w(T)$. Recall from Corollary 2.4 that $\pi_0(T) = \pi_{00}(T)$. Applying [10, Theorem 2.3] , it follows that T satisfies Weyl's Theorem.

Now we show that T^* holds Weyl's theorem. Since T satisfies Weyl's theorem, T^* satisfies Browder's theorem. Then

$$\sigma(T^*) \setminus \sigma_w(T^*) = \pi_0(T^*).$$

The inclusion $\pi_0(T^*) \subseteq \pi_{00}(T^*)$ holds for all $T \in B(H)$. To prove the opposite inclusion, let $\lambda \in \pi_{00}(T^*)$ then $\lambda \in$iso$\sigma(T)$ and so

$$H = (T - \lambda)^{-1}(0) \oplus (T - \lambda)H.$$

Thus

$$H^* = (T^* - \lambda I^*)H^* \oplus (T^* - \lambda I^*)^{-1}(0).$$

It follows that λ is the simple pole of the resolvent of T^* and so $T^* - \lambda I^*$ has closed range. Since T^* has SVEP and $0 \leq \alpha(T^* - \lambda I^*) < \infty$, both $\mathrm{asc}(T^* - \lambda I^*)$ and $\mathrm{des}(T^* - \lambda I^*)$ are finite. Then $\beta(T^* - \lambda I^*) < \infty$ by [14, Proposition 38.6]. Hence $T^* - \lambda I^*$ is Browder and so $\lambda \in \pi_0(T^*)$. Thus T^* satisfies Weyl's theorem. □

Theorem 2.9. *Let $T \in HAP$. Then both T and T^* holds a-Weyl's theorem.*

Proof. By Theorem 2.8, Weyl's theorem holds for T. Since T^* has SVEP, a-Weyl's theorem holds for T by [2, Theorem 3.6]. From Theorem 2.8, T^* satisfies Weyl's theorem and by Theorem 2.7, T has SVEP. Applying [2, Theorem 3.6], T^* satisfies a-Weyl's theorem. □

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