Bulletin of Mathematical Analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 3 Issue 3(2011), Pages 141-148.

A FIXED POINT THEOREM FOR KANNAN TYPE MAPPINGS IN 2-MENGER SPACE USING A CONTROL FUNCTION

(COMMUNICATED BY DENNY H. LEUNG)

BINAYAK S. CHOUDHURY, KRISHNAPADA DAS AND SAMIR KUMAR BHANDARI

Abstract. In this paper we have obtained a fixed point theorem for Kannan type mappings in 2-Menger space using a control function. Our result generalizes some existing fixed point theorems in 2-metric spaces. Our result is supported with an example.

1. Introduction

The purpose of this paper is to establish a Kannan type fixed point result in 2-Menger spaces. Kannan type of mappings are considered to be important in metric fixed point theory for several reasons. We mention two mathematical reasons in the following.

Banach contraction is continuous. A natural question is that whether there exists a class of mappings satisfying some contractive inequality which necessary have fixed points in complete metric spaces but need not necessarily be continuous. Kannan type mappings are such mappings to be first discovered [15, 16]. Another reason is its connection with metric completeness. A Banach contraction mapping may have a fixed point in metric space which is not complete. In fact, Connell in [5] has given an example of a metric space which is not complete but every Banach contraction defined on which has a fixed point. It has been established in [29] that the metric completeness is implied by the necessary existence of fixed points of the class of Kannan type mappings.

Definition 1.1. [15, 16] *Let (X, d) be a metric space and f be a mapping on X. The mapping f is called a Kannan type mapping if there exists $0 \leq \alpha < \frac{1}{2}$ such that*
$$d(fx, fy) \leq \alpha[d(x, fx) + d(y, fy)] \text{ for all } x, y \in X.$$

There are a large number of works dealing with Kannan type mappings. Several examples of these works are noted in [4, 17, 18, 28].

The concept of metric spaces has been extended in various ways. One such

---

2000 *Mathematics Subject Classification.* 47H10.
Key words and phrases. 2-Menger space, Cauchy sequence, fixed point, Φ-function, Ψ-function.

Submitted May 11, 2011. Published June 30, 2011.

extension has been made by Gähler [7] in which a positive real number is assigned to every three elements of the space.

Definition 1.2. ***2-metric space*** [7, 8]
Let X be a non empty set. A real valued function d on $X \times X \times X$ is said to be a 2-metric on X if

(i) given distinct elements x, y of X, there exists an element z of X such that $d(x, y, z) \neq 0$,
(ii) $d(x, y, z) = 0$ when at least two of x, y, z are equal,
(iii) $d(x, y, z) = d(x, z, y) = d(y, z, x)$ for all x, y, z in X and
(iv) $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)$ for all x, y, z, w in X.

When d is a 2-metric on X, the ordered pair (X, d) is called a 2-metric space.

Probabilistic metric spaces are probabilistic generalizations of metric spaces in which every pair of elements is assigned to a distribution function. The theory of these spaces is an important part of stochastic analysis. Schweizer and Sklar in their book noted in [26] have given a comprehensive account of several aspects of such spaces.

Definition 1.3. ***Probabilistic metric space*** [11, 26]
A probabilistic metric space (briefly, a PM-space) is an ordered pair (X, F), where X is a non empty set and F is a mapping from $X \times X$ into the set of all distribution functions. We denote the distribution function $F(x, y)$ by $F_{x,y}$ and $F_{x,y}(t)$ represents the value of $F_{x,y}$ at $t \in R$. The function $F_{x,y}$ is assumed to satisfy the following conditions for all $x, y, z \in X$,

(i) $F_{x,y}(0) = 0$,
(ii) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,
(iii) $F_{x,y}(t) = F_{y,x}(t)$ for all $t > 0$,
(iv) if $F_{x,y}(t_1) = 1$ and $F_{y,z}(t_2) = 1$ then $F_{x,z}(t_1 + t_2) = 1$, for $t_1, t_2 > 0$.

A particular type of probabilistic metric space is Menger space in which the triangular inequality is postulated with the help of a t-norm.

Definition 1.4. ***n-th order t-norm*** [30]
A mapping $T : \Pi_{i=1}^{n}[0, 1] \to [0, 1]$ is called a n-th order t-norm if the following conditions are satisfied :

(i) $T(0, 0, ..., 0) = 0$, $T(a, 1, 1, ..., 1) = a$ for all $a \in [0, 1]$,
(ii) $T(a_1, a_{2,}, a_3, ..., a_n) = T(a_2, a_1, a_3, ..., a_n) = T(a_2, a_3, a_1, ..., a_n)$
$= .... = T(a_2, a_3, a_4, ..., a_n, a_1)$,
(iii) $a_i \geq b_i$, i=1,2,3,....,n implies $T(a_1, a_2, a_3, ..., a_n) \geq T(b_1, b_2, b_3, ..., b_n)$,
(iv) $T(T(a_1, a_2, a_3, ..., a_n), b_2, b_3, ...b_n)$
$= T(a_1, T(a_2, a_3, ..., a_n, b_2), b_3, ..., b_n)$
$= T(a_1, a_2, T(a_3, a_4...., a_n, b_2, b_3), b_4, ..., b_n)$
=..............................
$= T(a_1, a_2, ..., a_{n-1}, T(a_n, b_2, b_3, ..., b_n))$.

When $n = 2$, we have a binary t-norm, which is commonly known as t-norm.

Definition 1.5. ***Menger space*** [11, 26]
A Menger space is a triplet (X, F, Δ), where X is a non empty set, F is a function defined on $X \times X$ to the set of distribution functions and Δ is a t-norm, such that the following are satisfied:

(i) $F_{x,y}(0) = 0$ *for all* $x, y \in X$,
(ii) $F_{x,y}(s) = 1$ *for all* $s > 0$ *and* $x, y \in X$ *if and only if* $x = y$,
(iii) $F_{x,y}(s) = F_{y,x}(s)$ *for all* $x, y \in X$, $s > 0$ *and*
(iv) $F_{x,y}(u+v) \geq \Delta\left(F_{x,z}(u), F_{z,y}(v)\right)$ *for all* $u, v \geq 0$ *and* $x, y, z \in X$.

The first fixed point result in probabilistic metric spaces proved by Sehgal and Bharucha-Reid [27]. After that a lot of results appeared in the literature. A comprehensive survey upto 2001 is given by Hadzic and Pap in [11].

Probabilistic generalization of 2-metric spaces has been done following the same ideas behind the introduction of probabilistic metric spaces.

Definition 1.6. ***2-probabilistic metric space*** [32]
A probabilistic 2-metric space is an order pair (X, F) *where* X *is an arbitrary set and* F *is a mapping from* X^3 *into the set of distribution functions. The distribution function* $F_{x,y,z}(t)$ *will denote the value of* $F_{x,y,z}$ *at the real number* t. *The function* $F_{x,y,z}$ *are assumed to satisfy the following conditions.*

(i) $F_{x,y,z}(t) = 0$ *for all* $t \leq 0$ *for all* $x, y, z \in X$,
(ii) $F_{x,y,z}(t) = 1$ *for all* $t > 0$ *iff at least two of the three points* x, y, z *are equal,*
(iii) For distinct points $x, y \in X$ *there exists a point* $z \in X$ *such that* $F_{x,y,z}(t) \neq 1$ *for* $t > 0$,
(iv) $F_{x,y,z}(t) = F_{x,z,y}(t) = F_{z,y,x}(t)$ *for all* $x, y, z \in X$ *and* $t > 0$,
(v) $F_{x,y,w}(t_1) = 1$, $F_{x,w,z}(t_2) = 1$ *and* $F_{w,y,z}(t_3) = 1$ *then* $F_{x,y,z}(t_1 + t_2 + t_3) = 1$, *for all* $x, y, z, w \in X$ *and* $t_1, t_2, t_3 > 0$.

A special case of the above definition is the following.

Definition 1.7. ***2-Menger space*** [10]
Let X *be any nonempty set and* D *the set of all left-continuous distribution functions. A triplet* (X, F, Δ) *is said to be a 2-Menger space if* F *is a mapping from* X^3 *into* D *satisfying the following conditions where the value of* F *at* $x, y, z \in X^3$ *is represented by* $F_{x,y,z}$ *or* $F(x, y, z)$ *for all* $x, y, z \in X$ *such that*

(i) $F_{x,y,z}(0) = 0$,
(ii) $F_{x,y,z}(t) = 1$ *for all* $t > 0$ *if and only if at least two of* $x, y, z \in X$ *are equal,*
(iii) $F_{x,y,z}(t) = F_{x,z,y}(t) = F_{z,y,x}(t)$, *for all* $x, y, z \in X$ *and* $t > 0$,
(iv) $F_{x,y,z}(t) \geq \Delta(F_{x,y,w}(t_1), F_{x,w,z}(t_2), F_{w,y,z}(t_3))$

where $t_1, t_2, t_3 > 0$, $t_1 + t_2 + t_3 = t$ *and* $x, y, z, w \in X$ *and* Δ *is the 3rd order t norm.*

Fixed point theory has developed rapidly in these spaces. Several results of metric fixed point theory was extended to these spaces. Some of the important fixed point theorems in 2-metric spaces are [12, 13, 19, 21, 23] while the references [9, 10, 31] are some fixed point results in 2-probabilistic metric spaces.

Definition 1.8. *A sequence* $\{x_n\}$ *in a 2-Menger space* (X, F, Δ) *is said to be converge with limit* x *if* $\lim_{n \to \infty} F_{x_n,x,a}(t) = 1$ *for all* $t > 0$ *and for every* $a \in X$.

Definition 1.9. *A sequence* $\{x_n\}$ *in a 2-Menger space* (X, F, Δ) *is said to be a Cauchy sequence in* X *if given* $\epsilon > 0, \lambda > 0$ *there exists a positive integer* $N_{\epsilon,\lambda}$

such that

$$F_{x_n,x_m,a}(\epsilon) > 1 - \lambda \tag{1.1}$$

for all $m, n > N_{\epsilon,\lambda}$ *and for every* $a \in X$.

Definition 1.10. *A 2-Menger space* (X, F, Δ) *is said to be complete if every Cauchy sequence is convergent in* X.

In [14] Khan, Swaleh and Sessa introduced a new category of contractive fixed point problems in metric space. They introduced the concept of "altering distance function", which is a control function that alters the distance between two points in a metric space. This concept was further generalized in a number of works. There are several works in fixed point theory involving altering distance function, some of these are noted in [22, 24] and [25].

Recently first two authors of the present paper had extended the concept of altering distance function in the context of Menger spaces in [1]. The definition is as follows:

Definition 1.11. Φ-***function*** [1]
A function $\phi : R \to R^+$ *is said to be a* Φ*-function if it satisfies the following conditions:*

(i) $\phi(t) = 0$ *if and only if* $t = 0$,
(ii) $\phi(t)$ *is strictly monotone increasing and* $\phi(t) \to \infty$ *as* $t \to \infty$,
(iii) ϕ *is left continuous in* $(0, \infty)$,
(iv) ϕ *is continuous at* 0.

In [1] Chowdhury and Das introduced a new type of contraction mapping in Menger spaces which is known as ϕ-contraction. The idea of control function has opened possibilities of proving new fixed point results in Menger spaces. This concept has also applied to a coincidence point result. Some recent results using Φ-function are noted in [2, 3, 6] and [20].

Definition 1.12. Ψ-***function*** [4]
A function $\psi : [0, 1] \times [0, 1] \to [0, 1]$ *is said to be a* Ψ*-function if*

(i) ψ*-is monotone increasing and continuous,*
(ii) $\psi(x, x) > x$ *for all* $0 < x < 1$,
(iii) $\psi(1, 1) = 1, \psi(0, 0) = 0$.

Definition 1.13. *Let* (X, F, Δ) *be a complete 2-Menger space, where* Δ *is the 3rd order minimum t-norm and the mapping* $T : X \to X$ *be a self mapping which satisfies the following inequality for all* $x, y, p \in X$,

$$F_{Tx,Ty,p}(\phi(t)) \geq \psi(F_{x,Tx,p}(\phi(\tfrac{t_1}{a})), F_{y,Ty,p}(\phi(\tfrac{t_2}{b}))) \tag{1.2}$$

where $t_1, t_2, t > 0$ *with* $t = t_1 + t_2$, $a, b > 0$ *with* $0 < a + b < 1$, ψ *is a* Ψ*-function and* ϕ *is a* Φ*-function. Then the mapping* T *is called a generalized Kannan type mapping.*

The purpose of this paper is to apply the control function mentioned above to establish a generalized Kannan type fixed point result in a 2-Menger space. Our result extends a result of [19] to 2-Menger spaces and is supported with an example.

2. Main Result

Theorem 2.1. *Let (X, F, Δ) be a complete 2-Menger space, where Δ is the 3rd order minimum t-norm and the mapping $T : X \to X$ be a self mapping which satisfies the following inequality for all $x, y, p \in X$,*

$$F_{Tx,Ty,p}(\phi(t)) \geq \psi(F_{x,Tx,p}(\phi(\tfrac{t_1}{a})), F_{y,Ty,p}(\phi(\tfrac{t_2}{b}))) \tag{2.1}$$

where $t_1, t_2, t > 0$ with $t = t_1 + t_2$, $a, b > 0$ with $0 < a + b < 1$, ψ is a Ψ-function and ϕ is a Φ-function. Then T has a unique fixed point in X.

Proof: Let $x_0 \in X$. We now construct a sequence $\{x_n\}$ as follows:

$$x_n = Tx_{n-1},\ n \in N,$$

where N is the set of all positive integers.
Now we have for $t, t_1, t_2 > 0$ with $t = t_1 + t_2$,

$$\begin{aligned} F_{x_{n+1},x_n,p}(\phi(t)) &= F_{Tx_n,Tx_{n-1},p}(\phi(t)) \\ &\geq \psi(F_{x_n,Tx_n,p}(\phi(\tfrac{t_1}{a})), F_{x_{n-1},Tx_{n-1},p}(\phi(\tfrac{t_2}{b}))) \\ &= \psi(F_{x_n,x_{n+1},p}(\phi(\tfrac{t_1}{a})), F_{x_{n-1},x_n,p}(\phi(\tfrac{t_2}{b}))) \\ &= \psi(F_{x_{n+1},x_n,p}(\phi(\tfrac{t_1}{a})), F_{x_n,x_{n-1},p}(\phi(\tfrac{t_2}{b}))). \end{aligned} \tag{2.2}$$

Let $t_1 = \frac{at}{a+b}$, $t_2 = \frac{bt}{a+b}$ and $c = a + b$, then obviously we have $0 < c < 1$.
Then we have from (2.2),

$$F_{x_{n+1},x_n,p}(\phi(t)) \geq \psi(F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})), F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c}))). \tag{2.3}$$

We now claim that for all $t > 0$,

$$F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})) \geq F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c})). \tag{2.4}$$

If possible, let for some $t > 0$,

$$F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})) < F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c})),$$

then we have,

$$\begin{aligned} F_{x_{n+1},x_n,p}(\phi(t)) &\geq \psi(F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})), F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c}))) \\ &> F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})) \\ &\geq F_{x_{n+1},x_n,p}(\phi(t)), \end{aligned}$$

which is a contradiction, since $0 < c < 1$, ϕ is strictly increasing and F is non-decreasing.

Therefore for all $t > 0$,

$$F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})) \geq F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c})).$$

Hence using (2.4) we have from (2.3),

$$\begin{aligned} F_{x_{n+1},x_n,p}(\phi(t)) &\geq \psi(F_{x_{n+1},x_n,p}(\phi(\tfrac{t}{c})), F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c}))) \\ &\geq \psi(F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c})), F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c}))) \\ &\geq F_{x_n,x_{n-1},p}(\phi(\tfrac{t}{c})). \end{aligned} \tag{2.5}$$

By repeated application of (2.5) we have after n steps,

$$F_{x_{n+1},x_n,p}(\phi(t)) \geq F_{x_1,x_0,p}(\phi(\tfrac{t}{c^n})). \tag{2.6}$$

Therefore,

$$\lim_{n\to\infty} F_{x_{n+1},x_n,p}(\phi(t)) = 1 \text{ for all } t > 0. \tag{2.7}$$

By virtue of property of ϕ and F we can choose $s > 0$ such that $s > \phi(t)$. Then for all $p \in X$ and $t > 0$ we have,

$$\lim_{n\to\infty} F_{x_n,x_{n+1},p}(s) = 1. \tag{2.8}$$

We next prove that $\{x_n\}$ is a Cauchy sequence. If possible, let $\{x_n\}$ be not a Cauchy sequence. Then there exist $\epsilon > 0$ and $0 < \lambda < 1$ for which we can find subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ with $m(k) > n(k) > k$ such that

$$F_{x_{m(k)},x_{n(k)},p}(\epsilon) \leq 1 - \lambda. \tag{2.9}$$

We take $m(k)$ corresponding to $n(k)$ to be the smallest integer satisfying (2.9), so that

$$F_{x_{m(k)-1},x_{n(k)},p}(\epsilon) > 1-\lambda. \tag{2.10}$$

If $\epsilon_1 < \epsilon$ then we have,

$$F_{x_{m(k)},x_{n(k)},p}(\epsilon_1) \leq F_{x_{m(k)},x_{n(k)},p}(\epsilon).$$

We conclude that it is possible to construct $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ with $m(k) > n(k) > k$ and satisfying (2.9), (2.10) whenever ϵ is replaced by a smaller positive value. As ϕ is continuous at 0 and strictly monotone increasing with $\phi(0) = 0$, it is possible to obtain $\epsilon_2 > 0$ such that $\phi(\epsilon_2) < \epsilon$.

Then, by the above argument, it is possible to obtain an increasing sequence of integers $\{m(k)\}$ and $\{n(k)\}$ with $m(k) > n(k) > k$ such that

$$F_{x_{m(k)},x_{n(k)},p}(\phi(\epsilon_2)) \leq 1-\lambda, \tag{2.11}$$

and

$$F_{x_{m(k)-1},x_{n(k)},p}(\phi(\epsilon_2)) > 1-\lambda. \tag{2.12}$$

Now from (2.11) we get,

$$\begin{aligned}1-\lambda &\geq F_{x_{m(k)},x_{n(k)},p}(\phi(\epsilon_2))\\ &= F_{Tx_{m(k)-1},Tx_{n(k)-1},p}(\phi(\epsilon_2))\\ &\geq \psi(F_{x_{m(k)-1},Tx_{m(k)-1},p}(\phi(\tfrac{\epsilon'}{a})), F_{x_{n(k)-1},Tx_{n(k)-1},p}(\phi(\tfrac{\epsilon''}{b}))) \text{ (where } \epsilon_2 = \epsilon' + \epsilon'')\\ &= \psi(F_{x_{m(k)-1},x_{m(k)},p}(\phi(\tfrac{\epsilon'}{a})), F_{x_{n(k)-1},x_{n(k)},p}(\phi(\tfrac{\epsilon''}{b}))).\end{aligned} \tag{2.13}$$

By the property of ϕ, we can choose $\eta_1, \eta_2 > 0$ such that $\phi(\frac{\epsilon'}{a}) = \eta_1$, $\phi(\frac{\epsilon''}{b}) = \eta_2$. Then from (2.13) we have,

$$1-\lambda \geq \psi(F_{x_{m(k)-1},x_{m(k)},p}(\eta_1), F_{x_{n(k)-1},x_{n(k)},p}(\eta_2)).$$

By (2.8) we can choose λ_1 with $0 < \lambda_1 < \lambda < 1$ such that,

$$F_{x_{m(k)-1},x_{m(k)},p}(\eta_1) \geq 1-\lambda_1 \text{ and } F_{x_{n(k)-1},x_{n(k)},p}(\eta_2) \geq 1-\lambda_1.$$

Therefore,

$$1-\lambda \geq \psi(1-\lambda_1, 1-\lambda_1) > 1-\lambda_1 > 1-\lambda \text{ (by the property of } \psi),$$

which is a contradiction.

Hence $\{x_n\}$ is a Cauchy sequence.

As (X, F, Δ) is a complete 2-Menger space we have $\{x_n\}$ is convergent in X.

Let

$$\lim_{n\to\infty} x_n = z. \tag{2.14}$$

We now show that $Tz = z$. If possible, let $0 < F_{z,Tz,p}(\phi(t)) < 1$ for some $t > 0$. By virtue of the property of ϕ we can choose $\xi_1, \xi_2, t_1, t_2 > 0$ such that $\phi(t) = \xi_1 + \xi_2 + \phi(t_1 + t_2)$.

Again since $0 < b < 1$, we can get $\phi(\frac{t_2}{b}) > \phi(t)$.

Then we have,

$$\begin{aligned}F_{z,Tz,p}(\phi(t)) &\geq \Delta(F_{z,Tz,x_{n+1}}(\xi_1), F_{z,x_{n+1},p}(\xi_2), F_{x_{n+1},Tz,p}(\phi(t_1+t_2)))\\ &= \Delta(F_{z,x_{n+1},Tz}(\xi_1), F_{z,x_{n+1},p}(\xi_2), F_{Tx_n,Tz,p}(\phi(t_1+t_2)))\\ &\geq \Delta(F_{z,x_{n+1},Tz}(\xi_1), F_{z,x_{n+1},p}(\xi_2), \psi(F_{x_n,x_{n+1},p}(\phi(\tfrac{t_1}{a})), F_{z,Tz,p}(\phi(\tfrac{t_2}{b}))))\\ &\geq \Delta(F_{z,x_{n+1},Tz}(\xi_1), F_{z,x_{n+1},p}(\xi_2), \psi(F_{x_n,x_{n+1},p}(\phi(\tfrac{t_1}{a})), F_{z,Tz,p}(\phi(t)))).\end{aligned}$$

(2.15)

By (2.8), (2.14) and (2.15), there exists a positive integer N_1 such that

$F_{z,x_{n+1},Tz}(\xi_1), F_{z,x_{n+1},p}(\xi_2), F_{x_n,x_{n+1},p}(\phi(\frac{t_1}{a})) > F_{z,Tz,p}(\phi(t))$ for all $n > N_1$.

Then we have from (2.15),

$F_{z,Tz,p}(\phi(t)) > F_{z,Tz,p}(\phi(t))$,

which is a contradiction.

Hence

$F_{z,Tz,p}(\phi(t)) = 1$ for all $t > 0$ which implies that $z = Tz$.

For uniqueness, let z and u be two fixed points. Therefore, for all $t > 0$,

$$\begin{aligned} F_{z,u,p}(\phi(t)) &= F_{Tz,Tu,p}(\phi(t)) \\ &\geq \psi(F_{z,Tz,p}(\phi(\tfrac{t_1}{a})), F_{u,Tu,p}(\phi(\tfrac{t_2}{b}))) \text{ (for } t_1, t_2 > 0 \text{ and } t_1 + t_2 = t) \\ &= \psi(F_{z,z,p}(\phi(\tfrac{t_1}{a})), F_{u,u,p}(\phi(\tfrac{t_2}{b}))) \\ &= \psi(1,1) = 1. \end{aligned}$$

Therefore, $z = u$.

Now we give the following example to validate our result.

Example 2.1. *Let $X = \{\alpha, \beta, \gamma, \delta\}$, the t-norm Δ is a 3rd order minimum t-norm and F be defined as*

$$F_{\alpha,\beta,\gamma}(t) = F_{\alpha,\beta,\delta}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.40, & \text{if } 0 < t < 4, \\ 1, & \text{if } t \geq 4, \end{cases}$$

$$F_{\alpha,\gamma,\delta}(t) = F_{\beta,\gamma,\delta}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 1, & \text{if } t > 0. \end{cases}$$

Then (X, F, Δ) is a complete 2-Menger space. If we define $T : X \to X$ as follows: $T\alpha = \delta, T\beta = \gamma, T\gamma = \gamma, T\delta = \gamma$ then the mapping T satisfies all the conditions of the Theorem 2.1 when

$$\phi(t) = \begin{cases} \sqrt{t}, & \text{if } t > 0, \\ 0, & \text{if } t \leq 0, \end{cases}$$

and γ is the unique fixed point of T.

Acknowledgments. The work is partially supported by CSIR, Govt. of India, Research Project No - 25(0168)/09/EMR-II. The support is gratefully acknowledged.

References

[1] B.S. Choudhury and K.P. Das, A new contraction principle in Menger spaces, Acta Mathematica Sinica, English Series, 24 (2008), 1379-1386.

[2] B.S. Choudhury, P.N. Dutta and K.P. Das, A fixed point result in Menger spaces using a real function, Acta. Math. Hungar., 122 (2008), 203-216.

[3] B.S. Choudhury and K.P. Das, A coincidence point result in Menger spaces using a control function, Chaos, Solitons and Fractals, 42 (2009), 3058-3063.

[4] B.S. Choudhury and K.P. Das, Fixed points of generalized Kannan type mappings in generalized Menger spaces, Commun. Korean Math. Soc., 24 (2009), 529-537.

[5] E.H. Connell, Properties of fixed point spaces, Proc. Amer. Math. Soc., 10 (1959), 974-979.

[6] P.N. Dutta, B.S. Choudhury and K.P. Das, Some fixed point results in Menger spaces using a control function, Surveys in Mathematics and its Applications, 4 (2009), 41-52.

[7] S. Gähler, 2-metrische Räume and ihre topologische strucktur, Math. Nachr., 26 (1963), 115-148.

[8] S. Gähler, Uber die unifromisieberkeit 2-metrischer Raume, Math. Nachr., 28 (1965), 235-244.

[9] I. Golet, A fixed point theorems in probabilistic 2-metric spaces, Sem. Math. Phys. Inst. Polit. Timisoara, 1988, 21-26.

[10] O. Hadzic, A fixed point theorem for multivalued mappings in 2-Menger spaces, Univ. u Novom Sadu, Zb. Rad. Prirod. Mat. Fak., Ser. Mat., 24 (1994), 1-7.

[11] O. Hadzic and E. Pap, Fixed Point Theory in Probabilistic Metric Spaces, Kluwer Academic Publishers, 2001.
[12] K. Iseki, Fixed point theorems in 2-metric space, Math. Sem. Notes, Kobe Univ., 3 (1975), 133-136.
[13] M.S. Khan, On the convergence of sequences of fixed points in 2-metric spaces, Indian J. Pure Appl. Math., 10 (1979), 1062-1067.
[14] M.S. Khan, M. Swaleh and S. Sessa, Fixed point theorems by altering distances between the points, Bull. Austral. Math. Soc., 30 (1984), 1-9.
[15] R. Kannan, Some results on fixed point, Bull. Cal. Math. Soc., 60 (1968), 71-76.
[16] R. Kannan, Some results on fixed point II, Amer. Math. Monthly, 76 (1969), 405-408.
[17] O. Kada, T. Suzuki, W. Takahashi, Nonconvex minimization theorems and fixed point theorems in complete metric spaces, Math. Japanica, 44 (1996), 381-391.
[18] M. Kikkawa and T. Suzuki, Some similarity between contractions and Kannan mappings, Fixed Point Theory and Applications, 2008 (2008), Article ID 649749.
[19] S.N. Lal and A.K. Singh , An analogue of Banach's contraction principle for 2-metric spaces, Bull. Austral. Math. Soc., 18 (1978), 137-143.
[20] D. Mihet, Altering distances in probabilistic Menger spaces, Nonlinear Analysis, 71 (2009), 2734-2738.
[21] S.V.R. Naidu, Some fixed point theorems in metric and 2-metric spaces, Int. J. Math. Math. Sci., 28:11 (2001), 625-638.
[22] S.V.R. Naidu, Some fixed point theorems in metric spaces by altering distances, Czechoslovak Mathematical Journal, 53 (2003), 205-212.
[23] B.E. Rhoades, Contraction type mapping on a 2-metric spaces, Math. Nachr., 91 (1979), 151-155.
[24] K.P.R. Sastry and G.V.R. Babu, Some fixed point theorems by altering distances between the points, Indian J. Pure. Appl. Math., 30(6) (1999), 641-647.
[25] K.P.R. Sastry, S.V.R. Naidu, G.V.R. Babu and G.A. Naidu, Generalisation of common fixed point theorems for weakly commuting maps by altering distances, Tamkang Journal of Mathematics, 31(3) (2000), 243-250.
[26] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, Elsevier, North-Holland, (1983).
[27] V.M. Sehgal and A.T. Bharucha-Reid, Fixed point of contraction mappings on PM space, Math. Sys. Theory, 6(2) (1972), 97-100.
[28] P. K. Saha and R. Tiwari, An alternative proof of Kannan's fixed point theorem in generalized metric space, News Bull. Cal. Math. Soc. 31 (2008), 15-18.
[29] P.V. Subrahmanyam, Completeness and fixed points, Monatsh. Math., 80 (1975), 325-330.
[30] Y. Shi, L. Ren and X. Wang, The extension of fixed point theorems for set valued mapping, J. Appl. Math. Computing, 13 (2003), 277-286.
[31] S.L. Singh, Rekha Talwar and Wen-Zhi Zeng, Common fixed point theorems in 2-Menger spaces and applications, Math. Student, 63 (1994), 74-80.
[32] Wen-Zhi Zeng, Probabilistic 2-metric spaces, J. Math. Research Expo., 2 (1987), 241-245.

Binayak S. Choudhury
Department of Mathematics
Bengal Engineering and Science University, Shibpur,
Howrah-711103, India.
E-mail address: binayak12@yahoo.co.in

Krishnapada Das
Department of Mathematics
Bengal Engineering and Science University, Shibpur,
Howrah-711103, India.
E-mail address: kestapm@yahoo.co.in

Samir Kumar Bhandari
Department of Mathematics
Bengal Engineering and Science University, Shibpur,
Howrah-711103, India.
E-mail address: skbhit@yahoo.co.in