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ON A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS IN $\mathbb{R}^N$ INVOLVING CRITICAL SOBOLEV EXPONENTS

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Abstract. We study here a class of quasilinear elliptic systems involving the p-Laplacian operator. Under some suitable assumptions on the nonlinearities, we show the existence result by using a fixed point theorem.

1. Introduction and Preliminaries

This paper is concerned with the existence of nontrivial solution to the quasilinear elliptic system of the form

$$\begin{cases} -\Delta_p u = f(x) \mid u \mid^{p^*-2} u + \lambda \frac{\partial F}{\partial u}(x,u,v), & \text{in } \mathbb{R}^N, \\ -\Delta_q v = g(x) \mid v \mid^{q^*-2} v + \mu \frac{\partial F}{\partial v}(x,u,v), & \text{in } \mathbb{R}^N, \\ u(x), v(x) \to 0, \quad \text{as } \mid x \mid \to +\infty \end{cases} \tag{1.1}$$

where Δ_p is the so called p-Laplacian operator, i.e. $\Delta_p u = div(\mid \nabla u \mid^{p-2} \nabla u)$. f, g and F are real-valued functions satisfying some assumptions; u and v are unknown real valued functions defined in $\mathbb{R}^N$ and belonging to appropriate function spaces; λ and μ are positive parameters, which can be taken equal to 1, and the parameters p and q are real numbers satisfying $2 \leq p, q < N$. The real number $p^* = \frac{Np}{N-p}$ designates the critical Sobolev exponent of p.
In recent years, several authors use different methods to solve quasilinear equations or systems defined in bounded or unbounded domains. Djellit and Tas [6] investigated a system such as (1.1) by employing variational approach.
In this work, motivated by [A. Djellit, S. Tas. On some nonlinear elliptic systems. Nonl. Anal. 59 (2004), 695-706], we show an existence result by using a fixed point theorem due to Bohnenblust-Karlin.
This paper is divided into three sections, organized as follows: in Section 2,we give some notation and hypotheses; Section 3 is devoted to establish an existence theorem.

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2. Notation and hypotheses

We denote by $D^{1,m}(\mathbb{R}^N)$ the completion of $C_0^\infty(\mathbb{R}^N)$ in the norm

$$\| u \|_{1,m} \equiv \| \nabla u \|_m = (\int_{\mathbb{R}^N} | \nabla u |^m \, dx)^{\frac{1}{m}}; \quad 1 < m < N.$$

It is well known that $D^{1,m}(\mathbb{R}^N)$ is a uniformly convex Banach space and may be written as

$$D^{1,m}(\mathbb{R}^N) = \{u \in L^{m^*}(\mathbb{R}^N); \quad \nabla u \in (L^m(\mathbb{R}^N))^N\}.$$

Moreover, we have the following Sobolev constant defined by

$$S_m \equiv C^{-m}(N,m) = \inf \left\{ \frac{\| u \|_{1,m}^m}{\| u \|_{m^*}^m}, \quad u \in D^{1,m}(\mathbb{R}^N) \setminus \{0\} \right\}.$$

We denote Z by the product space $Z \equiv D^{1,p}(\mathbb{R}^N) \times D^{1,q}(\mathbb{R}^N)$ with the norm $\| (u,v) \|_Z = \| u \|_{1,p} + \| v \|_{1,q}$; Z^* is the dual space of Z equipped with the dual norm $\| . \|_*$.
In addition, let T and N be two operators defined from Z into Z^* by

$$T(u,v)(w,z) = \int_{\mathbb{R}^N} | \nabla u |^{p-2} \nabla u \nabla w dx + \int_{\mathbb{R}^N} | \nabla v |^{q-2} \nabla v \nabla z dx,$$

and

$$\begin{aligned} N(u,v)(w,z) &= \int_{\mathbb{R}^N} [(f(x) \mid u \mid^{p^*-2} u + \lambda \frac{\partial F}{\partial u}(x,u,v))w \\ &\quad + (g(x) \mid v \mid^{q^*-2} v + \mu \frac{\partial F}{\partial v}(x,u,v))z]dx, \quad \forall \ (u,v),(w,z) \in Z. \end{aligned}$$

Now, we recall the fixed point theorem due to Bohnenblust-Karlin (see [11]).

Theorem 2.1.([11]) Let Z be a Banach space, let $B \subset Z$ be a nonempty, closed, convex set and let $S : B \to 2^B$ be a set-valued mapping satisfying
(a) for each $U \in Z$, the set SU is nonempty, closed and convex,
(b) S is closed,
(c) the set $S(B) = \bigcup_{U \in B} SU$ is relatively compact.
Then S has a fixed point in B i.e. there is $U \in B$ such that $U \in SU$.

Our aim is to find the condition of the above theorem. The fixed points of the set-valued mapping S are precisely the weak solutions of system (1.1). In other words, we state the existence of a pair $(u,v) \in Z$ such that $T(u,v)(w,z) = N(u,v)(w,z)$, $\forall (w,z) \in Z$, under the following assumptions.
(H1) f and g are positive and bounded functions.
(H2) $F \in C^1(\mathbb{R}^N, \mathbb{R}, \mathbb{R})$ and $F(x,0,0) = 0$.
(H3) For all $U = (u,v) \in \mathbb{R}^2$ and for almost every $x \in \mathbb{R}^N$

$$\mid \frac{\partial F}{\partial u}(x,U) \mid \leq a_1(x) \mid U \mid^{p_1-1} + a_2(x) \mid U \mid^{p_2-1}$$
$$\mid \frac{\partial F}{\partial v}(x,U) \mid \leq b_1(x) \mid U \mid^{q_1-1} + b_2(x) \mid U \mid^{q_2-1}$$
where $1 < p_1, q_1 < \min(p,q), \quad \max(p,q) < p_2, q_2 < \min(p^*, q^*)$
$$a_i \in L^{\alpha_i}(\mathbb{R}^N) \cap L^{\beta_i}(\mathbb{R}^N), \quad b_i \in L^{\gamma_i}(\mathbb{R}^N) \cap L^{\delta_i}(\mathbb{R}^N), \quad i = 1, 2.$$
$$\alpha_i = \frac{p^*}{p^* - p_i}, \quad \gamma_i = \frac{q^*}{q^* - q_i}, \quad \beta_i = \frac{p^* q^*}{p^* q^* - p^*(p_i - 1) - q^*},$$
$$\delta_i = \frac{p^* q^*}{p^* q^* - q^*(q_i - 1) - p^*}.$$

3. Existence of solutions

The goal of this section is to establish the following result.

Theorem 3.1. Under hypotheses $(H_1) - (H_3)$, the equation $T(u,v) = N(u,v)$ has a solution in Z.

First, two preliminary results. The first one concerns the properties of the operator T while the second one describes the property of the operator N.

Lemma 3.2. The operator T is monotone, hemicontinuous, coercive and satisfies the following property:

$$[(u_n, v_n) \rightharpoonup (u,v), T(u_n, v_n) \to T(u,v)] \Rightarrow (u_n, v_n) \to (u,v). \tag{3.1}$$

Proof. Let us denote by T_p the operator defined from $D^{1,p}(\mathbb{R}^N)$ into $(D^{1,p}(\mathbb{R}^N))^*$ by

$$T_p(u)w = \int_{\mathbb{R}^N} \mid \nabla u \mid^{p-2} \nabla u \cdot \nabla w dx, \qquad \forall u, w \in D^{1,p}(\mathbb{R}^N)$$

and T_q the corresponding one with p replaced by q.
Observe that $T(u,v)(w,z) = T_p(u)w + T_q(v)z$, $\forall (u,v), (w,z) \in Z$. T_p, T_q are duality mappings on $D^{1,p}(\mathbb{R}^N)$ and $D^{1,q}(\mathbb{R}^N)$ corresponding to the Guage functions $\Phi_p(T) = t^{p-1}$ and $\Phi_q(t) = t^{q-1}$, respectively. Hence T_p, T_q are demicontinuous(see[3,p.175]).
So, for $(u_n, v_n) \to (u,v)$ in Z, we have $T_p(u_n) \rightharpoonup T_p(u)$ in $(D^{1,p}(\mathbb{R}^N))^*$ and $T_q(v_n) \rightharpoonup T_q(v)$ in $(D^{1,q}(\mathbb{R}^N))^*$. Since $D^{1,p}(\mathbb{R}^N)$ and $D^{1,q}(\mathbb{R}^N)$ are reflexive, and the dual space of any reflexive space is reflexive. we get $T(u_n, v_n) = T_p(u_n) + T_q(v_n) \rightharpoonup T_p(u) + T_q(v) = T(u,v)$ in Z^*, i.e. T is demicontinuous. So, it is hemicontinuous.
We note according to [2] that $\forall \lambda, \mu \in \mathbb{R}^N$

$$\mid \lambda - \mu \mid^p \leq (\mid \lambda \mid^{p-2} \lambda - \mid \mu \mid^{p-2} \mu) \cdot (\lambda - \mu) \qquad \text{if } p \geq 2.$$

Replacing λ and μ by ∇u, ∇v respectively and integrating over $\mathbb{R}^N$, we obtain

$$\int_{\mathbb{R}^N} \mid \nabla u - \nabla v \mid^p \leq \int_{\mathbb{R}^N} (\mid \nabla u \mid^{p-2} \nabla u - \mid \nabla v \mid^{p-2} \nabla v) \cdot (\nabla u - \nabla v) \quad \text{if } p \geq 2. \tag{3.2}$$

By virtue of (3.2) we show that T_p (similarly T_q) is monotone, indeed,

$$\begin{aligned}(T_pu - T_pw)(u-w) &= T_pu(u-w) - T_pw(u-w)\\ &= \int_{\mathbb{R}^N} (\mid \nabla u \mid^{p-2} \nabla u \nabla(u-w)dx\\ &\quad - \int_{\mathbb{R}^N} (\mid \nabla w \mid^{p-2} \nabla w \nabla(u-w)dx\\ &= \int_{\mathbb{R}^N} (\mid \nabla u \mid^{p-2} \nabla u - \mid \nabla w \mid^{p-2} \nabla w)(\nabla u - \nabla w)dx\\ &\geq \int_{\mathbb{R}^N} \mid \nabla u - \nabla w \mid^p = \| u-w \|_{1,p}^p \geq 0.\end{aligned}$$

So, T is monotone.On the other hand, T is coercive since $T(u,v)(u,v) = \| u \|_{1,p}^p + \| v \|_{1,q}^q$. Now we show that T satisfies property (3.1).
Let us take a sequence $(u_n, v_n) \in Z$ such that $(u_n, v_n) \rightharpoonup (u,v)$ in Z and $T(u_n, v_n) \to T(u,v)$ in Z^*. Then $T(u_n,v_n)(u_n,v_n) \to T(u,v)(u,v)$. So $\| u_n \|_{1,p}^p + \| v_n \|_{1,q}^q \to \| u \|_{1,p}^p + \| v \|_{1,q}^q$. According to the uniform convexity of Z, $(u_n, v_n) \to (u,v)$ in Z. □

Lemma 3.3. Under hypothesis $(H_1) - (H_3)$, the operator N is compact.
Proof. Let B_R be the ball of radius R, centered at the origin of $\mathbb{R}^N$. We put $B'_R = \mathbb{R}^N - B_R$ and we designate N_R the operator defined from $Z_R \equiv D^{1,p}(B_R) \times D^{1,q}(B_R)$ into Z_R^* by

$$\begin{aligned}N_R(u,v)(w,z) &= \int_{B_R} [(f(x) \mid u \mid^{p^*-2} u + \lambda \frac{\partial F}{\partial u}(x,u,v))w\\ &\quad + (g(x) \mid v \mid^{q^*-2} v + \mu \frac{\partial F}{\partial v}(x,u,v))z]dx.\end{aligned}$$

Let $\{(u_n, v_n)\}$ be a bounded sequence in Z.There is a subsequence denoted again as $\{(u_n, v_n)\}$, weakly convergent to (u,v) in Z. For $(w,z) \in Z$, we have

$$\begin{aligned}&\mid N(u_n,v_n)(w,z) - N(u,v)(w,z) \mid\\ &= \mid N_R(u_n,v_n)(w,z) - N_R(u,v)(w,z) \mid\\ &+ \mid \int_{B'_R} f(x)(\mid u_n \mid^{p^*-2} u_n - \mid u \mid^{p^*-2} u)wdx \mid\\ &+ \mid \int_{B'_R} g(x)(\mid v_n \mid^{q^*-2} v_n - \mid v \mid^{q^*-2} v)zdx \mid\\ &+ \mid \int_{B'_R} \lambda (\frac{\partial F}{\partial u}(x,u_n,v_n) - \frac{\partial F}{\partial u}(x,u,v))wdx \mid\\ &+ \mid \int_{B'_R} \mu (\frac{\partial F}{\partial v}(x,u_n,v_n) - \frac{\partial F}{\partial v}(x,u,v))zdx \mid . \qquad (3.3)\end{aligned}$$

Since the restriction operator $(u,v) \to (u,v)|_{B_R}$ is continuous from $D^{1,p}(\mathbb{R}^N) \times D^{1,q}(\mathbb{R}^N)$ into $D^{1,p}(B_R) \times D^{1,q}(B_R)$, we have $(u_n, v_n) \rightharpoonup (u,v)$ in $D^{1,p}(B_R) \times D^{1,q}(B_R)$.We have also that the embeddings $D^{1,p}(B_R) \hookrightarrow L^p(B_R)$ and $D^{1,q}(B_R) \hookrightarrow L^q(B_R)$ are compact, so

$$\begin{aligned}u_n \to u &\qquad \text{a.e. in } B_R,\\ v_n \to v &\qquad \text{a.e. in } B_R.\end{aligned}$$

Hypothesis (H_3) gives

$$\begin{aligned} \left| \frac{\partial F}{\partial u}(x,u_n,v_n)w \right| &\leq [a_1(x)(| u_n |^{p_1-1} + | v_n |^{p_1-1}) \\ &\quad + a_2(x)(| u_n |^{p_2-1} + | v_n |^{p_2-1})] \mid w \mid, \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} \left| \frac{\partial F}{\partial v}(x,u_n,v_n)z \right| &\leq [b_1(x)(| u_n |^{q_1-1} + | v_n |^{q_1-1}) \\ &\quad + b_2(x)(| u_n |^{q_2-1} + | v_n |^{q_2-1})] \mid z \mid . \end{aligned} \tag{3.5}$$

Using Holder's inequality and Sobolev's imbedding, and the fact that $a_i \in L^{\alpha_i}(\mathbb{R}^N) \bigcap L^{\beta_i}(\mathbb{R}^N)$, $b_i \in L^{\gamma_i}(\mathbb{R}^N) \bigcap L^{\delta_i}(\mathbb{R}^N)$, we get that the right hand side of inequalities (3.4), (3.5) belong to $L^1(B_R)$. Hence under hypotheses $(H_1)-(H_3)$ and by using Holder's inequality and Sobolev's imbedding, according to Dominated convergence theorem, we obtain,the first expression on the right hand side of the inequality (3.3) tends to 0 as $n \to +\infty$; Taking (H_1) and (H_3) into account, and the fact that for $i = 1, 2$,

$$\| a_i \|_{L^{\alpha_i}(B'_R)} + \| a_i \|_{L^{\beta_i}(B'_R)} \to 0,$$
$$\| b_i \|_{L^{\gamma_i}(B'_R)} + \| b_i \|_{L^{\delta_i}(B'_R)} \to 0,$$

as $R \to +\infty$; we obtain, the other expressions tend also to 0 as R sufficiently large. So, the compactness of N follows. □

Lemma 3.4. Suppose that (H_1) and (H_3) hold. There is a constant $k > 0$ such that $T(u,v) = N(\sigma,\rho)$ and $\| (\sigma,\rho) \|_Z \leq k$ implies $\| (u,v) \|_Z \leq k$.
Proof. Let (u,v), $(\sigma,\rho) \in Z$ be such that $T(u,v) = N(\sigma,\rho)$, then

$$T(u,v)(w,z) = N(\sigma,\rho)(w,z), \qquad \forall (w,z) \in Z.$$

In particular, we have $T(u,v)(u,0) = N(\sigma,\rho)(u,0)$ i.e.

$$\| u \|_{1,p}^p = \int_{\mathbb{R}^N} | \nabla u |^p \, dx = \int_{\mathbb{R}^N} (f(x) \mid \sigma \mid^{p^*-2} \sigma + \lambda \frac{\partial F}{\partial u}(x,\sigma,\rho)) u dx. \tag{3.6}$$

In view of (H_1) and (H_3), by using Holder's inequality and Sobolev's imbedding we obtain

$$\begin{aligned} \int_{\mathbb{R}^N} f(x) \mid \sigma \mid^{p^*-2} \sigma u dx &\leq c' \int_{\mathbb{R}^N} | \sigma |^{p^*-1} | u | \, dx \\ &\leq c' \| u \|_{p^*} \| \sigma \|_{p^*}^{p^*-1} \leq c_1 \| u \|_{1,p} \| \sigma \|_{1,p}^{p^*-1}, \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} \int_{\mathbb{R}^N} \lambda \frac{\partial F}{\partial u}(x,\sigma,\rho) u dx &\leq c_1 \| u \|_{1,p} (\| a_1 \|_{\alpha_1} \| \sigma \|_{1,p}^{p_1-1} + \| a_1 \|_{\beta_1} \| \rho \|_{1,q}^{p_1-1} \\ &\quad + \| a_2 \|_{\alpha_2} \| \sigma \|_{1,p}^{p_2-1} + \| a_2 \|_{\beta_2} \| \rho \|_{1,q}^{p_2-1}) \end{aligned} \tag{3.8}$$

So, by virtue of (3.6), (3.7) and (3.8) we get

$$\begin{aligned} \| u \|_{1,p}^{p-1} &\leq c_1 \lambda (\| \sigma \|_{1,p}^{p^*-1} + \| a_1 \|_{\alpha_1} \| \sigma \|_{1,p}^{p_1-1} + \| a_1 \|_{\beta_1} \| \rho \|_{1,q}^{p_1-1} \\ &\quad + \| a_2 \|_{\alpha_2} \| \sigma \|_{1,p}^{p_2-1} + \| a_2 \|_{\beta_2} \| \rho \|_{1,q}^{p_2-1}). \end{aligned} \tag{3.9}$$

In the same way, we have

$$\begin{aligned}\| v \|_{1,q}^{q-1} &\leq c_2\,\mu(\| \rho \|_{1,q}^{q^*-1} + \| b_1 \|_{\delta_1} \| \sigma \|_{1,p}^{q_1-1} + \| b_1 \|_{\gamma_1} \| \rho \|_{1,q}^{q_1-1} \\ &\quad + \| b_2 \|_{\delta_2} \| \sigma \|_{1,p}^{q_2-1} + \| b_2 \|_{\gamma_2} \| \rho \|_{1,q}^{q_2-1}). \end{aligned} \tag{3.10}$$

If $\| (\sigma, \rho) \|_Z = \| \sigma \|_{1,p} + \| \rho \|_{1,q} \leq k$, we have $\| \sigma \|_{1,p} \leq k$ and $\| \rho \|_{1,q} \leq k$. So, in view of (3.9), (3.10) we get

$$\begin{aligned}\| u \|_{1,p}^{p-1} &\leq c\,\lambda(k^{p^*-1} + k^{p_1-1} + k^{p_2-1}), \\ \| v \|_{1,q}^{q-1} &\leq c\,\mu(k^{q^*-1} + k^{q_1-1} + k^{q_2-1}).\end{aligned}$$

Since $p_1 < p_2 < p^*$ and $q_1 < q_2 < q^*$, there is a $k > 0$ such that $c\,(k^{p^*-1} + k^{p_1-1} + k^{p_2-1}) \leq (\frac{k}{2})^{p-1}$ and $c\,(k^{q^*-1} + k^{q_1-1} + k^{q_2-1}) \leq (\frac{k}{2})^{q-1}$. So, $\| \sigma \|_{1,p} + \| \rho \|_{1,q} \leq k$ implies $\| u \|_{1,p} + \| v \|_{1,q} \leq k$. □

We have on the following proposition, which is standard in the theory of monotone operators.

Proposition 3.5. Let X be a real normed space, $T : X \to X^*$ be a monotone, hemicontinuous operator and let $w \in X$, $f \in X^*$.
The following two assertions are equivalent
(a) $Tw = f$
(b) $\langle Tz - f, z - w\rangle \geq 0$ for all $z \in X$.
Now, we are ready to give the following proof.

Proof of Theorem 3.1. In view of lemma 3.4, let $B \subset Z$ be the closed ball of radius k centered at the origin. We define the operator S from B into 2^B by

$$(\sigma, \rho) \mapsto S(\sigma, \rho) = \{(u, v); \quad T(u, v) = N(\sigma, \rho)\}.$$

By virtue of lemma 3.2, T is monotone, hemicontinuous and coercive, then according to Browder's Theorem (see[13,p.557]), $S(\sigma, \rho)$ is nonempty, convex, closed and bounded for every $(\sigma, \rho) \in B$. Furthermore, the operator S is closed, indeed, let $\{(\sigma_n, \rho_n)\} \subset B$; $(\sigma_n, \rho_n) \to (\sigma, \rho) \in Z$, and $\{(u_n, v_n)\} \subset Z$ such that $(u_n, v_n) \in S(\sigma_n, \rho_n)$ and $(u_n, v_n) \to (u, v)$ in Z.
Since N is continuous, it is demicontinuous. We have also that T is demicontinuous, so we can write

$$\begin{aligned}T(u_n, v_n) &\rightharpoonup T(u, v), \\ N(\sigma_n, \rho_n) &\rightharpoonup N(\sigma, \rho).\end{aligned}$$

Since $(u_n, v_n) \in S(\sigma_n, \rho_n)$, we have $T(u_n, v_n) = N(\sigma_n, \rho_n)$. Hence $T(u_n, v_n) \rightharpoonup N(\sigma, \rho)$ .Since the weak limit is unique, we get

$$T(u, v) = N(\sigma, \rho).$$

On the other hand , B is closed, consequently $(\sigma, \rho) \in B$ and then $(u, v) \in S(\sigma, \rho)$.
Now, let us show that $S(B) = \bigcup_{(\sigma,\rho)\in B} S(\sigma, \rho)$ is relatively compact.
Let $(u_n, v_n) \subset \bigcup_{(\sigma,\rho)\in B} S(\sigma, \rho)$ and $(\sigma_n, \rho_n) \subset B$ be such that

$$T(u_n, v_n) = N(\sigma_n, \rho_n). \tag{3.11}$$

In view of lemma 3.3, $N(B)$ is relatively compact. So there exists $H \in Z^*$ such that $N(\sigma_n, \rho_n) \to H$, Hence by (3.11) we have $T(u_n, v_n) \to H$. Consequently $T(u_n, v_n)$ is bounded. Since T is coercive, (u_n, v_n) is also bounded; otherwise, if

$\| (u_n, v_n) \| \to \infty$, we have $T(u_n, v_n) \to \infty$, which is a contradiction. Hence, we may choose a subsequence denoted again by $\{(u_n, v_n)\}$, weakly convergent to (u_0, v_0) in Z.
The monotonicity of T leads to $(T(u,v) - T(u_n, v_n))(u - u_n, v - v_n) \geq 0, \forall (u,v) \in Z$, and passing to the limit, we obtain

$$(T(u,v) - H)(u - u_0, v - v_0) \geq 0, \qquad \forall (u,v) \in Z,$$

i.e. $\langle T(u,v) - H, (u,v) - (u_0, v_0) \rangle \geq 0, \forall (u,v) \in Z$. So by virtue of proposition 3.5, we have $T(u_0, v_0) = H$. Taking the condition (3.1) into account, we obtain the convergence of (u_n, v_n) to (u_0, v_0). Finally, by Bohnenblust-Karlin fixed point theorem, S possesses a fixed point.i.e. there exist $(\sigma_0, \rho_0) \in B$ such that $T(\sigma_0, \rho_0) = N(\sigma_0, \rho_0)$. □

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