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NEW CRITERIA FOR GENERALIZED WEIGHTED COMPOSITION OPERATORS FROM MIXED NORM SPACES INTO BLOCH-TYPE SPACES

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Abstract. New criteria for the boundedness and the compactness of the generalized weighted composition operators from mixed norm spaces into Bloch-type spaces are given in this paper.

1. Introduction

Let $\mathbb{D}$ denote the open unit disk in the complex plane $\mathbb{C}$, i.e., $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Let $H(\mathbb{D})$ be the space of all analytic functions on $\mathbb{D}$.

Let $0 < p, q < \infty$, $\gamma > -1$. An $f \in H(\mathbb{D})$ is said to belong to the mixed norm space, denoted by $H_{p,q,\gamma} = H_{p,q,\gamma}(\mathbb{D})$, if

$$\|f\|^q_{H_{p,q,\gamma}} = \int_0^1 \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right)^{\frac{q}{p}} (1-r)^\gamma dr < \infty.$$

Let $\alpha \in (0, \infty)$. The Bloch-type space (or α-Bloch space), denoted by $\mathcal{B}^\alpha$, consists of all $f \in H(\mathbb{D})$ such that

$$\beta^\alpha(f) = \sup_{z \in \mathbb{D}} (1-|z|^2)^\alpha |f'(z)| < \infty.$$

Under the natural norm $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + \beta^\alpha(f)$, $\mathcal{B}^\alpha$ is a Banach space. See [21] for more information about Bloch-type space.

Let $\varphi : \mathbb{D} \to \mathbb{D}$ be an analytic self-map. The composition operator, denoted by C_φ, is defined as

$$C_\varphi(f) = f \circ \varphi, \quad f \in H(\mathbb{D}).$$

Let u be a fixed analytic function on $\mathbb{D}$. The weighted composition operator uC_φ, which induced by φ and u, is defined as follows.

$$(uC_\varphi f)(z) = u(z) f(\varphi(z)), \quad f \in H(\mathbb{D}).$$

We refer [2] for the theory of the composition operator on function spaces.

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The generalized weighted composition operator $D^n_{\varphi,u}$, which induced by Zhu (see [22–24]), is defined as follows.

$$(D^n_{\varphi,u}f)(z) = u(z) \cdot f^{(n)}(\varphi(z)), \quad f \in H(\mathbb{D}),\ z \in \mathbb{D}.$$

Here $f^{(n)}$ denote the n-th differentiation of f. This operator includes many known operators. If $n = 0$, then it is just the weighted composition operator uC_φ. If $n = 0$ and $u(z) \equiv 1$, then we obtain the composition operator C_φ. If $n = 1$, $u(z) = \varphi'(z)$, then $D^n_{\varphi,u} = DC_\varphi$, which was studied in [6, 9, 16, 19]. When $n = 1$ and $u(z) = 1$, then $D^n_{\varphi,u} = C_\varphi D$, which was studied in [6, 16, 19]. If we put $n = 1$ and $\varphi(z) = z$, then $D^n_{\varphi,u} = M_u D$. See [17, 18, 22–24] for the study of the generalized weighted composition operator on various function spaces.

Composition operators and weighted composition operators between Bloch-type spaces and some other spaces in one, as well as, in several complex variables were studied, for example, in [1, 4, 5, 7–13, 15–17, 19, 20, 22–25].

In [17], the author studied the generalized weighted composition operators $D^n_{\varphi,u}$ from $H_{p,q,\gamma}$ into weighted-type spaces. In [18], the author studied the generalized weighted composition operators $D^n_{\varphi,u}$ from $H_{p,q,\gamma}$ into the mth weighted-type space. Among others, he obtained the following result.

Theorem A Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a non-negative integer. Assume that $0 < p, q < \infty$, $\gamma > -1$ and $0 < \alpha < \infty$. Then the following propositions hold:

(a) The operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is bounded if and only if

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha |u'(z)|}{(1-|\varphi(z)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}} < \infty \text{ and } \sup_{z\in\mathbb{D}} \frac{(1-|z|^2)^\alpha |\varphi'(z)||u(z)|}{(1-|\varphi(z)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n+1}} < \infty. \tag{1}$$

(b) The operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is compact if and only if

$$\lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)^\alpha |u'(z)|}{(1-|\varphi(z)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}} = \lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)^\alpha |\varphi'(z)||u(z)|}{(1-|\varphi(z)|^2)^{1+\frac{1}{p}+\frac{\gamma+1}{q}+n}} = 0. \tag{2}$$

In this paper we give a new criteria for the boundedness and compactness of the generalized weighted composition operators $D^n_{\varphi,u}$ from $H_{p,q,\gamma}$ into $\mathcal{B}^\alpha$.

Throughout this paper, constants are denoted by C, they are positive and may differ from one occurrence to the next. The notation $a \asymp b$ means that there is a positive constant C such that $C^{-1}b \le a \le Cb$.

2. Main results and proofs

In this section we give our main results and proofs. For this purpose, we need two lemmas as follows.

Lemma 1. [18] *Assume that* $0 < p, q < \infty$ *and* $\gamma > -1$. *Let* $f \in H_{p,q,\gamma}$. *Then there is a positive constant* C *independent of* f *such that*

$$|f^{(n)}(z)| \le C \frac{\|f\|_{H_{p,q,\gamma}}}{(1-|z|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}}.$$

The following criterion follows from standard arguments similar, for example, to those outlined in Proposition 3.11 of [2].

Lemma 2. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Assume that $0 < p, q < \infty$, $\gamma > -1$ and $0 < \alpha < \infty$. The operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is compact if and only if $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H_{p,q,\gamma}$ which converges to zero uniformly on compact subsets of $\mathbb{D}$, we have $\|D^n_{\varphi,u} f_k\|_{\mathcal{B}^\alpha} \to 0$ as $k \to \infty$.*

Fix $0 < p, q < \infty$, $\gamma > -1$. For $a \in \mathbb{D}$ and $b > \frac{\gamma+1}{q}$, set

$$f_a(z) = \frac{(1-|a|^2)^{b-\frac{\gamma+1}{q}}}{(1-\overline{a}z)^{\frac{1}{p}+b}}, \text{ and } g_a(z) = \left(\frac{1-|a|^2}{1-\overline{a}z}\right) f_a(z). \tag{3}$$

We use these two families of functions to characterize the generalized weighted composition operators $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$.

Theorem 1. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Assume that $0 < p, q < \infty$, $\gamma > -1$ and $0 < \alpha < \infty$. Then the following conditions are equivalent:*

(a) *The operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is bounded;*

(b) *$u\varphi \in \mathcal{B}^\alpha$, $u \in \mathcal{B}^\alpha$,*

$$A := \sup_{w\in\mathbb{D}} \|D^n_{\varphi,u} f_{\varphi(w)}\|_{\mathcal{B}^\alpha} < \infty \quad \textit{and } B := \sup_{w\in\mathbb{D}} \|D^n_{\varphi,u} g_{\varphi(w)}\|_{\mathcal{B}^\alpha} < \infty.$$

Proof. $(a) \Rightarrow (b)$. Assume $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is bounded. Taking the functions z^n and z^{n+1} and using the boundedness of $D^n_{\varphi,u}$ we see that $u\varphi \in \mathcal{B}^\alpha$ and $u \in \mathcal{B}^\alpha$. For each $a \in \mathbb{D}$ and $b > \frac{\gamma+1}{q}$, from [18] we know that $f_a, g_a \in H_{p,q,\gamma}$. Moreover $\|f_a\|_{H_{p,q,\gamma}}$ and $\|g_a\|_{H_{p,q,\gamma}}$ are bounded by constants independent of a. By the boundedness of $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$, we get

$$\sup_{a\in\mathbb{D}} \|D^n_{\varphi,u} f_{\varphi(a)}\|_{\mathcal{B}^\alpha} \le \|D^n_{\varphi,u}\| \sup_{a\in\mathbb{D}} \|f_{\varphi(a)}\|_{H_{p,q,\gamma}} \le C\|D^n_{\varphi,u}\| < \infty$$

and

$$\sup_{a\in\mathbb{D}} \|D^n_{\varphi,u} g_{\varphi(a)}\|_{\mathcal{B}^\alpha} \le \|D^n_{\varphi,u}\| \sup_{a\in\mathbb{D}} \|g_{\varphi(a)}\|_{H_{p,q,\gamma}} \le C\|D^n_{\varphi,u}\| < \infty,$$

as desired.

$(b) \Rightarrow (a)$. Suppose that $u\varphi \in \mathcal{B}^\alpha$, $u \in \mathcal{B}^\alpha$, A and B are finite. Now we need to show that the inequalities in (1) hold. A calculation shows that

$$f_a^{(n)}(a) = \prod_{j=0}^{n-1} \left(\frac{1}{p} + b + j\right) \frac{\overline{a}^n}{(1-|a|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}}, \tag{4}$$

$$g_a^{(n)}(a) = \prod_{j=1}^{n} \left(\frac{1}{p} + b + j\right) \frac{\overline{a}^n}{(1-|a|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}}. \tag{5}$$

From (4), for $w \in \mathbb{D}$, we have

$$\begin{aligned}(D^n_{\varphi,u} f_{\varphi(w)})'(w) \quad &= \quad \prod_{j=0}^{n-1} \left(\frac{1}{p} + b + j\right) \frac{u'(w)\overline{\varphi(w)}^n}{(1-|\varphi(w)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}} \\ &\quad + \prod_{j=0}^{n} \left(\frac{1}{p} + b + j\right) \frac{u(w)\varphi'(w)\overline{\varphi(w)}^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}.\end{aligned} \tag{6}$$

For simplicity, we denote $\prod_{j=0}^{n-1}\left(\frac{1}{p}+b+j\right)$ and $\prod_{j=0}^{n}\left(\frac{1}{p}+b+j\right)$ by Q and R, respectively. Therefore

$$\begin{aligned}
&\frac{(1-|w|^2)^\alpha|u'(w)||\varphi(w)|^n}{(1-|\varphi(w)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}}\\
\leq\ & \frac{1}{Q}(1-|w|^2)^\alpha|(D^n_{\varphi,u}f_{\varphi(w)})'(w)|+\frac{R}{Q}\frac{(1-|w|^2)^\alpha|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\\
\leq\ & \frac{A}{Q}+\frac{R}{Q}\frac{(1-|w|^2)^\alpha|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}. \qquad (7)
\end{aligned}$$

In addition,

$$\begin{aligned}
(D^n_{\varphi,u}g_{\varphi(w)})'(w) &= \prod_{j=1}^{n}\Big(\frac{1}{p}+b+j\Big)\frac{u'(w)\overline{\varphi(w)}^n}{(1-|\varphi(w)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}}}\\
&\quad+\prod_{j=1}^{n+1}\Big(\frac{1}{p}+b+j\Big)\frac{u(w)\varphi'(w)\overline{\varphi(w)}^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}}}. \qquad (8)
\end{aligned}$$

Therefore, subtracting (6) from (8) and taking the modulus, we obtain

$$\begin{aligned}
&\frac{|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\\
\leq\ & \frac{\frac{1}{p}+b+n}{R}|(D^n_{\varphi,u}f_{\varphi(w)})'(w)|+\frac{\frac{1}{p}+b}{R}|(D^n_{\varphi,u}g_{\varphi(w)})'(w)|, \qquad (9)
\end{aligned}$$

which yields

$$\frac{(1-|w|^2)^\alpha|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\leq\frac{\frac{1}{p}+b+n}{R}A+\frac{\frac{1}{p}+b}{R}B. \qquad (10)$$

Hence, by (7), we get

$$\frac{(1-|w|^2)^\alpha|u'(w)||\varphi(w)|^n}{(1-|\varphi(w)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}}\leq\frac{(\frac{1}{p}+b+1+n)}{Q}A+\frac{(\frac{1}{p}+b)}{Q}B. \qquad (11)$$

Fix $r\in(0,1)$. If $|\varphi(w)|>r$, then from (10) we obtain

$$\frac{(1-|w|^2)^\alpha|u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\leq\frac{1}{r^{n+1}}(\frac{\frac{1}{p}+b+n}{R}A+\frac{\frac{1}{p}+b}{R}B). \qquad (12)$$

On the other hand, if $|\varphi(w)|\leq r$, by the fact that

$$(1-|w|^2)^\alpha|u(w)\varphi'(w)|\leq\|u\varphi\|_{\mathcal{B}^\alpha}+\|u\|_{\mathcal{B}^\alpha},$$

we get

$$\frac{(1-|w|^2)^\alpha|u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\leq\frac{1}{(1-r^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}}\Big(\|u\varphi\|_{\mathcal{B}^\alpha}+\|u\|_{\mathcal{B}^\alpha}\Big). \qquad (13)$$

From (12) and (13) we see that the second inequality in (1) holds. Using similar arguments and (11) we can obtain that the first inequality in (1) holds as well. The proof of this theorem is finished. □

Theorem 2. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Assume that $p, q > 0$, $\gamma > -1$ and $0 < \alpha < \infty$. Suppose that the operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is bounded, then the following conditions are equivalent:*

(a) *The operator $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is compact;*

(b)

$$\lim_{|\varphi(w)|\to 1} \|D^n_{\varphi,u} f_{\varphi(w)}\|_{\mathcal{B}^\alpha} = 0 \quad \textit{and} \quad \lim_{|\varphi(w)|\to 1} \|D^n_{\varphi,u} g_{\varphi(w)}\|_{\mathcal{B}^\alpha} = 0.$$

Proof. $(a) \Longrightarrow (b)$. Assume that $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is compact. Let $\{w_k\}_{k\in\mathbb{N}}$ be a sequence in $\mathbb{D}$ such that $\lim_{k\to\infty} |\varphi(w_k)| = 1$. Since the sequences $\{f_{\varphi(w_k)}\}$ and $\{g_{\varphi(w_k)}\}$ are bounded in $H_{p,q,\gamma}$ and converge to 0 uniformly on compact subsets of $\mathbb{D}$, by Lemma 2, we get

$$\|D^n_{\varphi,u} f_{\varphi(w_k)}\|_{\mathcal{B}^\alpha} \to 0 \text{ and } \|D^n_{\varphi,u} g_{\varphi(w_k)}\|_{\mathcal{B}^\alpha} \to 0 \tag{14}$$

as $k \to \infty$, which means that (b) holds.

$(b) \Longrightarrow (a)$. Suppose that the limits in (b) are 0. Using the inequality (9), we get

$$\begin{aligned} & \frac{(1-|w|^2)^\alpha |u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}} \\ \leq \; & \frac{(\frac{1}{p}+b+n)\|D^n_{\varphi,u} f_{\varphi(w)}\|_{\mathcal{B}^\alpha} + (\frac{1}{p}+b)\|D^n_{\varphi,u} g_{\varphi(w)}\|_{\mathcal{B}^\alpha}}{R|\varphi(w)|^{n+1}} \to 0 \end{aligned} \tag{15}$$

as $|\varphi(w)| \to 1$. Moreover, using (7), we deduce

$$\frac{(1-|w|^2)^\alpha |u'(w)|}{(1-|\varphi(w)|^2)^{\frac{\gamma+1}{q}+\frac{1}{p}+n}} \leq \frac{\|D^n_{\varphi,u} f_{\varphi(w)}\|_{\mathcal{B}^\alpha}}{Q|\varphi(w)|^n} + \frac{R}{Q} \frac{(1-|w|^2)^\alpha |u(w)\varphi'(w)||\varphi(w)|}{(1-|\varphi(w)|^2)^{1+\frac{\gamma+1}{q}+\frac{1}{p}+n}} \to 0,$$

as $|\varphi(w)| \to 1$. By Theorem A we see that $D^n_{\varphi,u} : H_{p,q,\gamma} \to \mathcal{B}^\alpha$ is compact. The proof of this theorem is complete. □

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