

COINCIDENCE AND FIXED POINTS IN G -METRIC SPACES

(COMMUNICATED BY DANIEL PELLEGRINO)

M. ALAMGIR KHAN AND SUNNY CHAUHAN

ABSTRACT. The intent of this paper is to extend the notions of occasionally weakly compatible mappings, subcompatibility and subsequential continuity in framework of generalized metric spaces and prove some common fixed point theorems. We give some examples which demonstrate the validity of the hypotheses and degree of generality of our results. Our results are independent of the continuity requirement of the involved mappings and completeness (or closedness) of the underlying space (or subspaces). Several known results are generalized in this note.

1. INTRODUCTION

In 1992, Dhage [8] introduced the concept of D -metric spaces. Mustafa and Sims [19] shown that most of the results concerning Dhage's D -metric spaces are invalid and thereafter, they introduced a new generalized metric space structure and called it, G -metric space. In this type of spaces a non-negative real number is assigned to every triplet of elements. Many mathematicians studied extensively various results on G -metric spaces by using the concept of weak commutativity, compatibility, non-compatibility and weak compatibility for single valued mappings satisfying different contractive conditions (see [3, 4, 5, 7, 14, 15, 16, 20, 21, 17, 18, 22, 23, 24, 25]).

In 2008, Al-Thagafi and Shahzad [1] weakened the concept of compatibility by giving a new notion of occasionally weakly compatible mappings which is most general among all the commutativity concepts. No wonder that the notion of occasionally weakly compatible mappings has become an area of interest for specialists in fixed point theory.

In this paper, first we prove a common fixed point theorem for a pair of occasionally weakly compatible mappings in symmetric G -metric space. We also prove a fixed point theorem for two pairs of self mappings by using the notions of compatibility and subsequentially continuity (alternately subcompatibility and reciprocally continuity) in G -metric space.

Our improvements in this paper are four-fold as:

- (1) relaxed the continuity of mappings completely,

⁰2000 Mathematics Subject Classification: 47H10, 54H25.

Keywords and phrases. G -metric space, Occasionally weakly compatible mappings, Subcompatible mappings, Subsequential continuity, Fixed point.

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Submitted July 3, 2012. Published October 29, 2012.

- (2) completeness of the space removed,
- (3) minimal type contractive condition used,
- (4) weakened the concept of compatibility by a more general concept of occasionally weakly compatible mappings, subcompatible mappings and subsequential continuity.

2. PRELIMINARIES

Definition 2.1. [20] *Let X be a nonempty set. Let $G : X \times X \times X \rightarrow \mathbb{R}^+$ be a function satisfying the following properties:*

- (G-1) $G(x, y, z) = 0$ if $x = y = z$,
- (G-2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
- (G-3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
- (G-4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ (symmetry in all three variables),
- (G-5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$, (rectangle inequality).

The function G is called a generalized metric or more specifically, a G -metric on X and the pair (X, G) is called a G -metric space.

If condition (G-6) is also satisfied then (X, G) is called symmetric G -metric space.

- (G-6) $G(x, x, y) = G(x, y, y)$.

For more details on G -metric spaces, we refer to the papers [20, 21].

Definition 2.2. [2] *Let (X, G) be a symmetric G -metric space and f and g be self mappings on X . A point x in X is called a coincidence point of f and g iff $fx = gx$. In this case, $w = fx = gx$ is called a point of coincidence of f and g .*

Definition 2.3. [2] *A pair of self mappings (f, g) of a symmetric G -metric space (X, G) is said to be weakly compatible if they commute at the coincidence points, that is, if $fu = gu$ for some u in X , then $fgu = gfu$.*

Definition 2.4. *A pair of self mappings (f, g) of a symmetric G -metric space (X, G) is said to be occasionally weakly compatible iff there is a point x in X which is coincidence point of f and g at which f and g commute.*

It is easy to see that the notion of occasionally weak compatibility is more general than weak compatibility. For details, we refer to [1, 13].

The following Lemma plays a key role in what follows.

Lemma 2.1. [11] *Let X be a non-empty set, and let f and g be two self mappings of X have a unique point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f and g .*

3. RESULTS

Let Φ be the set of all functions ϕ such that $\phi : [0, \infty) \rightarrow [0, \infty)$ be a non-decreasing function with $\lim_{n \rightarrow \infty} \phi^n(t) = 0$ for all $t \in [0, \infty)$.

If $\phi \in \Phi$, then ϕ is called ϕ -mapping then it is an easy to show that $\phi(t) < t$ for all $t \in [0, \infty)$ and $\phi(0) = 0$.

From now unless otherwise stated, we mean by ϕ the ϕ -mapping. Now, we state and prove our main results.

Theorem 3.1. *Let (X, G) be a symmetric G -metric space. If f and g are occasionally weakly compatible self mappings on X satisfying*

$$G(fx, fy, fy) \leq \phi \left(\max \left\{ \begin{array}{l} G(gx, gy, gy), G(gx, fy, fy), \\ G(gy, fx, fx), G(gy, fy, fy) \end{array} \right\} \right), \quad (1)$$

for all x, y in X and $\phi \in \Phi$. Then f and g have a unique common fixed point in X .

Proof. Since f and g are occasionally weakly compatible, there exists a point u in X such that $fu = gu$ and $fgu = gfu$. Hence, $ffu = fgu = gfu = ggu$.

We claim that fu is the unique common fixed point of f and g . First we assert that fu is a fixed point of f .

For, if $ffu \neq fu$, then from inequality (1), we get

$$\begin{aligned} G(fu, ffu, ffu) &\leq \phi \left(\max \left\{ \begin{array}{l} G(gu, gfu, gfu), G(gu, ffu, ffu), \\ G(gfu, fu, fu), G(gfu, ffu, ffu) \end{array} \right\} \right) \\ &= \phi \left(\max \left\{ \begin{array}{l} G(fu, ffu, ffu), G(fu, ffu, ffu), \\ G(ffu, fu, fu), G(ffu, ffu, ffu) \end{array} \right\} \right) \\ &= \phi (\max \{G(fu, ffu, ffu), G(fu, ffu, ffu), G(ffu, fu, fu)\}) \\ &= \phi (G(fu, ffu, ffu)) \\ &< G(fu, ffu, ffu), \end{aligned}$$

which contradicts. Then we have $ffu = fu$ and so $ffu = gfu = fu$. Thus, fu is a common fixed point of f and g .

Now, we prove uniqueness. Suppose that u, v in X such that $fu = gu = u$ and $fv = gv = v$ and $u \neq v$. Then from inequality (1), we have

$$\begin{aligned} G(u, v, v) = G(fu, fv, fv) &\leq \phi \left(\max \left\{ \begin{array}{l} G(gu, gv, gv), G(gu, fv, fv), \\ G(gv, fu, fu), G(gv, fv, fv) \end{array} \right\} \right) \\ &= \phi \left(\max \left\{ \begin{array}{l} G(u, v, v), G(u, v, v), \\ G(v, u, u), G(v, v, v) \end{array} \right\} \right) \\ &= \phi (\max \{G(u, v, v), G(u, v, v), G(v, u, u)\}) \\ &= \phi (G(u, v, v)) \\ &< G(u, v, v), \end{aligned}$$

which contradicts. Then we get $u = v$. Therefore, the common fixed point of f and g is unique. \square

Theorem 3.2. *Let (X, G) be a symmetric G -metric space. Suppose that f, g, h and k are self mappings on X and the pairs (f, h) and (g, k) are each occasionally weakly compatible satisfying*

$$G(fx, gy, gy) < \max \left\{ \begin{array}{l} G(hx, ky, ky), G(hx, fx, fx), G(ky, gy, gy), \\ G(hx, gy, gy), G(ky, fx, fx) \end{array} \right\}, \quad (2)$$

for all x, y in X . Then f, g, h and k have a unique common fixed point in X .

Proof. By hypothesis, there exist points x, y in X such that $fx = hx$, $fhx = hfx$ and $gy = ky$, $gky = kgy$. We claim that $fx = gy$. If $fx \neq gy$, then by inequality

(2), we have

$$\begin{aligned}
G(fx, gy, gy) &< \max \left\{ \begin{array}{c} G(hx, ky, ky), G(hx, fx, fx), G(ky, gy, gy), \\ G(hx, gy, gy), G(ky, fx, fx) \end{array} \right\} \\
&= \max \left\{ \begin{array}{c} G(fx, gy, gy), G(fx, fx, fx), G(gy, gy, gy), \\ G(fx, gy, gy), G(gy, fx, fx) \end{array} \right\} \\
&= \max\{G(fx, gy, gy), G(fx, gy, gy), G(gy, fx, fx)\} \\
&= G(fx, gy, gy),
\end{aligned}$$

which is a contradiction. This implies that $fx = gy$. So $fx = hx = gy = ky$. Moreover, if there is another point z such that $fz = Sz$, then, using inequality (2) it follows that $fz = hz = gy = ky$ or $fx = fz$ and $w = fx = hx$ is the unique point of coincidence of f and h . Then by Lemma 2.1, it follows that w is the unique common fixed point of f and h . By symmetry, there is a unique common fixed point z in X such that $z = gz = Tz$. Now, we claim that $w = z$. Suppose that $w \neq z$. Using inequality (2), we have

$$\begin{aligned}
G(w, z, z) &= G(fw, gz, gz) \\
&< \max \left\{ \begin{array}{c} G(hw, kz, kz), G(hw, fw, fw), G(kz, gz, gz), \\ G(hw, gz, gz), G(kz, fw, fw) \end{array} \right\} \\
&= \max \left\{ \begin{array}{c} G(w, z, z), G(w, w, w), G(z, z, z), \\ G(w, z, z), G(z, w, w) \end{array} \right\} \\
&= \max\{G(w, z, z), G(w, z, z), G(z, w, w)\} \\
&= G(w, z, z),
\end{aligned}$$

which is a contradiction. Therefore, $w = z$ and w is a unique point of coincidence of f, g, h and k . By Lemma 2.1, w is the unique common fixed point of f, g, h and k . \square

Corollary 3.1. *Let (X, G) be a symmetric G -metric space. Suppose that f, g, h and k are self mappings on X and that the pairs (f, h) and (g, k) are each occasionally weakly compatible satisfying*

$$G(fx, gy, gy) \leq cm(x, y, y), \quad (3)$$

where

$$m(x, y, y) = \max \left\{ \begin{array}{c} G(hx, ky, ky), G(hx, fx, fx), G(ky, gy, gy), \\ \frac{[G(hx, gy, gy) + G(ky, fx, fx)]}{2} \end{array} \right\},$$

for all x, y in X and $0 \leq c < 1$, then f, g, h and k have a unique common fixed point in X .

Proof. Since inequality (3) is a special case of inequality (2), the result follows immediately from Theorem 3.2. \square

In 2009 Bouhadjera and Godet-Thobie [6] enlarged the class of compatible (reciprocally continuous) pairs by introducing the concept of subcompatible (subsequential continuous) pair which is substantially weaker than compatibility (reciprocal continuity). Since then, Imdad et al. [12] improved the results of Bouhadjera and Godet-Thobie [6] and showed that these results can easily recovered by replacing subcompatibility with compatibility or subsequential continuity with reciprocally continuity.

Motivated by the results of Imdad et al. [12], we extend the notions of subcompatible mappings and subsequential continuity in framework of G -metric spaces and prove a common fixed point theorems for four self mappings in G -metric spaces.

Definition 3.1. [3] *Two self mappings f and g on G -metric space (X, G) are said to be compatible if $\lim_{n \rightarrow \infty} G(fgx_n, gfx_n, gfx_n) = 1$ whenever there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \in X$, for some $z \in X$.*

Definition 3.2. *Two self mappings f and g on G -metric space (X, G) are said to be subcompatible iff there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \in X$, for some $z \in X$ and $\lim_{n \rightarrow \infty} G(fgx_n, gfx_n, gfx_n) = 0$.*

Obviously two occasionally weakly compatible mappings are subcompatible mappings, however the converse is not true in general as shown in the following example.

Example 3.1. *Let $X = [0, \infty)$ be equipped with G -metric defined by*

$$G(x, y, z) = |x - y| + |y - z| + |z - x|,$$

for all $x, y, z \in X$. Then pair (X, G) is a G -metric space on X . Define the mappings $f, g : X \rightarrow X$ by

$$f(x) = \begin{cases} x^2, & \text{if } x < 1; \\ 2x - 1, & \text{if } x \geq 1. \end{cases} \quad g(x) = \begin{cases} 3x - 2, & \text{if } x < 1; \\ x + 3, & \text{if } x \geq 1. \end{cases}$$

Define a sequence $\{x_n\}_{n \in \mathbb{N}} = \{1 - \frac{1}{n}\}_{n \in \mathbb{N}}$, then $fx_n = (1 - \frac{1}{n})^2 \rightarrow 1$, $gx_n = 3 - \frac{3}{n} - 2 = (1 - \frac{3}{n}) \rightarrow 1$ as $n \rightarrow \infty$.

$$fgx_n = f(1 - \frac{3}{n}) = (1 - \frac{3}{n})^2 = 1 + \frac{9}{n^2} - \frac{6}{n}$$

and

$$gfx_n = g(1 - \frac{1}{n})^2 = 3(1 - \frac{1}{n})^2 - 2 = 3[1 + (\frac{1}{n})^2 - \frac{2}{n}] - 2 = 1 + (\frac{1}{n})^2 - \frac{6}{n}.$$

Here, $\lim_{n \rightarrow \infty} G(fgx_n, gfx_n, gfx_n) \rightarrow 1$, that is, the pair (f, g) is subcompatible. It is noted that f and g are not occasionally weakly compatible

$$f(4) = 7 = g(4) \text{ and } fg(4) = f(7) = 13 \neq gf(4) = 10.$$

It is also interesting to see the following one way implication.

Commuting \Rightarrow Weakly commuting \Rightarrow Compatibility \Rightarrow Weak compatibility \Rightarrow Occasionally weak compatibility \Rightarrow Subcompatibility.

Definition 3.3. *Two self mappings f and g on a G -metric space are called reciprocal continuous if $\lim_{n \rightarrow \infty} fgx_n = fz$ and $\lim_{n \rightarrow \infty} gfx_n = gz$ for some $z \in X$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \in X$.*

Definition 3.4. *Two self mappings f and g on a G -metric space are said to be subsequentially continuous iff there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$ for some $z \in X$ and satisfy $\lim_{n \rightarrow \infty} fgx_n = fz$ and $\lim_{n \rightarrow \infty} gfx_n = gz$.*

If f and g are both continuous or reciprocally continuous then they are obviously subsequentially continuous [6]. The next example shows that subsequentially continuous pairs of mappings which are neither continuous nor reciprocally continuous.

Example 3.2. *Let $X = \mathbb{R}$ endowed with G -metric defined by*

$$G(x, y, z) = |x - y| + |y - z| + |z - x|,$$

for all $x, y, z \in X$. Then (X, G) is a G -metric space on X . Define the mappings $f, g : X \rightarrow X$ by

$$f(x) = \begin{cases} 2, & \text{if } x < 3; \\ x, & \text{if } x \geq 3. \end{cases} \quad g(x) = \begin{cases} 2x - 4, & \text{if } x < 3; \\ 3, & \text{if } x \geq 3. \end{cases}$$

Consider a sequence $\{x_n\}_{n \in \mathbb{N}} = \{3 + \frac{1}{n}\}_{n \in \mathbb{N}}$, then $fx_n = (3 + \frac{1}{n}) \rightarrow 3$, $gx_n \rightarrow 3$ and $gfx_n = g(3 + \frac{1}{n}) = 3 \neq g(3) = 2$ as $n \rightarrow \infty$. Thus f and g are not reciprocally continuous but if we consider a sequence $\{x_n\}_{n \in \mathbb{N}} = \{3 - \frac{1}{n}\}_{n \in \mathbb{N}}$ then $fx_n \rightarrow 2$, $gx_n = 2(3 - \frac{1}{n}) - 4 = (2 - \frac{2}{n}) \rightarrow 2$ and $fgx_n = f(2 - \frac{2}{n}) = 2 = f(2)$ and $gfx_n = g(2) = 0$ as $n \rightarrow \infty$. Therefore, f and g are subsequentially continuous.

Theorem 3.3. Let f, g, h and k be four self mappings of a G -metric space (X, G) . If the pairs (f, h) and (g, k) are compatible and subsequentially continuous (alternately subcompatible and reciprocally continuous), then

- (a) f and h have a coincidence point,
- (b) g and k have a coincidence point.

Also, assume that

$$\psi \left(\begin{array}{c} G(fx, gy, gy), G(hx, ky, ky), G(fx, hx, hx), \\ G(gy, ky, ky), G(hx, gy, gy), G(fx, ky, ky) \end{array} \right) \leq 0, \quad (4)$$

for all x, y in X and $(\mathbb{R}^+)^6 \rightarrow \mathbb{R}$ be an upper semi-continuous function such that $\psi(u, u, 0, 0, u, u) > 0$, $u > 0$. Then f, g, h and k have a unique common fixed point in X .

Proof. Case I: Since, the pair (f, h) (also (g, k)) is subsequentially continuous and compatible mappings, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} hx_n = z, \text{ for some } z \in X,$$

and

$$\lim_{n \rightarrow \infty} G(fhx_n, hfx_n, hfx_n) = G(fz, hz, hz) = 0,$$

then $fz = hz$, whereas in respect of the pair (g, k) , there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} gy_n = \lim_{n \rightarrow \infty} ky_n = z', \text{ for some } z' \in X,$$

and

$$\lim_{n \rightarrow \infty} G(gky_n, kgy_n, kgy_n) = G(gz', kz', kz') = 0,$$

then $gz' = kz'$. Hence z is a coincidence point of the pair (f, h) , whereas z' is a coincidence point of the pair (g, k) .

Now, we prove that $z = z'$. Indeed, by inequality (4), we have

$$\psi \left(\begin{array}{c} G(fx_n, gy_n, gy_n), G(hx_n, ky_n, ky_n), G(fx_n, hx_n, hx_n), \\ G(gy_n, ky_n, ky_n), G(hx_n, gy_n, gy_n), G(fx_n, ky_n, ky_n) \end{array} \right) \leq 0.$$

Since, ψ is upper semi-continuous, taking the limit as $n \rightarrow \infty$ yields

$$\psi \left(\begin{array}{c} G(z, z', z'), G(z, z', z'), G(z, z, z), \\ G(z', z', z'), G(z, z', z'), G(z, z', z') \end{array} \right) \leq 0,$$

and so,

$$\psi \left(G(z, z', z'), G(z, z', z'), 0, 0, G(z, z', z'), G(z, z', z') \right) \leq 0,$$

which contradicts if $z \neq z'$. Hence, $z = z'$.

We claim that $fz = z$. If $fz \neq z$, using inequality (4), we get

$$\psi \left(\begin{array}{l} G(fz, gy_n, gy_n), G(hz, ky_n, ky_n), G(fz, fz, fz), \\ G(gy_n, ky_n, ky_n), G(hz, gy_n, gy_n), G(fz, ky_n, ky_n) \end{array} \right) \leq 0.$$

Since, ψ is upper semi-continuous, taking the limit as $n \rightarrow \infty$ yields

$$\psi \left(\begin{array}{l} G(fz, z, z), G(fz, z, z), G(fz, fz, fz), \\ G(z, z, z), G(fz, z, z), G(fz, z, z) \end{array} \right) \leq 0,$$

or, equivalently,

$$\psi (G(fz, z, z), G(fz, z, z), 0, 0, G(fz, z, z), G(fz, z, z)) \leq 0,$$

which contradicts. Therefore, $z = fz = fz$. Again, suppose that $gz \neq z$, using inequality (4), we get

$$\psi \left(\begin{array}{l} G(fz, gz, gz), G(hz, kz, kz), G(fz, fz, fz), \\ G(gz, kz, kz), G(hz, gz, gz), G(fz, kz, kz) \end{array} \right) \leq 0,$$

and so,

$$\psi \left(\begin{array}{l} G(z, gz, gz), G(z, gz, gz), G(z, z, z), \\ G(gz, gz, gz), G(z, gz, gz), G(z, gz, gz) \end{array} \right) \leq 0,$$

or, equivalently,

$$\psi (G(z, gz, gz), G(z, gz, gz), 0, 0, G(z, gz, gz), G(z, gz, gz)) \leq 0,$$

which contradicts. Hence, $z = gz = kz$. Therefore, $z = fz = gz = kz = kz$, i.e., z is common fixed point of f, g, h and k . Uniqueness of such common fixed point is an easy consequence of inequality (4).

Case II: Since, the pair (f, h) (also (g, k)) is subcompatible and reciprocally continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} hx_n = z \text{ for some } z \in X,$$

and

$$\lim_{n \rightarrow \infty} G(fhx_n, hfx_n, hfx_n) = G(fz, fz, fz) = 0,$$

which implies $fz = fz$. In respect of the pair (g, k) , there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} gy_n = \lim_{n \rightarrow \infty} ky_n = z' \text{ for some } z' \in X,$$

and

$$\lim_{n \rightarrow \infty} G(gky_n, kgy_n, kgy_n) = G(gz', kz', kz') = 0,$$

then $gz' = kz'$. Hence z is a coincidence point of the pair (f, h) , whereas z' is a coincidence point of the pair (g, k) . The rest of the proof can be completed on the lines of Case I. \square

On setting $h = k$, in Theorem 3.3, we get the following result:

Corollary 3.2. *Let f, g and h be three self mappings of a G -metric space (X, G) . If the pairs (f, h) and (g, h) are compatible and subsequentially continuous (alternately subcompatible and reciprocally continuous), then*

- (a) f and h have a coincidence point;
- (b) g and h have a coincidence point.

Further, suppose that

$$\psi \left(\begin{array}{l} G(fx, gy, gy), G(hx, hy, hy), G(fx, hx, hx), \\ G(gy, hy, hy), G(hx, gy, gy), G(fx, hy, hy) \end{array} \right) \leq 0, \quad (5)$$

for all x, y in X and $(\mathbb{R}^+)^6 \rightarrow \mathbb{R}$ be an upper semi-continuous function such that $\psi(u, u, 0, 0, u, u) > 0, u > 0$. Then f, g and h have a unique common fixed point in X .

If we put $f = g$ and $h = k$ in Theorem 3.3 then we deduce the following natural result for a pair of self mappings.

Corollary 3.3. *Let f and h be two self mappings of a G -metric space (X, G) . If the pair (f, h) is compatible and subsequentially continuous (alternately subcompatible and reciprocally continuous), then f and h have a coincidence point. Suppose that*

$$\psi \left(\begin{array}{l} G(fx, fy, fy), G(hx, hy, hy), G(fx, hx, hx), \\ G(fy, hy, hy), G(hx, fy, fy), G(fx, hy, hy) \end{array} \right) \leq 0, \quad (6)$$

for all x, y in X and $(\mathbb{R}^+)^6 \rightarrow \mathbb{R}$ be an upper semi-continuous function such that $\psi(u, u, 0, 0, u, u) > 0, u > 0$. Then f and h have a unique common fixed point in X .

CONCLUSION

Our results unify and generalize various results to a more general class of non-commuting and non-compatible mappings. These results have wide range of applications for solving differential and integral equations [24], partial differential equations, in dynamic programming [10] in game theory and economic theory [9].

ACKNOWLEDGEMENT

The authors are thankful to Professor M. Imdad for a reprint of his valuable paper [12].

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M. ALAMGIR KHAN

DEPARTMENT OF NATURAL RESOURCES ENGINEERING AND MANAGEMENT,
UNIVERSITY OF KURDISTAN, HEWLER, IRAQ.

E-mail address: alam3333@gmail.com

SUNNY CHAUHAN

NEAR NEHRU TRAINING CENTRE,
H. NO: 274, NAI BASTI B-14,
BLJNOR-246 701, UTTAR PRADESH, INDIA.

E-mail address: sun.gkv@gmail.com