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Nonexistence results of solutions for some fractional p-Laplacian equations in $\mathbb{R}^N$

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Abstract. In the present paper, we study the nonexistence of nontrivial weak solutions to a class of fractional p-Laplacian equation in two cases which are $sp > N$ and $sp < N$. In each of these cases, by using fractional Laplacian theory and inequality techniques, we obtain concrete range of parameter for which nontrivial weak solution of the problem does not exist. Our work complements the known nonexistence results in this direction.

Keywords: fractional p-Laplacian equation, nonexistence, weak solution.

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1 Introduction

In this paper, we investigate the following fractional p-Laplacian equation of the type

$$\begin{cases} (-\Delta)_p^s u + \lambda V(x)|u|^{q-2}u = m(x)|u|^{r-2}u, & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $s \in (0,1)$, p, q, r are positive numbers satisfying $1 < p < r < q < \infty$ or $1 < q < r < p < \infty$, $m, V \in L^1(\Omega)$ are positive functions and λ is a positive parameter.

The fractional p-Laplacian operator is defined as

$$(-\Delta)_p^s u(x) = 2 \lim_{\epsilon \searrow 0} \int_{\mathbb{R}^N \setminus B_\epsilon(x)} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))}{|x - y|^{N+sp}} \, dy, \quad x \in \mathbb{R}^N,$$

where $B_\epsilon(x) = \{y \in \mathbb{R}^N : |x - y| < \epsilon\}$.

In recent years, many papers have been devoted to the study of the fractional p-Laplacian equations due to their interesting applications, such as game theory, image processing, optimization and so on (see [3–5]). In particular, the existence, nonexistence, multiplicity and

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some other properties of solutions to the following type of fractional p-Laplacian equation where $sp < N$

$$\begin{cases} (-\Delta)_p^s u + V(x)|u|^{p-2}u = f(x,u), & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.2}$$

have been widely studied by many scholars (see [1,2,6,8,9,12–16] and the references therein). For instance, Goyal and Sreenadh [6] obtained some results on the existence and nonexistence of solutions for the following equation with respect to the parameter λ

$$\begin{cases} (-\Delta)_p^s u - \lambda V(x)|u|^{p-2}u = m(x)|u|^{r-2}u, & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.3}$$

where $sp < N$ and $1 < r < p$ or $p < r < p_s^* = \frac{Np}{N-sp}$.

Wu and Chen [15] studied the following equation

$$(-\Delta)_p^s u + V(x)|u|^{p-2}u = |u|^{r-2}u + \lambda|u|^{q-2}u, \quad \text{in } \mathbb{R}^N, \tag{1.4}$$

for the case $sp < N$ and $1 < q < p < r$. They deduced some existence results of nontrivial solution for some range of λ.

However, as far as we know, in the case $sp > N$, there have been rarely any existence or nonexistence results for problem (1.2). Inspired by the above mentioned papers, our purpose is to establish some results on the nonexistence of nontrivial weak solution for the problem (1.1) in both cases $sp > N$ and $sp < N$ under the assumptions $1 < p < r < q < \infty$ or $1 < q < r < p < \infty$. More precisely, we aim to obtain concrete range of parameter for which nontrivial weak solution of the problem does not exist in the case $sp > N$ and the case $sp < N$, respectively.

The rest of our paper is organized as follows. In Section 2, we will introduce some necessary lemmas and properties, which will be used in the sequel. In Section 3, we derive somewhat sharp nonexistence conditions of nontrivial solutions for (1.1) in both cases: $sp > N$ and $sp < N$.

2 Preliminaries

To state our results, we introduce some notations. Let $s \in (0,1)$ and $1 < p < \infty$ be real numbers. The fractional Sobolev space $W^{s,p}(\mathbb{R}^N)$ is defined as follows:

$$W^{s,p}(\mathbb{R}^N) := \{u \in L^p(\mathbb{R}^N) : [u]_{s,p} < \infty\}$$

equipped with the norm

$$\|u\|_{s,p} := \left(\|u\|^p_{L^p(\mathbb{R}^N)} + [u]^p_{s,p}\right)^{1/p},$$

where

$$[u]_{s,p} = \Big(\int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}} dxdy\Big)^{1/p}$$

is the Gagliardo seminorm of a measurable function $u : \mathbb{R}^N \to \mathbb{R}$.

Let Ω be a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. We shall work on the space

$$W_0^{s,p}(\Omega) := \left\{u \in W^{s,p}(\mathbb{R}^N) : u = 0 \text{ a.e. in } \mathbb{R}^N \setminus \Omega\right\},$$

which can be equivalently renormed by $[u]_{s,p}$.

Lemma 2.1 ([10]). *Let $\Omega \subset \mathbb{R}^N$ be bounded and open, $sp > N$ and $s \in (0,1)$. Then there is a constant $C_M > 0$ such that for all $u \in W_0^{s,p}(\Omega)$,*

$$|u(x) - u(y)| \leq C_M |x-y|^{\beta} [u]_{s,p}, \qquad x, y \in \mathbb{R}^N,$$

where $\beta = \frac{sp-N}{p}$.

Lemma 2.2 ([4]). *Let $\Omega \subset \mathbb{R}^N$ be bounded and open, $s \in (0,1)$, $1 < p < \infty$ with $sp < N$. Then, there exists a constant $C_H > 0$ such that*

$$\|u\|^p_{L^{p_s^*}(\mathbb{R}^N)} \leq C_H [u]^p_{s,p}, \qquad u \in W_0^{s,p}(\Omega),$$

where $p_s^* = \frac{Np}{N-sp}$.

Lemma 2.3 ([4]). *Let $\Omega \subset \mathbb{R}^N$ be an extension domain for $W^{s,p}$ with no external cusps and let $p \in [1, +\infty)$, $s \in (0,1)$ be such that $sp > N$. Then, there exists $C > 0$, depending on N, s, p and Ω, such that*

$$\|u\|_{C^{0,\alpha}(\Omega)} \leq C\Big(\|u\|^p_{L^p(\Omega)} + \int_\Omega \int_\Omega \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}} dxdy \Big)^{\frac{1}{p}},$$

for any $u \in L^p(\Omega)$, with $\alpha = (sp - N)/p$.

Lemma 2.4 ([7]). *Let $s \in (0,1)$ and $1 < p < \infty$ be such that $sp < N$. Assume that $\Omega \subset \mathbb{R}^N$ is a (bounded) uniform domain with a (locally) (s,p)-uniformly fat boundary. Then Ω admits an (s,p)-Hardy inequality, that is, there is a constant $C_K > 0$ such that*

$$\int_\Omega \frac{|u(x)|^p}{d(x,\partial\Omega)^{sp}} dx \leq C_K [u]^p_{s,p}, \qquad u \in W_0^{s,p}(\Omega),$$

where $d(x, \partial\Omega) = \inf\{|x-y| : \ y \in \partial\Omega\}$.

Lemma 2.5 ([11]). *Let $M > 0, L > 0, p > 0$, $q > 0$ and $r > 0$ be given. If*

(i) $1 < p < r < q$;

or

(ii) $1 < q < r < p$,

then for each $x \geq 0$,

$$Mx^r - Lx^q \leq \frac{M(q-r)}{q-p} \left(\frac{(r-p)M}{(q-p)L} \right)^{\frac{r-p}{q-r}} x^p$$

holds.

Definition 2.6. We say that $u \in W_0^{s,p}(\Omega)$ is a weak solution of (1.1) if

$$\int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))(v(x)-v(y))}{|x-y|^{N+ps}} dxdy + \lambda \int_\Omega V(x)|u(x)|^{q-2}u(x)v(x)dx = \int_\Omega m(x)|u(x)|^{r-2}u(x)v(x)dx, \quad (2.1)$$

for all $v \in W_0^{s,p}(\Omega)$.

3 Main results

In this section, we suppose that $\Omega \subset \mathbb{R}^N$ is a bounded domain satisfying the regularities required by the fractional Sobolev inequalities given by Lemmas 2.1–2.4.

3.1 The case $sp > N$

Theorem 3.1. *Suppose that $sp > N$ and $m\left(\frac{m}{V}\right)^{\frac{r-p}{q-r}} \in L^1(\Omega)$. If*

$$\lambda > \frac{r-p}{q-p}\left(C_M^p R_\Omega^{sp-N}\frac{q-r}{q-p}\right)^{\frac{q-r}{r-p}}\left[\int_\Omega m(x)\left(\frac{m(x)}{V(x)}\right)^{\frac{r-p}{q-r}}dx\right]^{\frac{q-r}{r-p}}, \tag{3.1}$$

then problem (1.1) has no nontrivial weak solution $u \in W_0^{s,p}(\Omega)$, where C_M is given in Lemma 2.1 and $R_\Omega = \max\{d(x,\partial\Omega) : x \in \Omega\}$.

Proof. Suppose on the contrary that problem (1.1) has a nontrivial weak solution $u \in W_0^{s,p}(\Omega)$. Taking $v = u$ in (2.1) and from Lemma 2.5, we have

$$\begin{aligned}
&\int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}}dxdy \\
&\quad = \int_\Omega \left[m(x)|u(x)|^{r-2}u(x) - \lambda V(x)|u(x)|^{q-2}u(x)\right]u(x)dx \\
&\quad \le \int_\Omega \left[m(x)|u(x)|^r - \lambda V(x)|u(x)|^q\right]dx \\
&\quad \le \int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}|u(x)|^p dx,
\end{aligned}$$

i.e.,

$$[u]_{s,p}^p \le \int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}|u(x)|^p dx. \tag{3.2}$$

By $sp > N$ and Lemma 2.3, we get u is continuous in $\mathbb{R}^N$, in particular in $\overline{\Omega}$. Then there is some $\xi \in \Omega$ such that

$$|u(\xi)| = \max\left\{|u(x)| : x \in \mathbb{R}^N\right\} > 0.$$

From Lemma 2.1, there is a constant C_M such that

$$|u(x)-u(y)| \le C_M|x-y|^{\frac{sp-N}{p}}[u]_{s,p}, \qquad x, y \in \mathbb{R}^N.$$

Taking $x = \xi$ in the above inequality, we obtain

$$|u(\xi)| \le C_M|\xi-y|^{\frac{sp-N}{p}}[u]_{s,p}, \qquad y \in \partial\Omega,$$

i.e.,

$$|u(\xi)| \le C_M R_\Omega^{\frac{sp-N}{p}}[u]_{s,p}. \tag{3.3}$$

Combining (3.2) with (3.3), we obtain

$$\begin{aligned}
|u(\xi)| &\le C_M R_\Omega^{\frac{sp-N}{p}}\left(\int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}|u(x)|^p dx\right)^{\frac{1}{p}} \\
&\le C_M R_\Omega^{\frac{sp-N}{p}}\left(\int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}dx\right)^{\frac{1}{p}}|u(\xi)|,
\end{aligned}$$

which yields

$$1 \le C_M R_\Omega^{\frac{sp-N}{p}} \left(\int_\Omega \frac{q-r}{q-p} \left(\frac{r-p}{q-p} \right)^{\frac{r-p}{q-r}} m(x) \left(\frac{m(x)}{\lambda V(x)} \right)^{\frac{r-p}{q-r}} dx \right)^{\frac{1}{p}}.$$

Thus

$$\lambda^{\frac{p-r}{q-r}} \frac{q-r}{q-p} \left(\frac{r-p}{q-p} \right)^{\frac{r-p}{q-r}} \int_\Omega m(x) \left(\frac{m(x)}{V(x)} \right)^{\frac{r-p}{q-r}} dx \ge \frac{1}{C_M^p R_\Omega^{sp-N}},$$

which implies that

$$\lambda^{\frac{p-r}{q-r}} \ge \frac{1}{C_M^p R_\Omega^{sp-N} \frac{q-r}{q-p} \left(\frac{r-p}{q-p} \right)^{\frac{r-p}{q-r}} \int_\Omega m(x) \left(\frac{m(x)}{V(x)} \right)^{\frac{r-p}{q-r}} dx}.$$

Hence, from $\frac{p-r}{q-r} < 0$ we obtain

$$\lambda \le \frac{r-p}{q-p} \Big(C_M^p R_\Omega^{sp-N} \frac{q-r}{q-p} \Big)^{\frac{q-r}{r-p}} \Big[\int_\Omega m(x) \Big(\frac{m(x)}{V(x)} \Big)^{\frac{r-p}{q-r}} dx \Big]^{\frac{q-r}{r-p}}, \tag{3.4}$$

which contradicts to (3.1). This completes the proof. ☐

3.2 The case $sp < N$

Theorem 3.2. *Suppose that* $sp < N$, $m\big(\frac{m}{V}\big)^{\frac{r-p}{q-r}} \in L^\mu(\Omega)$ *and* $\frac{N}{sp} < \mu < \infty$. *Assume that*

$$\lambda > \frac{r-p}{q-p} \left(C_K^{1-\frac{N}{\mu sp}} C_H^{\frac{N}{\mu sp}} R_\Omega^{sp-\frac{N}{\mu}} \frac{q-r}{q-p} \right)^{\frac{q-r}{r-p}} \left[\int_\Omega m(x) \left(\frac{m(x)}{V(x)} \right)^{\frac{r-p}{q-r}} dx \right]^{\frac{q-r}{r-p}}, \tag{3.5}$$

then problem (1.1) *has no nontrivial weak solution* $u \in W_0^{s,p}(\Omega)$, *where* C_H *and* C_K *are given in Lemmas* 2.2 *and* 2.4, *and* $R_\Omega = \max\{d(x, \partial\Omega) : x \in \Omega\}$.

Proof. Suppose on the contrary that problem (1.1) has a nontrivial weak solution $u \in W_0^{s,p}(\Omega)$. From the proof of Theorem 3.1, we have (3.2) holds. Let $\eta = \frac{1}{\mu-1}(\mu - \frac{N}{sp})$, $\theta = \eta p + (1-\eta)p_s^*$ where $p_s^* = \frac{Np}{N-sp}$. By a straightforward computation, we have $0 < \eta < 1$, $\theta = p\nu$, where $\frac{1}{\mu} + \frac{1}{\nu} = 1$. On the other hand, we get

$$\frac{1}{R_\Omega^{\eta sp}} \int_\Omega |u(x)|^\theta dx \le \int_\Omega \frac{|u(x)|^\theta}{d(x, \partial\Omega)^{\eta sp}} dx, \tag{3.6}$$

and by Hölder's inequality, Lemma 2.2, Lemma 2.4 and (3.2), we obtain

$$\begin{aligned}
&\int_\Omega \frac{|u(x)|^\theta}{d(x,\partial\Omega)^{\eta sp}}dx \\
&\quad = \int_\Omega \frac{|u(x)|^{\eta p}|u(x)|^{(1-\eta)p_s^*}}{d(x,\partial\Omega)^{\eta sp}}dx \\
&\quad \le \left[\int_\Omega \frac{|u(x)|^p}{d(x,\partial\Omega)^{sp}}dx\right]^\eta \left[\int_\Omega |u(x)|^{p_s^*}dx\right]^{1-\eta} \\
&\quad \le C_K^\eta [u]_{s,p}^{p\eta} C_H^{\frac{(1-\eta)p_s^*}{p}} [u]_{s,p}^{(1-\eta)p_s^*} \\
&\quad = C[u]_{s,p}^{p\eta+(1-\eta)p_s^*} \\
&\quad = C[u]_{s,p}^{p\frac{p\eta+(1-\eta)p_s^*}{p}} \\
&\quad = C[u]_{s,p}^{p\frac{\theta}{p}} \\
&\quad \le C\left(\int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}|u(x)|^p dx\right)^{\frac{\theta}{p}} \\
&\quad = C\left(\int_\Omega \frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}|u(x)|^p dx\right)^{\nu} \\
&\quad \le C\left(\int_\Omega \left[\frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}\right]^\mu dx\right)^{\frac{\nu}{\mu}} \int_\Omega |u(x)|^\theta dx,
\end{aligned} \tag{3.7}$$

where $C = C_K^\eta C_H^{\frac{(1-\eta)p_s^*}{p}}$. Thus, by (11) and (12), we have

$$\frac{1}{R_\Omega^{\eta sp}} \le C\left(\int_\Omega \left[\frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}\right]^\mu dx\right)^{\frac{\nu}{\mu}}.$$

Accordingly,

$$\int_\Omega \left[\frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}} m(x)\left(\frac{m(x)}{\lambda V(x)}\right)^{\frac{r-p}{q-r}}\right]^\mu dx \ge \frac{1}{C^{\frac{\mu}{\nu}} R_\Omega^{\mu sp-N}}. \tag{3.8}$$

Therefore,

$$\lambda^{\mu\frac{p-r}{q-r}}\left[\frac{q-r}{q-p}\left(\frac{r-p}{q-p}\right)^{\frac{r-p}{q-r}}\right]^\mu \left[\int_\Omega m(x)\left(\frac{m(x)}{V(x)}\right)^{\frac{r-p}{q-r}}\right]^\mu \ge \frac{1}{C^{\frac{\mu}{\nu}} R_\Omega^{\mu sp-N}}. \tag{3.9}$$

Hence, from $\frac{p-r}{q-r} < 0$ we obtain

$$\lambda \le \frac{r-p}{q-p}\left(C^{\frac{1}{\nu}} R_\Omega^{sp-\frac{N}{\mu}} \frac{q-r}{q-p}\right)^{\frac{q-r}{r-p}} \left[\int_\Omega m(x)\left(\frac{m(x)}{V(x)}\right)^{\frac{r-p}{q-r}} dx\right]^{\frac{q-r}{r-p}}. \tag{3.10}$$

Combining the definition of C and the inequality (3.10), we have

$$\lambda \le \frac{r-p}{q-p}\left(C_K^{1-\frac{N}{\mu sp}} C_H^{\frac{N}{\mu sp}} R_\Omega^{sp-\frac{N}{\mu}} \frac{q-r}{q-p}\right)^{\frac{q-r}{r-p}} \left[\int_\Omega m(x)\left(\frac{m(x)}{V(x)}\right)^{\frac{r-p}{q-r}} dx\right]^{\frac{q-r}{r-p}},$$

which contradicts to (3.5). This completes the proof. □

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References

[1] M. Bhakta, S. Chakraborty, O. H. Miyagaki, P. Pucci, Fractional elliptic systems with critical nonlinearities, *Nonlinearity* **34**(2021), No. 11, 7540–7573. https://doi.org/10.1088/1361-6544/ac24e5; MR4331246; Zbl 1476.35296

[2] C. Brändle, E. Colorado, A. de Pablo, U. Sánchez, A concave-convex elliptic problem involving the fractional Laplacian, *Proc. Roy. Soc. Edinburgh Sect. A* **143**(2013), No. 1, 39–71. https://doi.org/10.1017/S0308210511000175; MR3023003; Zbl 1290.35304

[3] L. Caffarelli, Non-local diffusions, drifts and games, *Nonlinear Partial Differ. Equ.* **7**(2012), 37–52. https://doi.org/10.1007/978-3-642-25361-4_3; MR3289358; Zbl 1266.35060

[4] E. Di Nezza, G. Palatucci, E. Valdinoci, Hitchhiker's guide to the fractional Sobolev spaces, *Bull. Sci. Math.* **136**(2012), No. 5, 521–573. https://doi.org/10.1016/j.bulsci.2011.12.004; MR2944369; Zbl 1252.46023

[5] G. Gilboa, S. Osher, Nonlocal operators with applications to image processing, *Multiscale Model. Simul.* **7**(2008), No. 3, 1005–1028. https://doi.org/10.1137/070698592; MR2480109; Zbl 1181.35006

[6] S. Goyal, K. Sreenadh, Existence of multiple solutions of p-fractional Laplace operator with sign-changing weight function, *Adv. Nonlinear Anal.* **4**(2015), No. 1, 37–58. https://doi.org/10.1515/anona-2014-0017; MR3305916; Zbl 1331.35368

[7] L. Ihnatsyeva, J. Lehrbäck, H. Tuominen, A. Vähäkangas, Fractional Hardy inequalities and visibility of the boundary, *Stud. Math.* **224**(2014), No. 1, 47–80. https://doi.org/10.4064/sm224-1-3; MR3277052; Zbl 1325.46040

[8] M. Jleli, M. Kirane, B. Samet, Lyapunov-type inequalities for fractional partial differential equations, *Appl. Math. Lett.* **66**(2017), No. 5, 30–39. https://doi.org/10.1016/j.aml.2016.10.013; MR3583856; Zbl 1489.35114

[9] N. Li, X. M. He, Positive solutions for a class of fractional p-Laplacian equation with critical Sobolev exponent and decaying potentials, *Acta Math. Appl. Sin. Engl. Ser.* **38**(2022), No. 2, 463–483. https://doi.org/10.1007/s10255-022-1090-8; MR4405544; Zbl 1489.35149

[10] E. Lindgren, P. Lindqvist, Fractional eigenvalues, *Calc. Var. Partial Differential Equations* **49**(2014), No. 1–2, 795–826. https://doi.org/10.1007/s00526-013-0600-1; MR3148135; Zbl 1292.35193

[11] H. D. Liu, C. Y. Li, F. C. Shen, A class of new nonlinear dynamic integral inequalities containing integration on infinite interval on time scales, *Adv. Differ. Equ.* **2019**, Paper No. 311, 11 pp. https://doi.org/10.1186/s13662-019-2236-8; MR3989979; Zbl 1485.34215

[12] H. L. Mi, X. Q. Deng, W. Zhang, Ground state solution for asymptotically periodic fractional p-Laplacian equation, *Appl. Math. Lett.* **120**(2021), 107280. https://doi.org/10.1016/j.aml.2021.107280; MR4243281; Zbl 1475.35396

[13] O. H. Miyagaki, S. I. Moreira, R. S. Vieira, Schrödinger equations involving fractional p-Laplacian with supercritical exponent, *Complex Var. Elliptic Equ.* **67**(2022), No. 5, 1273–1286. https://doi.org/10.1080/17476933.2020.1857370; MR4410290; Zbl 1489.35114

[14] C. E. Torres, Existence and symmetry result for fractional p-Laplacian in $\mathbb{R}^N$, *Commun. Pure Appl. Anal.* **16**(2017), No. 1, 99–113. https://doi.org/10.3934/cpaa.2017004; MR3583517; Zbl 1364.35426

[15] Z. J. Wu, H. B. Chen, An existence result for super-critical problems involving the fractional p-Laplacian in $\mathbb{R}^N$, *Appl. Math. Lett.* **135**(2023), 108422. https://doi.org/10.1016/j.aml.2022.108422; MR4479922; Zbl 1498.35599

[16] M. Q. Xiang, B. L. Zhang, V. D. Rădulescu, Existence of solutions for perturbed fractional p-Laplacian equations, *J. Differential Equations* **260**(2016), No. 2, 1392–1413. https://doi.org/10.1016/j.jde.2015.09.028; MR3419730; Zbl 1332.35387