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ADDENDUM TO METHODS OF EXTENDING LOWER ORDER PROBLEMS TO HIGHER ORDER PROBLEMS IN THE CONTEXT OF SMALLEST EIGENVALUE COMPARISONS

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Abstract. This paper serves as an addendum to the paper titled Methods of extending lower order problems to higher order problems in the context of smallest eigenvalue comparisons appearing in EJQTDE no. 99, 2011.

This addendum concerns the paper with the above title found in EJQTDE **99**, 2011. In the proof of Lemma 3.1, ϵ_0 is picked so $u'(0)-\epsilon_0>0$ and $u(x)-\epsilon_0>0$ for $0<x\le 1$. This would imply $u(0)\neq 0$, which is contradictory to the fact that $u\in\mathcal{B}$. The proof should read as follows:

Define
$$\Omega=\{u\in\mathcal{B}\mid u(x)>0 \text{ on } (0,1] \text{ and } u'(0)>0\}.$$
Note $\Omega\subset\mathcal{P}$. Choose $u\in\Omega$ and define $B_\epsilon(u)=\{v\in\mathcal{B}\mid ||u-v||<\epsilon\}$ for $\epsilon>0$. We will show that for sufficiently small ϵ, $B_\epsilon(u)\subset\Omega$.

Since $u'(0)>0$, we can choose $\epsilon_1>0$ such that $u'(0)-\epsilon_1>0$. Then, since $u(0)=0$ and $u'(0)>\epsilon_1$, there exists an a with $0<a\le 1$ such that $u(x)\ge x\epsilon_1$ on $[0,a]$. Also, since $u(x)>0$ on $(0,1]$, we can choose ϵ_2 such that $u(x)-\epsilon_2>0$ on $[a,1]$. Let $\epsilon_0=\min\{\frac{\epsilon_1}{2},\epsilon_2\}$. So for $v\in B_{\epsilon_0}(u)$, $||v-u||=\sup\limits_{0\le x\le 1}|v'(x)-u'(x)|<\epsilon_0$. So $v'(0)>u'(0)-\epsilon_0>0$. Now, for $x\in(0,a]$, $|v(x)-u(x)|\le ||v-u||x<x\epsilon_0$. So $v(x)>u(x)-x\epsilon_0\ge x\epsilon_1-x\epsilon_0\ge x\left(\epsilon_1-\frac{\epsilon_1}{2}\right)>0$, and so $v(x)>0$ on $(0,a]$. Finally, for $x\in[a,1]$, $|v(x)-u(x)|\le||v-u||<\epsilon_0$, so $v(x)>0$ on $[a,1]$ and hence $v(x)>0$ on $(0,1]$. So $v\in\Omega$. Therefore $B_{\epsilon_0}(u)\subset\Omega\subset\mathcal{P}$ and $\Omega\subset\mathcal{P}^\circ$, so $\mathcal{P}$ is solid in $\mathcal{B}$.

The proof to Lemma 4.1 should be changed in a similar manner. I regret any inconvenience this may have caused any reader.

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