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Permanence for Nicholson-type Delay Systems with Patch Structure and Nonlinear Density-dependent Mortality Terms*

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Abstract: In this paper, we study the Nicholson-type delay systems with patch structure and nonlinear density-dependent mortality terms. Under appropriate conditions, we establish some criteria to ensure the permanence of this model. Moreover, we give some examples to illustrate our main results.

Keywords: Nicholson-type delay system; permanence; patch structure; Nonlinear density-dependent mortality terms.

AMS(2000) Subject Classification: 34C25; 34K13.

1. Introduction

To reveal the rule of population of the Australian sheep blowfly that obtained in experimental data [1], Gurney et al [2] put forward the following Nicholson's blowflies model

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)}. \tag{1.1}$$

Here, $N(t)$ is the size of the population at time t, p is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. As a class of biological systems,

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Nicholson's blowflies model and its analogous equation have attracted much attention. There have been a large number of results about this model and its modifications. We refer the reader to [3-9] and the references cited therein. Moreover, the main focus of Nicholson's blowflies model is on the scalar equation and results about patch structure of this model are gained rarely (see e.g.[10-13] and the reference therein). On the other hand, L. Berezansky et al [9] pointed out that a new study indicates that a linear model of density-dependent mortality will be most accurate for populations at low densities and marine ecologists are currently in the process of constructing new fishery models with nonlinear density-dependent mortality rates. Consequently, B. Liu and S. Gong [14] and Liu [15] presented extensive results on the permanence of the following Nicholson's blowflies model with a nonlinear density-dependent mortality term

$$N'(t) = -D(N(t)) + PN(t-\tau)e^{-aN(t-\tau)} \tag{1.2}$$

where P is a positive constant and function D might have one of the following forms: $D(N) = \frac{aN}{N+b}$ or $D(N) = a - be^{-N}$ with positive constants $a, b > 0$.

However, to the best of our knowledge, there have been few publications concerned with the permanence for Nicholson-type delay system with patch structure and nonlinear density-dependent mortality terms. Motivated by this, the main purpose of this paper is to give the conditions to guarantee the permanence for the following Nicholson-type delay system with patch structure and nonlinear density-dependent mortality terms:

$$N_i'(t) = -D_{ii}(t, N_i(t)) + \sum_{j=1, j\neq i}^{n} D_{ij}(t, N_j(t)) + \sum_{j=1}^{l} c_{ij}(t)N_i(t-\tau_{ij}(t))e^{-\gamma_{ij}(t)N_i(t-\tau_{ij}(t))}, \tag{1.3}$$

where

$$D_{ij}(t,N) = \frac{a_{ij}(t)N}{b_{ij}(t)+N} \quad \text{or} \quad D_{ij}(t,N) = a_{ij}(t) - b_{ij}(t)e^{-N},$$

$a_{ij}, b_{ij}, c_{ik}, \gamma_{ik} : R \to (0, +\infty)$ are all continuous functions bounded above and below by positive constants, and $\tau_{ik}(t) \geq 0$ are bounded continuous functions, $r_i = \max\limits_{1\leq j\leq l}\{\sup_{t\in R}\tau_{ij}(t)\} > 0$, and $i, j = 1, 2\cdots, n$, $k = 1, 2\cdots, l$. Furthermore, in the case $D_{ij}(t,N) = a_{ij}(t) - b_{ij}(t)e^{-N}$, we assume that $a_{ij}(t) > b_{ij}(t)$ for $t \in R$ and $i, j = 1, 2\cdots, n$, which show the biological significance of the mortality terms.

For convenience, we introduce some notations. Throughout this paper, given a bounded continuous function g defined on R, let g^+ and g^- be defined as

$$g^- = \inf_{t\in R} g(t), \quad g^+ = \sup_{t\in R} g(t).$$

Let $R^n(R^n_+)$ be the set of all (nonnegative) real vectors, we will use $x=(x_1,\dots,x_n)^T\in R^n$ to denote a column vector, in which the symbol $()^T$ denotes the transpose of a vector. we let $|x|$ denote the absolute-value vector given by $|x|=(|x_1|,\dots,|x_n|)^T$ and define $||x||=\max_{1\le i\le n}|x_i|$. Denote $C=\prod\limits_{i=1}^{n}C([-r_i,0],R^1)$ and $C_+=\prod\limits_{i=1}^{n}C([-r_i,0],R^1_+)$ as Banach space equipped with the supremum norm defined by $||\varphi||=\sup\limits_{-r_i\le t\le 0}\max\limits_{1\le i\le n}|\varphi_i(t)|$ for all $\varphi(t)=(\varphi_1(t),\dots,\varphi_n(t))^T\in C$ (or $\in C_+$). If $x_i(t)$ is defined on $[t_0-r_i,\nu)$ with $t_0,\nu\in R^1$ and $i=1,\dots,n$, then we define $x_t\in C$ as $x_t=(x^1_t,\dots x^n_t)^T$ where $x^i_t(\theta)=x_i(t+\theta)$ for all $\theta\in[-r_i,0]$ and $i=1,\dots,n$.

The initial conditions associated with system (1.3) are of the form:

$$N_{t_0}=\varphi,\ \varphi=(\varphi_1,\dots,\varphi_n)^T\in C_+ \ \text{ and } \ \varphi_i(0)>0, i=1,\dots,n. \tag{1.4}$$

We write $N_t(t_0,\varphi)(N(t;t_0,\varphi))$ for a solution of the initial value problem (1.3) and (1.4) . Also, let $[t_0,\eta(\varphi))$ be the maximal right-interval of existence of $N_t(t_0,\varphi)$.

Definition 1.1. The system (1.3) with initial conditions (1.4) is said to be permanent, if there are positive constants k_i and K_i such that

$$k_i\le\liminf_{t\to+\infty}N_i(t;t_0,\varphi)\le\limsup_{t\to+\infty}N_i(t;t_0,\varphi)\le K_i,\ \ i=1,2\cdots,n.$$

The remaining part of this paper is organized as follows. In sections 2 and 3, we shall derive new sufficient conditions for checking the permanence of model (1.3). In Section 4, we shall give some examples and remarks to illustrate our results obtained in the previous sections.

2. Permanence of Nicholson-type delay systems with $D_{ij}(t,N)=\frac{a_{ij}(t)N}{b_{ij}(t)+N}(i,j=1,2,\cdots,n)$

Theorem 2.1. Assume that the following conditions are satisfied

$$\min_{1\le i\le n}\{a^-_{ii}\}>\sum_{i=1}^{n}\sum_{j=1,j\ne i}^{n}a^+_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{l}\frac{c^+_{ij}}{e\gamma^-_{ij}}, \tag{2.1}$$

$$\sup_{t\in R}\frac{a_{ii}(t)}{b_{ii}(t)\sum\limits_{j=1}^{n}c_{ij}(t)}<1,\quad i=1,2,\cdots,n. \tag{2.2}$$

Then, the model (1.3) and (1.4) with $D_{ij}(t,N)=\frac{a_{ij}(t)N}{b_{ij}(t)+N}(i,j=1,2,\cdots,n)$ is permanent.

Proof. Set $N(t) = N(t; t_0, \varphi)$ for all $t \in [t_0, \eta(\varphi))$. In view of $\varphi \in C_+$, using Theorem 5.2.1 in [16, p.81], we have $N_t(t_0, \varphi) \in C_+$ for all $t \in [t_0, \eta(\varphi))$. From(1.3) and the fact that $\frac{a_{ii}(t)N}{b_{ii}(t)+N} \le \frac{a_{ii}(t)N}{b_{ii}(t)}$ for all $t \in R, N \ge 0$, we obtain

$$\begin{aligned} N_i'(t) &= -D_{ii}(t, N_i(t)) + \sum_{j=1, j\neq i}^{n} D_{ij}(t, N_j(t)) + \sum_{j=1}^{l} c_{ij}(t) N_i(t - \tau_{ij}(t)) e^{-\gamma_{ij}(t) N_i(t-\tau_{ij}(t))} \\ &\ge -\frac{a_{ii}(t)N_i(t)}{b_{ii}(t)} + \sum_{j=1}^{l} c_{ij}(t) N_i(t - \tau_{ij}(t)) e^{-\gamma_{ij}(t) N_i(t-\tau_{ij}(t))}, \quad i = 1, 2, \cdots, n. \qquad (2.3) \end{aligned}$$

In view of $N_i(t_0) = \varphi_i(0) > 0$, integrating (2.3) from t_0 to t, we get

$$\begin{aligned} N_i(t) &\ge e^{-\int_{t_0}^{t} \frac{a_{ii}(u)}{b_{ii}(u)} du} N_i(t_0) + \\ & \quad e^{-\int_{t_0}^{t} \frac{a_{ii}(u)}{b_{ii}(u)} du} \int_{t_0}^{t} e^{\int_{t_0}^{s} \frac{a_{ii}(v)}{b_{ii}(v)} dv} \sum_{j=1}^{l} c_{ij}(s) N_i(s - \tau_{ij}(s)) e^{-\gamma_{ij}(s) N_i(s-\tau_{ij}(s))} ds \\ &> 0, \quad \text{for all } \ t \in [t_0, \eta(\varphi)), \quad i = 1, 2, \cdots, n. \end{aligned}$$

Let $y(t) = \sum\limits_{i=1}^{n} x_i(t)$, where $t \in [t_0 - r, \eta(\varphi)), r = \min\limits_{1\le i\le n} \{r_i\}$. Notice that $\max\limits_{x\ge 0} xe^{-x} = \frac{1}{e}$, we have

$$\begin{aligned} y'(t) &= -\sum_{i=1}^{n} \frac{a_{ii}(t)N_i(t)}{b_{ii}(t) + N_i(t)} + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \frac{a_{ij}(t)N_j(t)}{b_{ij}(t) + N_j(t)} + \\ & \quad \sum_{i=1}^{n} \sum_{j=1}^{l} c_{ij}(t) N_i(t - \tau_{ij}(t)) e^{-\gamma_{ij}(t) N_i(t-\tau_{ij}(t))} \\ &\le -\frac{\sum\limits_{i=1}^{n} a_{ii}(t) N_i(t)}{\sum\limits_{i=1}^{n} b_{ii}(t) + \sum\limits_{i=1}^{n} N_i(t)} + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij}(t) + \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{c_{ij}(t)}{e\gamma_{ij}(t)} \\ &\le -\frac{\sum\limits_{i=1}^{n} a_{ii}^{-} N_i(t)}{\sum\limits_{i=1}^{n} b_{ii}(t) + \sum\limits_{i=1}^{n} N_i(t)} + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij}^{+} + \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{c_{ij}^{+}}{e\gamma_{ij}^{-}} \\ &\le -\frac{\min\limits_{1\le i\le n} \{a_{ii}^{-}\} y(t)}{\sum\limits_{i=1}^{n} b_{ii}(t) + y(t)} + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij}^{+} + \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{c_{ij}^{+}}{e\gamma_{ij}^{-}}. \end{aligned}$$

For each $t \in [t_0 - r, \eta(\varphi))$, we define

$$M(t) = \max\{\xi : \xi \le t, y(\xi) = \max_{t_0 - r \le s \le t} y(s)\}.$$

We now claim that $y(t)$ is bounded on $[t_0, \eta(\varphi))$. In the contrary case, observe that $M(t) \to \eta(\varphi)$ as $t \to \eta(\varphi)$, we get

$$\lim_{t \to \eta(\varphi)} y(M(t)) = +\infty.$$

But $y(M(t)) = \max\limits_{t_0 - r \le s \le t} y(s)$, and so $y'(M(t)) \ge 0$ for all $M(t) > t_0$. Thus,

$$\begin{aligned} 0 &\le y'(M(t)) \\ &\le -\frac{\min\limits_{1\le i\le n}\{a_{ii}^-\} y(M(t))}{\sum\limits_{i=1}^n b_{ii}(M(t)) + y(M(t))} + \sum_{i=1}^n \sum_{j=1, j\ne i}^n a_{ij}^+ + \sum_{i=1}^n \sum_{j=1}^l \frac{c_{ij}^+}{e\gamma_{ij}^-}, \text{ for all } M(t) > t_0, \end{aligned}$$

which yields

$$\frac{\min\limits_{1\le i\le n}\{a_{ii}^-\} y(M(t))}{\sum\limits_{i=1}^n b_{ii}(M(t)) + y(M(t))} \le \sum_{i=1}^n \sum_{j=1, j\ne i}^n a_{ij}^+ + \sum_{i=1}^n \sum_{j=1}^l \frac{c_{ij}^+}{e\gamma_{ij}^-}, \text{ for all } M(t) > t_0. \tag{2.4}$$

Therefore, from the continuities and boundedness of the functions $b_{ij}(t)$, $i, j = 1, 2, \cdots, n$, we can select a sequence $\{T_n\}_{n=1}^{+\infty}$ such that

$$\lim_{n\to+\infty} T_n = \eta(\varphi), \quad \lim_{n\to+\infty} y(M(T_n)) = +\infty, \quad \lim_{n\to+\infty} b_{ij}(M(T_n)) = b_{ij}^*, \tag{2.5}$$

and

$$\frac{\min\limits_{1\le i\le n}\{a_{ii}^-\} y(M(T_n))}{\sum\limits_{i=1}^n b_{ii}(M(T_n)) + y(M(T_n))} \le \sum_{i=1}^n \sum_{j=1, j\ne i}^n a_{ij}^+ + \sum_{i=1}^n \sum_{j=1}^l \frac{c_{ij}^+}{e\gamma_{ij}^-}. \tag{2.6}$$

Letting $n \to +\infty$, (2.5) and (2.6) imply that

$$\min_{1\le i\le n}\{a_{ii}^-\} \le \sum_{i=1}^n \sum_{j=1, j\ne i}^n a_{ij}^+ + \sum_{i=1}^n \sum_{j=1}^l \frac{c_{ij}^+}{e\gamma_{ij}^-}.$$

which contradicts with (2.1). This implies that $y(t)$ is bounded on $[t_0, \eta(\varphi))$. From Theorem 2.3.1 in [17], we easily obtain $\eta(\varphi) = +\infty$.Thus, every solution $N(t; t_0, \varphi)$ of (1.3) and (1.4) is positive and bounded on $[t_0, +\infty)$. So there exist positive constants K_i, such that

$$0 < N_i(t) \le K_i, \quad \text{for all} \quad t > t_0 \quad, i = 1, 2, \cdots, n$$

It follows that

$$\lim_{t\to+\infty} \sup N_i(t) \le K_i, \quad i = 1, 2, \cdots, n. \tag{2.7}$$

We next prove that there exist positive constants k_i, such that

$$\lim_{t\to+\infty}\inf N_i(t) \geq k_i, \quad i=1,2,\cdots,n. \tag{2.8}$$

For $i=1,2,\cdots,n$, from (1.3) we have

$$N_i'(t) \geq -\frac{a_{ii}(t)N_i(t)}{b_{ii}(t)} + \sum_{j=1}^{l} c_{ij}(t)N_i(t-\tau_{ij}(t))e^{-\gamma_{ij}(t)N_i(t-\tau_{ij}(t))}, \tag{2.9}$$

where $t\in[t_0,+\infty)$. Suppose that (2.8) does not hold, that is,

$$\lim_{t\to+\infty}\inf N_i(t) = 0, i=1,2,\cdots,n.$$

For each $t\geq t_0$, we define

$$\theta_i(t) = \max\{\xi : \xi\leq t, N_i(\xi) = \min_{t_0\leq s\leq t} N_i(s),\}, i=1,2,\cdots,n.$$

Observe that $\theta_i(t)\to+\infty$ as $t\to+\infty$,$i=1,2,\cdots,n$, and

$$\lim_{t\to+\infty} N_i(\theta_i(t)) = 0, i=1,2,\cdots,n. \tag{2.10}$$

However, $N_i(\theta_i(t)) = \min_{t_0\leq s\leq t} N_i(s)$, and so $N_i'(\theta_i(t))\leq 0$, where $\theta_i(t)>t_0, i=1,2,\cdots,n$. According to (2.9), we have

$$\begin{aligned} 0 &\geq N_i'(\theta_i(t)) \\ &\geq -\frac{a_{ii}(t)N_i(\theta_i(t))}{b_{ii}(t)} + \sum_{j=1}^{l} c_{ij}(\theta_i(t))N_i(\theta_i(t)-\tau_{ij}(\theta_i(t)))e^{-\gamma_{ij}(\theta_i(t))N_i(\theta_i(t)-\tau_{ij}(\theta_i(t)))}, \end{aligned}$$

which is equivalent to

$$\frac{a_{ii}(\theta_i(t))}{b_{ii}(\theta_i(t))}N_i(\theta_i(t)) \geq \sum_{j=1}^{l} c_{ij}(\theta_i(t))N_i(\theta_i(t)-\tau_{ij}(\theta_i(t)))e^{-\gamma_{ij}(\theta_i(t))N_i(\theta_i(t)-\tau_{ij}(\theta_i(t)))}, \tag{2.11}$$

where $\theta_i(t)>t_0, i=1,2,\cdots,n$. This, together with (2.10), implies that

$$\lim_{t\to+\infty} N_i(\theta_i(t)-\tau_{ij}(\theta_i(t))) = 0, i=1,2,\cdots,n \tag{2.12}$$

Now we select a sequence $\{t_n\}_{n=1}^{+\infty}$ such that

$$\begin{cases} \theta_i(t_n)>t_0, \quad \lim\limits_{n\to+\infty} t_n = +\infty, \quad \lim\limits_{n\to+\infty} N_i(\theta_i(t_n)) = 0, \quad \lim\limits_{n\to+\infty} a_{ii}(\theta_i(t_n)) = a_{ii}^* \\ \lim\limits_{n\to+\infty} b_{ii}(\theta_i(t_n)) = b_{ii}^*, \quad \lim\limits_{n\to+\infty} c_{ij}(\theta_i(t_n)) = c_{ij}^*, \quad \lim\limits_{n\to+\infty} \gamma_{ij}(\theta_i(t_n)) = \gamma_{ij}^*, \end{cases} \tag{2.13}$$

where $i = 1, 2, \cdots, n, j = 1, 2, \cdots, l$. Thus, we obtain

$$\frac{a_{ii}(\theta_i(t_n))}{b_{ii}(\theta_i(t_n))} \geq \sum_{j=1}^{l} c_{ij}(\theta_i(t_n)) e^{-\gamma_{ij}(\theta_i(t_n))N_i(\theta_i(t_n)-\tau_{ij}(\theta_i(t_n)))}, \tag{2.14}$$

where $i = 1, 2, \cdots, n$. Letting $n \to +\infty$, from (2.12)-(2.14) we know that

$$\sup_{t\in R} \frac{a_{ii}(t)}{b_{ii}(t) \sum\limits_{j=1}^{l} c_{ij}(t)} \geq \lim_{n\to+\infty} \frac{a_{ii}(\theta_i(t_n))}{b_{ii}(\theta_i(t_n)) \sum\limits_{j=1}^{l} c_{ij}(\theta_i(t_n))} = \frac{a_{ii}^*}{b_{ii}^* \sum\limits_{j=1}^{l} c_{ij}^*} \geq 1,$$

which contradicts with (2.2). Hence, inequality of (2.8) holds. Combining (2.7) and (2.8) the whole proof of Theorem 2.1 is complete.

3. Permanence of Nicholson-type delay systems with $D_{ij}(t, N) = a_{ij}(t) - b_{ij}(t)e^{-N}(i, j = 1, 2, \cdots, n)$

Theorem 3.1. Assume that

$$a_{ii}^+ - b_{ii}^- < \sum_{j=1, j\neq i}^{n} (a_{ij}^- - b_{ij}^+), \quad i = 1, 2, \cdots, n, \tag{3.1}$$

$$\sum_{j=1, j\neq i}^{n} a_{ij}^+ + \sum_{j=1}^{l} \frac{c_{ij}^+}{e\gamma_{ij}^-} < a_{ii}^-, \quad i = 1, 2, \cdots, n. \tag{3.2}$$

Then, the model (1.3) and (1.4) with $D_{ij}(t, N) = a_{ij}(t) - b_{ij}(t)e^{-N}(i, j = 1, 2, \cdots, n)$ is permanent.

Proof. Let $N(t) = N(t; t_0, \varphi)$, we first claim that

$$N_i(t) > 0 \text{ for all } \ t \in [t_0, \eta(\varphi)), i = 1, 2, \cdots, n. \tag{3.3}$$

Contrarily, it must occur that there exist $t^* \in [t_0, \eta(\varphi))$ and $k \in \{1, 2, \cdots, n\}$ such that

$$N_k(t^*) = 0, \quad N_i(t) > 0 \quad \text{for all} \quad t \in [t_0, t^*), i = 1, 2, \cdots, n.$$

Then, we have

$$\begin{aligned}
0 \ &\geq \ N_k'(t^*) \\
&= \ -D_{kk}(t^*, N_i(t^*)) + \sum_{j=1, j\neq k}^{n} D_{kj}(t^*, N_j(t^*)) + \sum_{j=1}^{l} c_{kj}(t^*) N_k(t^* - \tau_{kj}(t^*)) e^{-\gamma_{kj}(t^*)N_k(t^*-\tau_{kj}(t^*))} \\
&\geq \ -a_{kk}(t^*) + b_{kk}(t^*) + \sum_{j=1, j\neq k}^{n} a_{kj}(t^*) - \sum_{j=1, j\neq k}^{n} b_{kj}(t^*) \\
&\geq \ -a_{kk}^+ + b_{kk}^- + \sum_{j=1, j\neq k}^{n} a_{kj}^- - \sum_{j=1, j\neq k}^{n} b_{kj}^+,
\end{aligned}$$

It follows that $a_{kk}^{+}-b_{kk}^{-}\geq\sum\limits_{j=1,j\neq k}^{n}(a_{kj}^{-}-b_{kj}^{+})$ which contradicts with inequality of (3.1). This implies that (3.3) holds. For all $t\in[t_0-r_i,\eta(\varphi))$, we define

$$m_i(t)=\max\{\xi:\xi\leq t,N_i(\xi)=\max_{t_0-r_i\leq s\leq t}N_i(s)\},\quad i=1,2,\cdots,n.$$

We now show that $N_i(t)$ are bounded on $[t_0,\eta(\varphi)),i=1,2,\cdots,n$. In the contrary case, it exists $k\in\{1,2,\cdots,n\}$ and observe that $m_k(t)\to\eta(\varphi)$ as $t\to\eta(\varphi)$, we get

$$\lim_{t\to\eta(\varphi))}N_k(m_k(t))=+\infty. \tag{3.4}$$

But $N_k(m_k(t))=\max\limits_{t_0-r_k\leq s\leq t}N_k(s)$, and so $N_k'(m_k(t))\geq 0$ for all $m_k(t)>t_0$. Thus,

$$\begin{aligned}
0 &\leq N_k'(m_k(t))\\
&\leq -a_{kk}(m_k(t))+b_{kk}(m_k(t))e^{-N_k(m_k(t))}+\sum_{j=1,j\neq k}^{n}a_{kj}(m_k(t))+\sum_{j=1}^{l}c_{kj}^{+}\frac{1}{e\gamma_{kj}^{-}}.
\end{aligned}\tag{3.5}$$

Letting $t\to\eta(\varphi)$, (3.5) implies that

$$\sum_{j=1,j\neq k}^{n}a_{kj}^{+}+\sum_{j=1}^{l}\frac{c_{kj}^{+}}{e\gamma_{kj}^{-}}\geq a_{kk}^{-}.$$

which contradicts with the inequality of (3.2). This shows that $N_i(t)$ are positive and bounded for all $t\in[t_0,\eta(\varphi))$, $i=1,2,\cdots,n$. From Theorem 2.3.1 in [17], we easily obtain $\eta(\varphi)=+\infty$. So there exist positive constants L_i such that

$$0<N_i(t)\leq L_i,i=1,2,\cdots,n.$$

It follows that

$$\lim_{t\to+\infty}\sup N_i(t)\leq L_i,i=1,2,\cdots,n. \tag{3.6}$$

In what follows, we prove that there exists a positive constant l_i such that

$$\lim_{t\to+\infty}\inf N_i(t)\geq l_i,\quad i=1,2,\cdots,n \tag{3.7}$$

Assume that (3.7) does not hold, then it exists $k\in\{1,2,\cdots,n\}$, such that

$$\lim_{t\to+\infty}\inf N_k(t)=0.$$

For each $t\geq t_0$, we define

$$\omega(t)=\max\{\xi:\xi\leq t,N_k(\xi)=\min_{t_0\leq s\leq t}N_k(s)\}.$$

Observe that $\omega(t) \to +\infty$ as $t \to +\infty$ and

$$\lim_{t\to+\infty} N_k(\omega(t)) = 0. \tag{3.8}$$

However, $N_k(\omega(t)) = \min_{t_0\le s\le t} N_k(s)$, and so $N_k'(\omega(t)) \le 0$, where $\omega(t) > t_0$. Then

$$\begin{aligned}
0 &\geq N_k'(\omega(t)) \\
&\geq -a_{kk}(\omega(t)) + b_{kk}(\omega(t))e^{-N_k(\omega(t))} + \sum_{j=1,j\neq k}^{n} (a_{kj}(\omega(t)) - b_{kj}(\omega(t))e^{-N_j(\omega(t))}) \\
&\geq -a_{kk}(\omega(t)) + b_{kk}(\omega(t))e^{-N_k(\omega(t))} + \sum_{j=1,j\neq k}^{n} (a_{kj}^- - b_{kj}^+).
\end{aligned} \tag{3.9}$$

Letting $t \to +\infty$, (3.9) implies that

$$a_{kk}^+ - b_{kk}^- \geq \sum_{j=1,j\neq k}^{n} (a_{kj}^- - b_{kj}^+),$$

which contradicts with the inequality of (3.1). This ends the proof of Theorem 3.1.

4. Some examples

In this section we present some examples to illustrate our results.

Example 4.1. Consider the following Nicholson-type delay system with patch structure and nonlinear density-dependent mortality terms:

$$\left\{\begin{aligned}
N_1'(t) &= -\tfrac{(13+|\cos\sqrt{3}t|)N_1(t)}{5+|\sin\sqrt{2}t|+N_1(t)} + \tfrac{(1+|\sin 2t|)N_2(t)}{3+|\cos 3t|+N_2(t)} + \tfrac{(1+|\cos 2t|)N_3(t)}{4+|\sin 3t|+N_3(t)} \\
&\quad +(1+\cos^2 t)N_1(t-2|\sin t|)e^{-4N_1(t-2|\sin t|)} \\
&\quad +(1+\sin^2 t)N_1(t-2|\cos t|)e^{-4N_1(t-2|\cos t|)} \\
N_2'(t) &= -\tfrac{(14+|\sin\sqrt{3}t|)N_2(t)}{6+|\cos\sqrt{2}t|+N_2(t)} + \tfrac{(1+|\cos 2t|)N_1(t)}{3+|\sin 3t|+N_1(t)} + \tfrac{(1+|\sin 2t|)N_3(t)}{4+|\cos 3t|+N_3(t)} \\
&\quad +(1+\sin^2 t)N_2(t-2|\cos t|)e^{-5N_2(t-2|\cos t|)} \\
&\quad +(1+\cos^2 t)N_2(t-2|\sin t|)e^{-5N_2(t-2|\sin t|)} \\
N_3'(t) &= -\tfrac{(15+|\sin\sqrt{5}t|)N_3(t)}{6+|\cos\sqrt{6}t|+N_3(t)} + \tfrac{(1+|\cos 3t|)N_1(t)}{3+|\sin 2t|+N_1(t)} + \tfrac{(1+|\sin 3t|)N_2(t)}{4+|\cos 2t|+N_2(t)} \\
&\quad +(1+\sin^2\sqrt{2}t)N_3(t-2|\cos 2t|)e^{-6N_3(t-2|\cos 2t|)} \\
&\quad +(1+\cos^2\sqrt{2}t)N_3(t-2|\sin 3t|)e^{-6N_3(t-2|\sin 3t|)},
\end{aligned}\right. \tag{4.1}$$

Obviously, $a_{11}^- = 13, a_{22}^- = 14, a_{33}^- = 15$, $a_{ij}^+ = 2, (i,j = 1,2,3, i \neq j)$, $c_{ij}^+ = 2, (i = 1,2,3, j = 1,2)$, $\gamma_{1j}^- = 4, \gamma_{2j}^- = 5, \gamma_{3j}^- = 6, (j = 1,2), r_i = 2, (i = 1,2,3)$. So

$$13=\min_{1\le i\le 3}\{a_{ii}^-\}>\sum_{i=1}^{3}\sum_{j=1,j\ne i}^{3}a_{ij}^+ +\sum_{i=1}^{3}\sum_{j=1}^{2}\frac{c_{ij}^+}{e\gamma_{ij}^-}=12+\frac{32}{15e},$$

and

$$\max_{1\le i\le 3}\{\sup_{t\in R}\frac{a_{ii}(t)}{b_{ii}(t)\sum\limits_{j=1}^{2}c_{ij}(t)}\}=\max\{\frac{14}{15},\frac{5}{6},\frac{8}{9}\}=\frac{14}{15}<1.$$

It follows that the Nicholson's blowflies model with patch structure and nonlinear density-dependent mortality terms (4.1) satisfies all the conditions in Theorem 2.1. Hence, from Theorem 2.1, the system (4.1) with initial conditions (1.4) is permanent.

Example 4.2. Consider the following Nicholson-type delay system with patch structure and nonlinear density-dependent mortality terms:

$$\left\{\begin{array}{rcl} N_1'(t) &=& -(9+|\cos t|)+(8+|\sin t|)e^{-N_1(t)}+(3+|\sin t|)-(0.5+|\cos t|)e^{-N_2(t)}\\ && +(3+|\cos t|)-(0.5+|\sin t|)e^{-N_3(t)}+(1+\cos^2 t)N_1(t-2|\sin t|)e^{-4N_1(t-2|\sin t|)}\\ && +(1+\sin^2 t)N_1(t-2|\cos t|)e^{-4N_1(t-2|\cos t|)}\\ N_2'(t) &=& -(9+|\sin t|)+(8+|\cos t|)e^{-N_2(t)}+(3+|\cos t|)-(0.5+|\sin t|)e^{-N_1(t)}\\ && +(3+|\sin t|)-(0.5+|\cos t|)e^{-N_3(t)}+(1+\sin^2 t)N_2(t-2|\cos t|)e^{-4N_2(t-2|\cos t|)}\\ && +(1+\cos^2 t)N_2(t-2|\sin t|)e^{-4N_2(t-2|\sin t|)}\\ N_3'(t) &=& -(9+|\sin 2t|)+(8+|\cos 2t|)e^{-N_3(t)}+(3+|\cos 2t|)-(0.5+|\sin 2t|)e^{-N_1(t)}\\ && +(3+|\sin 2t|)-(0.5+|\cos 2t|)e^{-N_2(t)}\\ && +(1+\sin^2 2t)N_2(t-2|\cos 2t|)e^{-4N_2(t-2|\cos 2t|)}\\ && +(1+\cos^2 2t)N_2(t-2|\sin 2t|)e^{-4N_2(t-2|\sin 2t|)}, \end{array}\right. \tag{4.2}$$

Obviously, $a_{ii}^+=10, a_{ii}^-=9, b_{ii}^+=9, b_{ii}^-=8, (i=1,2,3)$, $a_{ij}^+==4, b_{ij}^+=1.5, a_{ij}^-=3, b_{ij}^-=0.5, (i,j=1,2,3, i\ne j)$, $c_{ij}^+=2, \gamma_{ij}^-=4, (i=1,2,3, j=1,2)$, $r_i=2, (i=1,2,3)$. So

$$2=a_{ii}^+-b_{ii}^-<\sum_{j=1,j\ne i}^{n}(a_{ij}^--b_{ij}^+)=3,\quad i=1,2,3\quad ,$$

and

$$8+\frac{1}{e}=\sum_{j=1,j\ne i}^{n}a_{ij}^+ +\sum_{j=1}^{2}\frac{c_{ij}^+}{e\gamma_{ij}^-}<a_{ii}^-=9,\quad i=1,2,3.$$

Hence, from Theorem 3.1, the model (4.2) is permanent.

The above two examples that satisfy the conditions of Theorem 2.1 and Theorem 3.1 respectively are permanent. Next we shall give the example that does not satisfy the conditions of Theorem 2.1 is not permanent.

Example 4.3. Consider the following Nicholson-type delay system with patch structure and nonlinear density-dependent mortality terms:

$$\begin{cases} N_1'(t) &= -\frac{(25+|\cos t|)N_1(t)}{1+|\sin t|+N_1(t)} + \frac{(12+|\sin t|)N_2(t)}{13+|\cos t|+N_2(t)} + (1+\cos^2 t)N_1(t-|\sin t|)e^{-4N_1(t-|\sin t|)} \\ & +(1+\sin^2 t)N_1(t-|\cos t|)e^{-4N_1(t-|\cos t|)} \\ N_2'(t) &= -\frac{(25+|\sin t|)N_2(t)}{1+|\cos t|+N_2(t)} + \frac{(12+|\cos t|)N_1(t)}{13+|\sin t|+(1+\sin^2 t)N_1(t)} + N_2(t-|\cos t|)e^{-4N_2(t-|\cos t|)} \\ & +(1+\cos^2 t)N_2(t-|\sin t|)e^{-4N_2(t-|\sin t|)} \end{cases} \tag{4.3}$$

Obviously, $a_{11}^- = a_{22}^- = 25$, $a_{ij}^+ = 12, (i,j=1,2, i\neq j)$, $c_{ij}^+ = 2$, $\gamma_{ij}^- = 4$,$r_i = 1, (i,j=1,2)$. So

$$25 = \min_{1\leq i\leq 2}\{a_{ii}^-\} < \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2} a_{ij}^+ + \sum_{i=1}^{2}\sum_{j=1}^{2}\frac{c_{ij}^+}{e\gamma_{ij}^-} = 26 + \frac{2}{e},$$

and

$$\max_{1\leq i\leq 2}\{\sup_{t\in R}\frac{a_{ii}(t)}{b_{ii}(t)\sum_{j=1}^{2} c_{ij}(t)}\} = \frac{26}{3} > 1.$$

It follows that the Nicholson's blowflies model with patch structure and nonlinear density-dependent mortality terms (4.3) dose not satisfy the conditions of Theorem 2.1. Moreover, we shall prove the model (4.3) is not permanent with the initial condition φ^* satisfying $\varphi^* \in C_+, \varphi_i^*(0) > 0$ and $||\varphi^*|| < e$, $i = 1,2$. We write (4.3) as the following systems of delay differential equation:

$$N_i'(t) = f_i(t, N_t), \quad i = 1,2$$

where

$$\begin{cases} f_1(t,\varphi) &= -\frac{(25+|\cos t|)\varphi_1(0)}{1+|\sin t|+\varphi_1(0)} + \frac{(12+|\sin t|)\varphi_2(0)}{13+|\cos t|+\varphi_2(0)} + (1+\cos^2 t)\varphi_1(-|\sin t|)e^{-4\varphi_1(-|\sin t|)} \\ & +(1+\sin^2 t)\varphi_1(-|\cos t|)e^{-4\varphi_1(-|\cos t|)} \\ &= -\frac{a_{11}(t)\varphi_1(0)}{b_{11}(t)+\varphi_1(0)} + \frac{a_{12}(t)\varphi_2(0)}{b_{12}(t)+\varphi_2(0)} + \sum_{j=1}^{2} c_{1j}(t)\varphi_1(-\tau_{1j}(t))e^{-\gamma_{1j}(t)\varphi_1(-\tau_{1j}(t))} \\ f_2(t,\varphi) &= -\frac{(25+|\sin t|)\varphi_2(0)}{1+|\cos t|+\varphi_2(0)} + \frac{(12+|\cos t|)\varphi_1(0)}{13+|\sin t|+\varphi_1(0)} + (1+\sin^2 t)\varphi_2(-|\cos t|)e^{-4\varphi_2(-|\cos t|)} \\ & +(1+\cos^2 t)\varphi_2(-|\sin t|)e^{-4\varphi_2(-|\sin t|)} \\ &= -\frac{a_{22}(t)\varphi_2(0)}{b_{22}(t)+\varphi_2(0)} + \frac{a_{21}(t)\varphi_1(0)}{b_{21}(t)+\varphi_1(0)} + \sum_{j=1}^{2} c_{2j}(t)\varphi_2(-\tau_{2j}(t))e^{-\gamma_{2j}(t)\varphi_2(-\tau_{2j}(t))} \end{cases}$$

Let $N(t) = N(t; t_0, \varphi^*)$ be the solution of system (4.3) with the initial condition φ^* for all $t \in [t_0, \eta(\varphi))$. In view of $\varphi^* \in C_+$, using Theorem 5.2.1 in [16,p.81], we have $N_t(t, \varphi^*) \in C_+$ for all $t \in [t_0, \eta(\varphi))$. Now we prove that

$$||N(t)|| < e \quad \text{for all} \quad t \in [t_0, \eta(\varphi)) \quad \text{and} \quad \eta(\varphi) = +\infty. \tag{4.4}$$

In the contrary case, there are $i \in \{1, 2\}$ and $t_1 > t_0$ such that

$$N_i(t_1) = e, \ 0 \leq N_j(t) < e \ \text{ for all } \ t_0 \leq t < t_1, \quad j = 1, 2.$$

We have

$$\begin{aligned}
0 \ &\leq \ N_i'(t_1) = -\frac{a_{ii}(t_1)N_i(t_1)}{b_{ii}(t_1) + N_i(t_1)} + \sum_{j=1, j\neq i}^{2} \frac{a_{ij}(t_1)N_j(t_1)}{b_{ij}(t_1) + N_j(t_1)} \\
&\quad + \sum_{j=1}^{2} c_{ij}(t_1)N_i(t_1 - \tau_{ij}(t_1))e^{-\gamma_{ij}(t_1)N_i(t_1 - \tau_{ij}(t_1))} \\
&\leq \ -\frac{a_{ii}(t_1)N_i(t_1)}{b_{ii}(t_1) + N_i(t_1)} + \sum_{j=1, j\neq i}^{2} \frac{a_{ij}(t_1)N_j(t_1)}{b_{ij}(t_1)} + \sum_{j=1}^{2} \frac{c_{ij}(t_1)}{\gamma_{ij}(t_1)} \frac{1}{e} \\
&\leq \ -\frac{a_{ii}^- e}{b_{ii}^+ + e} + \sum_{j=1, j\neq i}^{2} \frac{a_{ij}^+ e}{b_{ij}^-} + \sum_{j=1}^{2} \frac{c_{ij}^+}{\gamma_{ij}^-} \frac{1}{e} \\
&= \ -\frac{25e}{2 + e} + e + \frac{1}{e} < 0,
\end{aligned}$$

which is a contradiction. This implies that (4.4) holds. Let $y(t) = N(t)e^{\lambda t}$, where $\lambda > 0$ and satisfying $\lambda - \frac{23-e}{2+e} + 4e^{\lambda} < 0$. We claim that

$$||y(t)|| < e \ \text{ for all } \ t \in [t_0, +\infty). \tag{4.5}$$

If this is not valid, there are $i \in \{1, 2\}$ and $t_2 > t_0$ such that

$$y_i(t_2) = e, \ 0 \leq y_j(t) < e \ \text{ for all } \ t_0 \leq t < t_2, \quad j = 1, 2.$$

We have

$$\begin{aligned}
0 \ &\leq \ y_i'(t_2) = \lambda N_i(t_2)e^{\lambda t_2} + e^{\lambda t_2} N_i'(t_2) \\
&= \ \lambda N_i(t_2)e^{\lambda t_2} - \frac{a_{ii}(t_2)N_i(t_2)e^{\lambda t_2}}{b_{ii}(t_2) + N_i(t_2)} + \sum_{j=1, j\neq i}^{2} \frac{a_{ij}(t_2)N_j(t_2)e^{\lambda t_2}}{b_{ij}(t_2) + N_j(t_2)}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{2} c_{ij}(t_2)e^{\lambda\tau_{ij}(t_2)}N_i(t_2-\tau_{ij}(t_2))e^{\lambda(t_2-\tau_{ij}(t_2))}e^{-\gamma_{ij}(t_2)N_i(t_2-\tau_{ij}(t_2))}\\
\leq\ & \lambda e-\frac{a_{ii}(t_2)e}{b_{ii}(t_2)+e}+\sum_{j=1,j\neq i}^{2}\frac{a_{ij}(t_2)e}{b_{ij}(t_2)}+\sum_{j=1}^{2}c_{ij}(t_2)e^{\lambda r_i}e\\
\leq\ & (\lambda-\frac{a_{ii}^{-}}{b_{ii}^{+}+e}+\sum_{j=1,j\neq i}^{2}\frac{a_{ij}^{+}}{b_{ij}^{-}}+\sum_{j=1}^{2}c_{ij}^{+}e^{\lambda r_i})e\\
=\ & (\lambda-\frac{23-e}{2+e}+4e^{\lambda})e<0,
\end{aligned}
$$

which is a contradiction. This implies that (4.5) holds and the system (4.3) with initial condition φ^* is not permanent but extinct.

Remark 4.1. To the best of our knowledge, few authors have considered the problems of the permanence of Nicholson's blowflies model with patch structure and nonlinear density-dependent mortality terms. It is clear that all the results in [12-15] and the references therein cannot be applicable to prove the permanence of (4.1) and (4.2). This implies that the results of this paper are new.

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