Electronic Journal of Qualitative Theory of Differential Equations
2007, No. **21**, 1-10; **http://www.math.u-szeged.hu/ejqtde/**

On global attractivity of solutions of a functional-integral equation

Mohamed Abdalla Darwish

Department of Mathematics, Faculty of Science,
Alexandria University at Damanhour, 22511 Damanhour, Egypt
e-mail: darwishma@yahoo.com, mdarwish@ictp.trieste.it

Abstract

We prove an existence theorem for a quadratic functional-integral equation of mixed type. The functional-integral equation studied below contains as special cases numerous integral equations encountered in nonlinear analysis. With help of a suitable measure of noncompactness, we show that the functional integral equation of mixed type has solutions being continuous and bounded on the interval $[0, \infty)$ and those solutions are globally attractive.

Key words and phrases: Functional-integral equation, existence, global attractivity, measure of noncompactness, fixed point theorem due to Darbo.
AMS (MOS) Subject Classifications: 45G10, 45M99, 47H09.

1 Introduction

Quadratic integral equations are often applicable in the theory of radiative transfer, kinetic theory of gases, in the theory of neutron transport and in the traffic theory. Especially, the so-called quadratic integral equation of Chandrasekhar type can be very often encountered in many applications (cf. [1, 2, 3, 6, 7, 8, 9, 10, 13, 14]). In this paper we study the functional integral equation of mixed type, namely

$$x(t) = f\left(t, x(t), \int_0^t u(t, s, x(s))ds, \int_0^\infty v(t, s, x(s))\, ds\right). \tag{1}$$

Eq.(1) contains as special cases numerous integral and functional-integral equations encountered in nonlinear analysis. Also, the famous Chandrasekhar's integral equation is considered as a special case.

Using the technique associated with a suitable measure of noncompactness, we show that Eq.(1) has solutions being continuous and bounded on the interval $[0, \infty)$ and those solutions are globally attractive. In fact, our result in this paper is motivated by the extension of the work of Hu and Yan [12].

2 Auxiliary facts and results

This section collects some definitions and results which will be needed further on. Assume that $(\mathrm{E}, \|.\|)$ is a Banach space with zero element θ. Let $B(x,r)$ denote the closed ball centered at x and with radius r. The symbol B_r stands for the ball $B(\theta, r)$.

If X is a subset of E, then $\bar{\mathrm{X}}$ and $ConvX$ denote the closure and convex closure of X, respectively. Moreover, we denote by $\mathcal{M}_{\mathrm{E}}$ the family of all nonempty and bounded subsets of E and $\mathcal{N}_{\mathrm{E}}$ its subfamily consisting of all relatively compact subsets.

Next we give the concept of a measure of noncompactness [5]:

Definition 2.1 *A mapping $\mu : \mathcal{M}_{\mathrm{E}} \to [0, +\infty)$ is said to be a measure of noncompactness in* E *if it satisfies the following conditions:*

1) *The family* $\mathrm{Ker}\mu = \{X \in \mathcal{M}_{\mathrm{E}} : \mu(X) = 0\}$ *is nonempty and* $\mathrm{Ker}\mu \subset \mathcal{N}_{\mathrm{E}}$.
2) $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$.
3) $\mu(\bar{X}) = \mu(ConvX) = \mu(X)$.
4) $\mu(\lambda X + (1-\lambda)Y) \leq \lambda\, \mu(X) + (1-\lambda)\, \mu(Y)$ *for* $0 \leq \lambda \leq 1$.
5) *If* $X_n \in \mathcal{M}_{\mathrm{E}}$, $X_n = \bar{X}_n$, $X_{n+1} \subset X_n$ *for* $n = 1,\ 2,\ 3,\ \ldots$ *and* $\lim_{n\to\infty} \mu(X_n) = 0$ *then* $\cap_{n=1}^{\infty} X_n \neq \phi$.

We recall the fixed point theorem due to Darbo [11]. Before quoting this theorem, we need the following definition:

Definition 2.2 *Let M be a nonempty subset of a Banach space* E, *and* $\mathrm{T} : M \to \mathrm{E}$ *be a continuous operator that transforms bounded sets onto bounded ones. We say that* T *satisfies the Darbo condition (with constant $c \geq 0$) with respect to a measure of noncompactness μ if for any bounded subset X of M we have*

$$\mu(\mathrm{T}X) \leq c\, \mu(X).$$

If T satisfies the Darbo condition with $c < 1$ then it is called a contraction operator with respect to μ.

Theorem 2.1 Let Q be a nonempty, bounded, closed and convex subset of the space E and let

$$H : Q \to Q$$

be a contraction with respect to the measure of noncompactness μ.

Then H has a fixed point in the set Q.

Remark 2.1 [5]
Under the assumptions of the above theorem the set Fix H of fixed points of H belonging to Q is a member of ker μ. In fact, as $\mu(H(\mathrm{Fix}\ H)) = \mu(\mathrm{Fix}\ H) \leq c\, \mu(\mathrm{Fix}\ H)$ and $0 \leq c < 1$, we deduce that $\mu(\mathrm{Fix}\ H) = 0$.

This observation allows us to characterize solutions of considered operator equation.

In what follows we will work in the Banach space $BC(\mathbb{R}_+)$ consisting of all real functions defined, bounded and continuous on $\mathbb{R}_+$. The space $BC(\mathbb{R}_+)$ is equipped with the standard norm

$$\| x \| = \sup\{|x(t)| : t \geq 0\}$$

Now, we recollect the construction of the measure of noncompactness which will be used in the next section (see [4])

Let us fix a nonempty and bounded subset X of $BC(\mathbb{R}_+)$ and let T be a positive number. For $x \in X$ and $\varepsilon \geq 0$ denote by $\omega^T(x, \varepsilon)$ the modulus of continuity of the function x on the interval $[0, T]$, i.e.,

$$\omega^T(x, \varepsilon) = \sup\{|x(t) - x(s)| : t,\ s \in [0, T],\ |t - s| \leq \varepsilon\}$$

Further, let us put

$$\begin{aligned} \omega^T(X, \varepsilon) &= \sup\{\omega^T(x, \varepsilon) : x \in X\}, \\ \omega_0^T(X) &= \lim_{\varepsilon \to 0} \omega^T(X, \varepsilon), \\ \omega_0(X) &= \lim_{T \to \infty} \omega_0^T(X). \end{aligned}$$

For a fixed number $t \geq 0$ we denote

$$X(t) = \{x(t) : x \in X\}$$

and

$$\operatorname{diam} X(t) = \sup\{|x(t) - y(t)| : x,\ y \in X\}.$$

Now, let us define the function μ on the family $\mathcal{M}_{BC(\mathbb{R}_+)}$ by the following formula

$$\mu(X) = \omega_0(X) + \limsup_{t \to \infty} \operatorname{diam} X(t).$$

It can be shown [4] that the function μ is a measure of noncompactness on the space $BC(\mathbb{R}_+)$.

Definition 2.3 The solution $x(t)$ of Eq.(1) is said to be globally attractive, if there are

$$\lim_{t \to +\infty} (x(t) - y(t)) = 0$$

for any solution $y(t)$ of Eq.(1).

3 Main Results

In this section, we will study Eq.(1) assuming that the following assumptions are satisfied:

$a_1)$ $f : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and the function $t \to f(t, 0, 0, 0)$ is an element of the space $BC(\mathbb{R}_+)$.

$a_2)$ There exist continuous functions $m_1,\ m_2 : \mathbb{R}_+ \to \mathbb{R}_+$ and a constant $0 \leq k < 1$ such that

$$|f(t, x_1, y_1, z_1) - f(t, x_2, y_2, z_2)| \leq k\ |x_1 - x_2| + m_1(t)|y_1 - y_2| + m_2(t)|z_1 - z_2|$$

for all $x_i,\ y_i,\ z_i \in \mathbb{R};\ i = 1, 2$ and $t \in \mathbb{R}_+$

$a_3)$ $u,\ v : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ are continuous functions such that

$$\lim_{t\to\infty} m_1(t) \int_0^t |u(t, s, x(s))|\ ds = 0,$$

$$\lim_{t\to\infty} m_2(t) \int_0^\infty |v(t, s, x(s))|\ ds = 0$$

uniformly with respect to $x \in BC(\mathbb{R}_+)$.

Now, we are in a position to state and prove our main result in the paper

Theorem 3.1 Let the assumptions $a_1) - a_3)$ be satisfied. Then Eq.(1) has at least one solution $x \in BC(\mathbb{R}_+)$ which is globally attractive.

Proof: Denote by $\mathcal{F}$ the operator associated with the right-hand side of Eq.(1), i.e., equation (1) takes the form

$$x = \mathcal{F}\ x, \tag{2}$$

where

$$(\mathcal{F}\ x)(t) = f\left(t, x(t), \int_0^t u(t, s, x(s))\ ds, \int_0^\infty v(t, s, x(s))\ ds\right),\ t \in \mathbb{R}_+ \tag{3}$$

Solving Eq.(1) is equivalent to finding a fixed point of the operator $\mathcal{F}$ defined on the space $BC(\mathbb{R}_+)$.

Clearly, in virtue of our assumptions the function $\mathcal{F}x$ is continuous on the interval $\mathbb{R}_+$ for any function $x \in BC(\mathbb{R}_+)$. Moreover, from our assumptions we have

$$\begin{aligned} |(\mathcal{F}\ x)(t)| &\leq \left| f\left(t, x(t), \int_0^t u(t, s, x(s))\ ds, \int_0^\infty v(t, s, x(s))\ ds\right) - f(t, 0, 0, 0))\right| \\ &\quad + |f(t, 0, 0, 0)| \\ &\leq k\ |x(t)| + m_1(t) \int_0^t |u(t, s, x(s))|\ ds + m_2(t) \int_0^\infty |v(t, s, x(s))|\ ds \\ &\quad + |f(t, 0, 0, 0)| \end{aligned}$$

Hence,

$$\|\mathcal{F}x\| \leq k\ \|x\| + A, \tag{4}$$

where

$$A = \sup\left\{ m_1(t) \int_0^t |u(t,s,x(s))|\, ds + \right.$$
$$\left. m_2(t) \int_0^\infty |v(t,s,x(s))|\, ds + |f(t,0,0,0)| : t \in \mathbb{R}_+ \right\} \in \mathbb{R}_+$$

Since $k < 1$, this implies that $\mathcal{F}(B_r) \subset B_r$ for $r = \frac{A}{1-k}$.

We **claim** that the operator $\mathcal{F}$ is continuous on B_r. To establish this claim, let us fix $\varepsilon > 0$ and take arbitrary $x,\ y \in B_r$ such that $\|x - y\| \le \varepsilon$. Then, for $t \in \mathbb{R}_+$, we have

$$\begin{aligned}
|(\mathcal{F}\, x)(t) - (\mathcal{F}\, y)(t)| &\le \left| f\left(t, x(t), \int_0^t u(t,s,x(s))\, ds, \int_0^\infty v(t,s,x(s))\, ds\right) \right. \\
&\qquad \left. - f\left(t, y(t), \int_0^t u(t,s,y(s))\, ds, \int_0^\infty v(t,s,y(s))\, ds\right) \right| \\
&\le k\, |x(t) - y(t)| + m_1(t) \int_0^t |u(t,s,x(s)) - u(t,s,y(s))|\, ds \\
&\quad + m_2(t) \int_0^\infty |v(t,s,x(s)) - v(t,s,y(s))|\, ds
\end{aligned} \tag{5}$$

By assumptions $a1) - a3)$ we choose $T > 0$ such that for $t \ge T$ the following inequalities hold

$$m_1(t) \int_0^t |u(t,s,x(s))|\, ds \le \frac{\varepsilon}{4}(1-k) \tag{6}$$

and

$$m_2(t) \int_0^\infty |v(t,s,x(s))|\, ds \le \frac{\varepsilon}{4}(1-k). \tag{7}$$

Thus for $t \ge T$, in view of (5)-(7) we obtain

$$\begin{aligned}
|(\mathcal{F}\, x)(t) - (\mathcal{F}\, y)(t)| &\le k\,\varepsilon + \frac{\varepsilon}{4}(1-k) + \frac{\varepsilon}{4}(1-k) + \frac{\varepsilon}{4}(1-k) + \frac{\varepsilon}{4}(1-k) \\
&= \varepsilon
\end{aligned}$$

On the other hand, taking into account the uniform continuity of the functions $u = u(t,s,x)$ and $v = v(t,s,x)$ on the set $[0,T] \times [0,T] \times [-r,r]$, we deduce that $\omega_1(\varepsilon),\ \omega_2(\varepsilon) \to 0$ as $\varepsilon \to 0$, where

$$\omega_1(\varepsilon) = \sup\left\{ |u(t,s,x) - u(t,s,y)| : t,\ s \in [0,\mathrm{T}],\ x, y \in [-r,r],\ |x-y| \le \varepsilon \right\}$$

and

$$\omega_2(\varepsilon) = \sup\left\{ |u(v,s,x) - v(t,s,y)| : t,\ s \in [0,\mathrm{T}],\ x, y \in [-r,r],\ |x-y| \le \varepsilon \right\}.$$

Thus,

$$\begin{aligned}
|(\mathcal{F}\, x)(t) - (\mathcal{F}\, y)(t)| &\le k\,\varepsilon + T\,\omega_1(\varepsilon)\ \sup\left\{ m_1(t) : t \in [0,T] \right\} \\
&\quad + \omega_2(\varepsilon)\ \sup\left\{ m_2(t) : t \in [0,T] \right\}
\end{aligned}$$

Hence, we established our claim.

Now, let X be a nonempty set of B_r. Then, for $x,\ y \in X$ and $t \in \mathbb{R}_+$, we get

$$\begin{aligned}
|(\mathcal{F}\, x)(t) - (\mathcal{F}\, y)(t)| &\leq k\,|x(t) - y(t)| + m_1(t)\int_0^t |u(t,s,x(s)) - u(t,s,y(s))|\, ds \\
&\quad + m_2(t)\int_0^\infty |v(t,s,x(s)) - v(t,s,y(s))|\, ds \\
&\leq k\,|x(t) - y(t)| + m_1(t)\int_0^t |u(t,s,x(s))|\, ds \\
&\quad + m_1(t)\int_0^t |u(t,s,y(s))|\, ds + m_2(t)\int_0^\infty |v(t,s,x(s))|\, ds \\
&\quad + m_2(t)\int_0^\infty |v(t,s,y(s))|\, ds
\end{aligned}$$

Hence

$$\begin{aligned}
\operatorname{diam}(\mathcal{F}\, X)(t) &\leq k\,\operatorname{diam} X(t) \\
&\quad + \sup_{x,\, y \in \mathrm{X}} \Big\{ m_1(t)\int_0^t |u(t,s,x(s))|\, ds + m_1(t)\int_0^t |u(t,s,y(s))|\, ds \\
&\qquad + m_2(t)\int_0^\infty |v(t,s,x(s))|\, ds + m_2(t)\int_0^\infty |v(t,s,y(s))|\, ds \Big\}
\end{aligned}$$

Therefore,

$$\limsup_{t\to\infty}\ \operatorname{diam}(\mathcal{F}\, X)(t) \leq k\ \limsup_{t\to\infty}\ \operatorname{diam} X(t), \tag{8}$$

thanks to assumption a_3).

For any $\mathrm{T} > 0$ and $\varepsilon > 0$. Choose a function $x \in X$ and $t,\ s \in [0, \mathrm{T}]$ such that $|t - s| \leq \varepsilon$, without loss of generality we may assume that $t > s$. Then, in view of our assumptions we have

$$\begin{aligned}
|(\mathcal{F}\, x)(t) - (\mathcal{F}\, x)(s)| &\leq k\,|x(t) - x(s)| \\
&\quad + m_1(t)\left|\int_0^t u(t,\tau,x(\tau))\, d\tau - \int_0^s u(s,\tau,x(\tau))\, d\tau\right| \\
&\quad + m_2(t)\left|\int_0^\infty v(t,\tau,x(\tau))\, d\tau - \int_0^\infty v(s,\tau,x(\tau))\, ds\right| \\
&\quad + \left| f\left(t, x(s), \int_0^s u(s,\tau,x(\tau))\, d\tau, \int_0^\infty v(s,\tau,x(\tau))\, d\tau\right)\right. \\
&\quad \left. - f\left(s, x(s), \int_0^s u(s,\tau,x(\tau))\, d\tau, \int_0^\infty v(s,\tau,x(\tau))\, d\tau\right)\right| \\
&\leq k\,|x(t) - x(s)| \\
&\quad + m_1(t)\left|\int_s^t u(t,\tau,x(\tau))\, d\tau\right| \\
&\quad + m_1(t)\left|\int_0^s u(t,\tau,x(\tau))\, d\tau - \int_0^s u(s,\tau,x(\tau))\, d\tau\right| \\
&\quad + m_2(t)\left|\int_0^\infty v(t,\tau,x(\tau))\, d\tau - \int_0^\infty v(s,\tau,x(\tau))\, ds\right|
\end{aligned}$$

$$
\begin{aligned}
&+\left|f\left(t, x(s), \int_0^s u(s,\tau,x(\tau))\,d\tau, \int_0^\infty v(s,\tau,x(\tau))\,d\tau\right)\right. \\
&\left.-f\left(s, x(s), \int_0^s u(s,\tau,x(\tau))\,d\tau, \int_0^\infty v(s,\tau,x(\tau))\,d\tau\right)\right|
\end{aligned}
$$

Hence

$$
\begin{aligned}
\omega^T(\mathcal{F}x,\varepsilon) \;\le\; & k\,\omega^T(x,\varepsilon) \\
& +m_1(t)\,\varepsilon\;\sup\{|u(t,s,x(s))| : t,\ s\in[0,T],\ \|x\|\le r\} \\
& +m_1(t)\,T\;\sup\{|u(t,\tau,x(\tau))-u(s,\tau,x(\tau))| : t,\ s,\ \tau\in[0,T],\ \|x\|\le r\} \\
& +m_2(t)\sup\{|v(t,\tau,x(\tau))-v(s,\tau,x(\tau))| : t,\ s,\ \tau\in[0,T],\ \|x\|\le r\} \\
& +\sup\left\{\left|f\left(t, x(s), \int_0^s u(s,\tau,x(\tau))\,d\tau, \int_0^\infty v(s,\tau,x(\tau))\,d\tau\right)\right.\right. \\
& \left.\left.-f\left(s, x(s), \int_0^s u(s,\tau,x(\tau))\,d\tau, \int_0^\infty v(s,\tau,x(\tau))\,d\tau\right)\right| : t,\ s\in[0,T],\ \|x\|\le r\right\}
\end{aligned}
$$

Since $f(t,x,y,z)$ is uniformly continuous on the set $[0,T]\times[-r,r]\times[-N,N]\times[-M,M]$ and the functions $u(t,s,x)$ and $v(t,s,x)$ are uniformly continuous on the set $[0,T]\times[0,T]\times[-r,r]$, where

$$
N=\sup\left\{\int_0^s u(s,\tau,x(\tau))\,d\tau : s\in[0,T],\ \|x\|\le r\right\}
$$

and

$$
M=\sup\left\{\int_0^\infty v(s,\tau,x(\tau))\,d\tau : s\in[0,T],\ \|x\|\le r\right\},
$$

we have

$$
\begin{aligned}
\sup\{|f(t,x,y,z)-f(s,x,y,z)| : t,\ s\in[0,T],\ |s-t|\le\varepsilon,\ \|x\|\le r,;\|y\|\le N,\ \|z\|\le M\} \\
\to 0 \text{ as } \varepsilon\to 0,
\end{aligned}
$$

$$
\sup\{|u(t,\tau,x)-u(s,\tau,x)| : t,\ s,\ \tau\in[0,T],\ |s-t|\le\varepsilon,\ \|x\|\le r\}\to 0 \text{ as } \varepsilon\to 0
$$

and

$$
\sup\{|v(t,\tau,x)-v(s,\tau,x)| : t,\ s,\ \tau\in[0,T],\ |s-t|\le\varepsilon,\ \|x\|\le r\}\to 0 \text{ as } \varepsilon\to 0.
$$

Hence

$$
\omega_0(\mathcal{F}X)\le k\,\omega_0(X). \tag{9}
$$

From (8) and (9), keeping in mind the definition of the measure of noncompactness in the space $BC(\mathbb{R}_+)$, we obtain

$$
\mu(\mathcal{F}X)\le k\,\mu(X). \tag{10}
$$

The above obtained inequality together with the fact that $k<1$ enable us to apply Theorem 2.1. Hence, we infer that Eq.(1) has at least one solution $x(t)$.

Now, for any other solution $y(t)$ of Eq.(1), we have

$$\begin{aligned}|x(t)-y(t)| &= |(\mathcal{F}x)(t)-(\mathcal{F}y)(t)| \\ &\leq k\,|x(t)-y(t)| + m_1(t)\int_0^t |u(t,s,x(s))|\,ds \\ &\quad + m_2(t)\int_0^t |u(t,s,y(s))|\,ds + m_2(t)\int_0^\infty |v(t,s,x(s))|\,ds \\ &\quad + m_2(t)\int_0^\infty |v(t,s,y(s))|\,ds.\end{aligned}$$

Thus

$$\begin{aligned}\lim_{t\to\infty}|x(t)-y(t)| &\leq \frac{1}{1-k}\left[\lim_{t\to\infty} m_1(t)\left(\int_0^t |u(t,s,x(s))|\,ds + \int_0^t |u(t,s,y(s))|\,ds\right)\right. \\ &\quad \left. + \lim_{t\to\infty} m_2(t)\left(\int_0^\infty |v(t,s,x(s))|\,ds + \int_0^t |v(t,s,y(s))|\,ds\right)\right] \\ &= 0.\end{aligned}$$

This complete the proof.

4 Examples

Example 4.1 If $f(t,x,y,z)=g(t)+y$, then Eq.(1) is the well-known Urysohn-Volterra integral equation

$$x(t)=g(t)+\int_0^t u(t,s,x(s))\,ds.$$

On the other hand, for $f(t,x,y,z)=h(t)+z$, Eq.(1) reduces to the well-known Urysohn integral equation

$$x(t)=h(t)+\int_0^\infty v(t,s,x(s))\,ds.$$

Example 4.2 If $f(t,x,y,z)=g(t,x,y)$, then Eq.(1) becomes a functional-integral equation

$$x(t)=g\left(t,x(t),\int_0^t u(t,s,x(s))\,ds\right). \tag{11}$$

In [12], the authors proved the existence of solutions to Eq.(11). These solutions continuous and bounded on the interval $[0,\infty)$ and are globally attractive.

Example 4.3 In the case $f(t,x,y,z)=1+x\,z$ and $v(t,s,x)=\frac{t}{t+s}\,\phi(s)\,x$, Eq.(1) has the form

$$x(t)=1+x(t)\int_0^\infty \frac{t}{t+s}\,\phi(s)\,x(s)\,ds. \tag{12}$$

Eq.(12) creates an unbounded version of the famous quadratic integral equation of Chandrasekhar type

$$x(t)=1+x(t)\int_0^1 \frac{t}{t+s}\,\phi(s)\,x(s)\,ds. \tag{13}$$

Eq.(13) considered in many papers and monographs (cf. [1, 3, 8, 13] for instance). Some Problems considered in the theory of radiative transfer, in the theory of neutron transport and in the kinetic theory of gases lead to Eq.(13) (cf. [2, 3, 6, 7, 8, 9, 10, 13, 14]).

Remark 4.1 In order to apply our technique to Eq.(12) we have to impose an additional condition that the characteristic function ϕ is continuous and satisfies $\phi(0) = 0$. This condition will ensure that the kernel $v(t, s, x)$ defined by

$$v(t,s,x) = \begin{cases} 0, & s = 0,\ t \geq 0,\ x \in \mathbb{R} \\ \frac{t}{t+s}\phi(s)\ x, & s \neq 0,\ t \geq 0,\ x \in \mathbb{R} \end{cases}$$

is continuous on $\mathrm{I} \times \mathrm{I} \times \mathbb{R}$ in accordance with assumption a_3), see [6].

Acknowledgement. The author would like to thank the referee for his corrections and valuable remarks. This work was completed when the author was visiting the ICTP, Trieste, Italy as a Regular Associate. It is a pleasure for him to express gratitude for its financial support and the warm hospitality.

References

[1] I.K. Argyros, *Quadratic equations and applications to Chandrasekhar's and related equations*, Bull. Austral. Math. Soc. **32** (1985), 275-292.

[2] J. Banaś and B. Rzepka, *On existence and asymptotic stability of a nonlinear integral equation*, J. Math. Anal. Appl. **284** (2003), 165-173.

[3] J. Banaś, M. Lecko and W.G. El-Sayed, *Existence theorems of some quadratic integral equations*, J. Math. Anal. Appl. **222** (1998), 276-285.

[4] J. Banaś, *Measures of noncompactness in the space of continuous tempered functions*, Demonstratio Math. **14** (1981), 127-133.

[5] J. Banaś and K. Goebel, *Measures of noncompactness in Banach spaces*, Lecture Notes in Pure and Applied Mathematics **60**, Marcel Dekker, New York, 1980.

[6] J. Caballero, A.B. Mingarelli and K. Sadarangani, *Existence of solutions of an integral equation of Chandrasekhar type in the theory of radiative transfer*, Electronic Journal of Differential Equations **2006 (57)** (2006), 1-11.

[7] K.M. Case and P.F. Zweifel, *Linear Transport Theory*, Addison-Wesley, Reading, MA 1967.

[8] S. Chandrasekher, *Radiative Transfer*, Dover Publications, New York, 1960.

[9] M.A. Darwish, *On quadratic integral equation of fractional orders*, J. Math. Anal. Appl. **311** (2005) 112-119.

[10] K. Deimling, *Nonlinear fuctional analysis*, Springer-Verlag, Berlin, 1985.

[11] J. Dugundji and A. Granas, *Fixed Point Theory*, Monografie Mathematyczne, PWN, Warsaw, 1982.

[12] X. Hu, J. Yan, *The global attractivity and asympototic stability of solution of a nonlinear integral equation*, J. Math. Anal. Appl. **321** (2006) 147-156.

[13] S. Hu, M. Khavani and W. Zhuang, *Integral equations arrising in the kinetic theory of gases*, Appl. Analysis, **34** (1989), 261-266.

[14] C.T. Kelly, *Approximation of solutions of some quadratic integral equations in transport theory*, J. Integral Eq. **4** (1982), 221-237.

(Received July 23, 2007)