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# Corrigendum to "On the stochastic Allen–Cahn equation on networks with multiplicative noise" [*Electron. J. Qual. Theory Differ. Equ.* 2021, No. 7, 1–24]

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**Abstract.** We reprove Proposition 3.8 in our paper that was published in [*Electron. J. Qual. Theory Differ. Equ.* **2021**, No. 7, 1–24], to fill a gap in the proof of Corollary 3.7 where the density of one of the embeddings does not follow by the original arguments. We further carry out some minor corrections in the proof of Corollary 3.7, in Remark 3.1 and in the formula (3.23) of the original paper.

**Keywords:** stochastic evolution equations, stochastic reaction-diffusion equations on networks, analytic semigroups, stochastic Allen–Cahn equation.

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## 1 Introduction

The proof of [4, Corollary 3.7] is incomplete as only the continuity of the embedding $E_p^\theta \hookrightarrow B$ is verified in the proof but the density of the embedding is not (for the explanation of notation we refer to [4]). Therefore, in the present note we first provide a new poof for [4, Proposition 3.8], which is Proposition 2.1 below, that is independent of [4, Corollary 3.7]. After that we may use Proposition 2.1 to fill in the gap in the proof of [4, Corollary 3.7], which is Corollary 3.1 below.

Furthermore, at the end of the note, we also correct some minor inaccuracies that use the injectivity of the system operator $A_p$ which is not true in general.

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## 2 New proof of Proposition 3.8

In contrast to the original paper [4] we now reprove [4, Proposition 3.8] but without using [4, Corollary 3.7].

**Proposition 2.1.** *For $p \in (1, \infty)$ the part of $(A_p, D(A_p))$ in $B$ generates a positive strongly continuous semigroup of contractions on $B$.*

*Proof.* 1. We first prove that the semigroup $(T_p(t))_{t\geq 0}$ leaves $B$ invariant. We take $u \in B \subset E_p$ and use that $(T_p(t))_{t\geq 0}$ is analytic on $E_p$ (see [4, Proposition 2.8]) to conclude that $T_p(t)u \in D(A_p)$. The explicit form [4, (2.11)] of $D(A_p)$ shows that $D(A_p) \subset B$ and hence also

$$T_p(t)u \in B$$

holds.

2. In the next step we prove that $(T_p(t)|_B)_{t\geq 0}$ is a strongly continuous semigroup. By [2, Proposition I.5.3], it is enough to prove that there exist $K > 0$ and $\delta > 0$ and a dense subspace $D \subset B$ such that

(a) $\|T_p(t)\|_{\mathcal{L}(B)} \leq K$ for all $t \in [0, \delta]$, where $\mathcal{L}(B)$ denotes the space of bounded linear operators on $B$ equipped with the operator norm, and

(b) $\lim_{t\downarrow 0} T_p(t)u = u$ for all $u \in D$.

To verify (a), we obtain by [4, Proposition 2.8] that for $u \in B$,

$$\|T_p(t)u\|_B = \|T_p(t)u\|_{E_\infty} = \|T_\infty(t)u\|_{E_\infty} \leq \|u\|_{E_\infty} = \|u\|_B,$$

hence

$$\|T_p(t)\|_{\mathcal{L}(B)} \leq 1 =: K, \quad t \geq 0,$$

showing also that $(T_p(t)|_B)_{t\geq 0}$ is a semigroup of contractions. To prove (b) we first set $p = 2$. Taking $\omega > 0$ arbitrary, we obtain that the form

$$\mathfrak{a}_\omega(u, v) := \mathfrak{a}(u, v) + \omega \cdot \langle u, v\rangle_{E_2}, \quad u, v \in V,$$

is coercive, symmetric and continuous, see [3, Remark 7.3.3] and [4, Proposition 2.4]. For the form-domain $V$ defined in [4, (2.9)], equipped with the usual $(H^1(0,1))^m$-norm, we have that

$$V = D((\omega - A_2)^{1/2})$$

holds with equivalence of norms (see e.g. [1, Proposition 5.5.1]). We also have

$$V = D((\omega - A_2)^{1/2}) = D((-A_2)^{1/2}) \tag{2.1}$$

with equivalent norms, where we used [3, Proposition 3.1.7] for the second equality and norm equivalence. Notice that the subspace $(C^\infty[0,1])^m \cap B$ (the infinitely many times differentiable functions on the edges that are continuous across the vertices) is contained in $V$ and is dense in $B$ by the Stone–Weierstrass theorem. Hence, $V$ and thus $D((-A_2)^{1/2})$ is dense in $B$. Defining $D := D((-A_2)^{1/2})$, for $u \in D$ there exist $C_1, C_2 > 0$ such that

$$\begin{aligned}\|T_2(t)u - u\|_B &\leq C_1 \cdot \|T_2(t)u - u\|_{(H^1(0,1))^m} \\ &\leq C_2 \cdot \left(\|T_2(t)(-A_2)^{1/2}u - (-A_2)^{1/2}u\|_{E_2} + \|T_2(t)u - u\|_{E_2}\right) \to 0, \quad t \downarrow 0.\end{aligned}$$

In the first inequality we have used Sobolev embedding and in the second one the norm equivalence in (2.1) and the fact that $T_2(t)$ and $(-A_2)^{1/2}$ commute on $D((-A_2)^{1/2})$. Summarizing 1. and 2., and using that clearly $B$ is continuously embedded in $E_p$, we can apply [2, Proposition in Section II.2.3] for $(A_2, D(A_2))$ and $Y = B$, and obtain that the part of $(A_2, D(A_2))$ in $B$ generates a positive strongly continuous semigroup of contractions on $B$. Since the semigroups in [4, Proposition 2.8] are consistent, the same is true for $(T_p(t))_{t\geq 0}$ for any $p \in (1, \infty)$. □

## 3 Further necessary changes

We may now present the complete proof of [4, Corollary 3.7] including the density of the embeddings in the statement using the above, independently proven, Proposition 2.1. We also made some minor changes in the proof so that the injectivity of the system operator $A_p$ is not used.

**Corollary 3.1.** *Let $E_p^\theta$ defined in [4, (3.5)]. If $\theta > \frac{1}{2p}$ then the following continuous, dense embeddings are satisfied:*

$$E_p^\theta \hookrightarrow B \hookrightarrow E_p.$$

*Proof.* By [4, Proposition 2.8] the operator $(A_p, D(A_p))$ generates a positive, contraction semigroup on $E_p$. It follows from [1, Theorem in §4.7.3] and [1, Proposition in §4.4.10] that for the complex interpolation spaces

$$D((\omega' - A_p)^\theta) \cong [D(\omega' - A_p), E_p]_\theta$$

holds for any $\omega' > 0$. Therefore,

$$E_p^\theta = D((\omega' - A_p)^\theta) \cong [D(\omega' - A_p), E_p]_\theta \cong [D(A_p), E_p]_\theta.$$

By defining the operator $(A_{p,\max}, D(A_{p,\max}))$ as in [4, (3.6),(3.7)] it follows that

$$D(A_p) \hookrightarrow D(A_{p,\max})$$

holds. Hence

$$E_p^\theta \hookrightarrow \left[D(A_{p,\max}), E_p\right]_\theta. \tag{3.1}$$

By [4, Lemma 3.5],

$$D(A_{p,\max}) \cong W_0(G) \times \mathbb{R}^n \tag{3.2}$$

holds, where $W_0(G)$ is defined in [4, (3.8)]. Since $E_p \cong E_p \times \{0_{\mathbb{R}^n}\}$, using general interpolation theory, see e.g. [5, Section 4.3.3], we have that for $\theta > \frac{1}{2p}$,

$$\left[W_0(G) \times \mathbb{R}^n, E_p \times \{0_{\mathbb{R}^n}\}\right]_\theta \hookrightarrow \left(\prod_{j=1}^m W_0^{2\theta,p}(0,1;\mu_j dx)\right) \times \mathbb{R}^n.$$

Thus, by (3.1) and (3.2),

$$E_p^\theta \hookrightarrow \left(\prod_{j=1}^m W_0^{2\theta,p}(0,1;\mu_j dx)\right) \times \mathbb{R}^n$$

holds. Hence,

$$E_p^\theta \hookrightarrow (C_0[0,1])^m \times \mathbb{R}^n$$

is true by Sobolev's embedding. Applying [4, Lemma 3.6] we obtain that for $\theta > \frac{1}{2p}$,

$$E_p^\theta \hookrightarrow B$$

is satisfied. The continuity of the embedding $B \hookrightarrow E_p$ is clear. It follows from Proposition 2.1 that $D(A_p)$ is a dense subspace of $B$ and then so is $E_p^\theta$ for $\theta > \frac{1}{2p}$. Since $B \cong (C_0[0,1])^m \times \mathbb{R}^n$ by [4, Lemma 3.6] and $E_p \cong E_p \times \{0_{\mathbb{R}^n}\}$, the space $B$ is also dense in $E_p$ and the claim follows. □

Next, we include some minor changes as follows:

- In [4, Remark 3.1] one has to omit "If $A$ is injective". The statement remains true without this assumption, see [3, Proposition 3.1.7], when $D((-A)^\alpha)$ is equipped with the graph norm.
- Formula (3.23) on page 16, the definition of the operator $G$ has to be modified as

  $$(\nu - A_p)^{-\kappa_G} G(t,u)h := \jmath\,\imath\,(\nu - A_2)^{-\kappa_G}\Gamma(t,u)h, \quad u \in B,\ h \in H,$$

  for $\nu > 0$ arbitrary.

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