UNIFORM BOUNDEDNESS AND GLOBAL EXISTENCE OF SOLUTIONS FOR REACTION-DIFFUSION SYSTEMS WITH A BALANCE LAW AND A FULL MATRIX OF DIFFUSION COEFFICIENTS.

SAID KOUACHI

ABSTRACT. The purpose of this paper is to prove uniform boundedness and so global existence of solutions for reaction-diffusion systems with a full matrix of diffusion coefficients satisfying a balance law. Our techniques are based on invariant regions and Lyapunov functional methods. The nonlinearity of the reaction term which we take positive in an invariant region has been supposed to be polynomial..

1. INTRODUCTION

We consider the following reaction-diffusion system

$$\frac{\partial u}{\partial t}\text{-}a\Delta u\text{-}b\Delta v = \text{-}\sigma f(u,v) \qquad in\ \mathbb{R}^+ \times \Omega \tag{1.1}$$

$$\frac{\partial v}{\partial t}\text{-}c\Delta u\text{-}a\Delta v = \rho f(u,v) \qquad in\ \mathbb{R}^+ \times \Omega, \tag{1.2}$$

with the boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \quad on\ \mathbb{R}^+ \times \partial\Omega, \tag{1.3}$$

and the initial data

$$u(0,x) = u_0(x), \qquad v(0,x) = v_0(x) \qquad in\ \Omega, \tag{1.4}$$

where Ω is an open bounded domain of class $\mathbb{C}^1$ in $\mathbb{R}^n$, with boundary $\partial\Omega$, and $\dfrac{\partial}{\partial\eta}$ denotes the outward normal derivative on $\partial\Omega$, σ, ρ, a, b and c are positive constants satisfying the condition $2a > (b+c)$ which reflects the parabolicity of the system. The initial data are assumed to be in the following region

$$\Sigma = \begin{cases} \left\{(u_0,v_0) \in IR^2 \text{ such that } v_0 \geq \sqrt{\frac{c}{b}}\,|u_0|\right\} \text{ when } \sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho}, \\ \\ \left\{(u_0,v_0) \in IR^2 \text{ such that } u_0 \geq \sqrt{\frac{b}{c}}\,|v_0|\right\} \text{ when } \sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}. \end{cases}$$

1991 *Mathematics Subject Classification.* 35K57, 35K45.

Key words and phrases. Reaction diffusion systems, Invariant regions, Lyapunov functionals, Global existence.

The function $f(r,s)$ is continuously differentiable, nonnegative on Σ with

$$(1.5)\qquad \begin{cases} f(-\sqrt{\frac{b}{c}}s,s)=0, \text{ for all } s\geq 0, \quad \text{when } \sqrt{\frac{b}{c}}<\dfrac{\sigma}{\rho} \\ \text{and} \\ f(r,\sqrt{\frac{c}{b}}r)=0, \text{ for all } r\geq 0, \quad \text{when } \sqrt{\frac{b}{c}}>\dfrac{\sigma}{\rho}, \end{cases}$$

and

$$(1.6)\qquad \begin{cases} \lim\limits_{s\to+\infty}\left[\frac{log(1+f(r,s))}{s}\right]=0, \text{ for any } r\geq 0, \text{when } \sqrt{\frac{b}{c}}<\dfrac{\sigma}{\rho}, \\ \text{and} \\ \lim\limits_{r\to+\infty}\left[\frac{log(1+f(r,s))}{r}\right]=0, \text{ for all } s\geq 0, \text{when } \sqrt{\frac{b}{c}}>\dfrac{\sigma}{\rho}. \end{cases}$$

The system (1.1)-(1.2) may be regarded as a perturbation of the simple and trivial case where $b=c=0$; for which nonnegative solutions exist globally in time.

When the coefficient of $-\Delta u$ in equation (1.1) is different from the one of $-\Delta v$ in equation (1.2), N. Alikakos [1] established global existence and L^∞-bounds of solutions for positive initial data for $f(u,v)=uv^\beta$ and $1<\beta<\frac{(n+2)}{n}$ and K. Masuda [16] showed that solutions to this system exist globally for every $\beta>1$ and converge to a constant vector as $t\to+\infty$. A. Haraux and A. Youkana [6] have generalized the method of K.Masuda to handle nonlinearities $uF(v)$ that are form a particular case of ours; since the hypothesis (1.5) is replaced automatically by $f(0,s)=0$ for any $s\geq 0$. Recently S. Kouachi and A. Youkana [14] have generalized the method of A. Haraux and A. Youkana to the case $c>0$ and the limit (1.6) is a small number strictly positive , hypothesis that is in fact, weaker than the last one.

The components $u(t,x)$ and $v(t,x)$ represent either chemical concentrations or biological population densities and system (1.1)-(1.2) is a mathematical model describing various chemical and biological phenomena (see E. L. Cussler [2], P. L. Garcia-Ybarra and P. Clavin [4], S. R. De Groot and P. Mazur [5], J. Jorne [9], J. S. Kirkaldy [13], A. I. Lee and J. M. Hill [15] and J. Savchik, b. Changs and H. Rabitz[18].

It is well known that, to establish a global existence of unique solutions for (1.1)-(1.3), usual techniques based on Lyapunov functionals wich need invariant regions(see M. Kirane and S. Kouachi [11], [12] and S. Kouachi and A. Youkana [14]) are not directly applicable. For this purpose we construct invariant regions.

2. **EXISTENCE.**

2.1. **Local existence.** The usual norms in spaces $\mathbb{L}^p(\Omega)$, $\mathbb{L}^\infty(\Omega)$ and $\mathbb{C}\left(\overline{\Omega}\right)$ are respectively denoted by :

$$\|u\|_p^p = \frac{1}{|\Omega|} \int_{\Omega} |u(x)|^p \, dx,$$

$$\|u\|_\infty = \max_{x\in\Omega} |u(x)| .$$

For any initial data in $\mathbb{C}\left(\overline{\Omega}\right)$ or $\mathbb{L}^p(\Omega)$, $p \in (1, +\infty)$; local existence and uniqueness of solutions to the initial value problem (1)-(4) follow from the basic existence theory for abstract semilinear differential equations (see A. Friedman [3], D. Henry [7] and Pazy [17]). The solutions are classical on $]0, T^*[$, where T^* denotes the eventual blowing-up time in $\mathbb{L}^\infty(\Omega)$.

2.2. **Invariant regions.**

Proposition 1. *Suppose that the function f is nonnegative on the region Σ and that the conditions (1.5) and (1.6) are satisfied, then for any (u_0, v_0) in Σ the solution $(u(t, .), v(t, .))$ of the problem (1.1)-(1.4) remains in Σ for any time and there exists a positive constant M such that*

$$\left\{ \begin{array}{c} \left\| \sqrt{c}u(t,.) + \sqrt{b}v(t,.) \right\|_\infty \leq M, \textit{ when } \sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho} \\ \textit{and} \\ \left\| \sqrt{c}u(t,.) - \sqrt{b}v(t,.) \right\|_\infty \leq M, \textit{ when } \sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}. \end{array} \right. \tag{2.1}$$

Proof. One starts with the case where $\sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho}$:

Multiplying equation (1.1) through by $\sqrt{c}$ and equation (1.2) by $\sqrt{b}$, subtracting the resulting equations one time and adding them an other time we have

$$\frac{\partial w}{\partial t} - (a + \sqrt{bc})\Delta w = (\rho\sqrt{b} - \sigma\sqrt{c})F(w, z) \qquad in \ ]0, T^*[\times \Omega \tag{2.2}$$

$$\frac{\partial z}{\partial t} - (a - \sqrt{bc})\Delta z = (\rho\sqrt{b} + \sigma\sqrt{c})F(w, z) \qquad in \ ]0, T^*[\times \Omega, \tag{2.3}$$

with the boundary conditions

$$\frac{\partial w}{\partial \eta} = \frac{\partial z}{\partial \eta} = 0 \qquad \text{on }]0, T^*[\times \partial\Omega \ , \tag{2.4}$$

and the initial data

$$w(0, x) = w_0(x), \qquad z(0, x) = z_0(x) \qquad in \ \Omega, \tag{2.5}$$

where,

$$w(t, x) = \sqrt{c}u(t, x) + \sqrt{b}v(t, x) \text{ and } z(t, x) = -\sqrt{c}u(t, x) + \sqrt{b}v(t, x), \tag{2.6}$$

for any (t, x) *in* $]0, T^*[\times \Omega$, and

$$F(w, z) = f(u, v).$$

First, let's notice that the condition of parabolicity of the system (1.1)-(1.2) implies the one of the (2.2)-(2.3) system; since $2a > (b+c) \Rightarrow a - \sqrt{bc} > 0$.
Now, it suffices to prove that the region

$$\left\{(w_0, z_0) \in IR^2 \text{ such that } w_0 \geq 0, z_0 \geq 0\right\} = IR^+ \times IR^+,$$

is invariant for system (2.2)-(2.3) and $w(t,x)$ is uniformly bounded in $]0, T^*[\times\Omega$. Since, from (1.5), $F(0,z) = f(-\sqrt{\frac{b}{c}}v, v) = 0$ for all $z \geq 0$ and all $v \geq 0$, then $w(t,x) \geq 0$ for all $(t,x) \in]0,T^*[\times\Omega$, thanks to the invariant region's method (see Smoller [19]) and because $F(w,z) \geq 0$ for all (w,z) in $IR^+ \times IR^+$ and $z_0(x) \geq 0$ in Ω, we can deduce by the same method applied to equation (2.3), that

$$z(t,x) = \sqrt{c}u(t,x) - \sqrt{b}v(t,x) \geq 0 \text{ in }]0,T^*[\times\Omega;$$

then Σ is an invariant region for the system (1.1)-(1.3).
At the end, to show that $w(t,x)$ is uniformly bounded on $]0,T^*[\times\Omega$, it is sufficient to apply the maximum's principle directly to equation (2.2).

For the case $\sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}$, the same reasoning with equations

$$(2.2)' \qquad \frac{\partial w}{\partial t} - (a - \sqrt{bc})\Delta w = -(\rho\sqrt{b} + \sigma\sqrt{c})F(w,z) \qquad in\ ]0,T^*[\times\Omega,$$

$$(2.3)' \qquad \frac{\partial z}{\partial t} - (a + \sqrt{bc})\Delta z = (\rho\sqrt{b} - \sigma\sqrt{c})F(w,z) \qquad in\ ]0,T^*[\times\Omega$$

with the same boundary condition (2.4) implies the invariance of $IR^+ \times IR^+$ and the uniform boundeness of $w(t,x)$ on $]0,T^*[\times\Omega$, where in this case we take

$$(2.6)' \qquad w(t,x) = \sqrt{c}u(t,x) - \sqrt{b}v(t,x) \text{ and } z(t,x) = \sqrt{c}u(t,x) + \sqrt{b}v(t,x),$$

for all (t,x) in $]0,T^*[\times\Omega$. ∎

Once, invariant regions are constructed, one can apply Lyapunov technique and establish global existence of unique solutions for (1.1)-(1.4).

2.3. **Global existence.** As the determinant of the linear algebraic system (2.6) or $(2.6)'$, with regard to variables u and v, is different from zero, then to prove global existence of solutions of problem (1.1)-(1.4) comes back in even to prove it for problem (2.2)-(2.5). To this subject, it is well known that (see Henry [7]) it suffices to derive an uniform estimate of $\|F(w,z)\|_p$ on $[0,T^*[$ for some $p > n/2$.

The main result and in some sense the heart of the paper is:

Theorem 2. *Let $(w(t,.), z(t,.))$ be any solution of system (2.2)-(2.3) (respectively $(2.2)' - (2.3)'$) with initial data in $IR^+ \times IR^+$ and boundary conditions (2.4), then the functional*

$$(2.7) \qquad t \longrightarrow L(t) = \int_\Omega (M\text{-}w(t,x))^{-\gamma} \exp \beta z(t,x) dx$$

is nonincreasing on $[0,T^*[$, for all positive constants β and γ such that

$$\beta M \frac{\mu}{\lambda} < \gamma < \frac{a^2 - bc}{bc}, \tag{2.8}$$

and all M satisfying

$$\|w_0\|_\infty < M, \tag{2.9}$$

where $\lambda = \sigma\sqrt{c} - \rho\sqrt{b}$ and $\mu = \sigma\sqrt{c} + \rho\sqrt{b}$ (respectively $\lambda = \rho\sqrt{b} + \sigma\sqrt{c}$ and $\mu = \rho\sqrt{b} - \sigma\sqrt{c}$) and $w(t,x)$ and $z(t,x)$ are given by (2.6) (respectively (2.6)$'$).

Proof. Let's demonstrate the theorem in the case $\sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho}$. Taking $\theta = a + \sqrt{bc}$ and $\varphi = a - \sqrt{bc}$.

Differentiating L with respect to t yields:

$$\begin{aligned}
\dot{L}(t) &= \int_\Omega \left[\gamma \left((M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z}\right) \frac{\partial w}{\partial t} + \left(\beta (M\text{-}w)^{\text{-}\gamma} e^{\beta z}\right) \frac{\partial z}{\partial t}\right] dx \\
&= \int_\Omega \left(\gamma (M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z}\right)(\theta \Delta w \text{-} \lambda F(w,z))\, dx + \\
&\quad \int_\Omega \left(\beta (M\text{-}w)^{\text{-}\gamma} e^{\beta z}\right)(\varphi \Delta z + \mu F(w,z))\, dx \\
&= \int_\Omega \left[\gamma\theta (M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z} \Delta w + \beta\varphi (M\text{-}w)^{\text{-}\gamma} e^{\beta z} \Delta z\right] dx \\
&\quad + \int_\Omega \left[\mu\beta (M\text{-}w)^{\text{-}\gamma} \text{-} \lambda\gamma (M\text{-}w)^{\text{-}\gamma\text{-}1}\right] e^{\beta z} F(w,z) dx
\end{aligned}$$

$= I + J.$

By simple use of Green's formula, we get

$$I = \text{-}\int_\Omega T(\nabla w, \nabla z)(M-w)^{\text{-}\gamma\text{-}2} e^{\beta z} dx,$$

where

$$\begin{aligned}
T(\nabla w, \nabla z) &= \theta\gamma(\gamma+1)\left|\nabla w\right|^2 + \\
&\quad \beta(M-w)(\theta+\varphi)\gamma \nabla w \nabla z + \\
&\quad \varphi\beta^2 (M-w)^2 \left|\nabla z\right|^2 .
\end{aligned}$$

The discriminant of T is given by:

$$
\begin{aligned}
D &= \left[((\theta+\varphi)\gamma)^2 - 4\varphi\theta\gamma(\gamma+1)\right]\beta^2(M-w)^2 \\
&= \left((\theta-\varphi)^2\gamma^2 - 4\theta\varphi\gamma\right)\beta^2(M-w)^2.
\end{aligned}
$$

$D < 0$, if

$$\gamma > 0 \text{ and } (\theta-\varphi)^2\gamma - 4\theta\varphi < 0.$$

Theses two last inequalities can be written as follows:

$$0 < \gamma < \frac{4\theta\varphi}{(\theta-\varphi)^2}.$$

Using the following inequality

$$\xi x^2 + \sigma xy + \rho y^2 \leq -\frac{\left(\sigma^2\text{-}4\xi\rho\right)}{2}\left[\frac{y^2}{4\xi} + \frac{x^2}{4\rho}\right] \quad \text{for all } (x,y) \in \mathbb{R}^2,$$

where ξ and ρ are two negative constants and $\sigma \in \mathbb{R}$, we can show that

$$I \leq -\int_\Omega \left(\mathbf{m_1}\left|\nabla w\right|^2 + \mathbf{m}_2\left|\nabla z\right|^2\right)(M\text{-}w)^{\text{-}\gamma\text{-}2}e^{\beta z}dx,$$

where the positive constants $\mathbf{m}_1$ and $\mathbf{m}_2$ are given by:

$$
\begin{aligned}
\mathbf{m}_1 &= \frac{\left(4\theta\varphi\text{-}\gamma\left(\theta\text{-}\varphi\right)^2\right)\gamma}{8\varphi} \\
\mathbf{m}_2 &= \frac{\left(4\theta\varphi\text{-}\gamma\left(\theta\text{-}\varphi\right)^2\right)\beta^2(M-\|w_0\|_\infty)^2}{8\theta(\gamma+1)}.
\end{aligned}
$$

$$= \int_\Omega \left((M\mu\beta\text{-}\lambda\gamma)\text{-}\mu\beta w\right)(M\text{-}w)^{\text{-}\gamma\text{-}1}e^{\beta z}F(w,z)dx,$$

if we choose

$$\beta < \frac{\lambda\gamma}{M\mu},$$

then

$$J \leq -C(\beta,\gamma,\lambda,\mu,M)\int_\Omega (M-w)^{\text{-}\gamma\text{-}1}e^{\beta z}F(w,z)dx,$$

where $C(\beta,\gamma,\lambda,\mu,M)$ is a positive constant.
Hence,

$$\dot{L}(t) \leq -\int_{\Omega} \left(\mathbf{m_1} \left|\nabla w\right|^2 + \mathbf{m}_2 \left|\nabla z\right|^2\right) (M\text{-}w)^{\text{-}\gamma\text{-}2} e^{\beta z} dx$$

$$-C(\beta, \gamma, \lambda, \mu, M) \int_{\Omega} (M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z} F(w, z) dx \leq 0.$$

Concerning the case $\sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}$, we take $\theta = a - \sqrt{bc}$, $\varphi = a + \sqrt{bc}$, $\lambda = \rho\sqrt{b} + \sigma\sqrt{c}$ and $\mu = \rho\sqrt{b} - \sigma\sqrt{c}$. The same reasoning with equations $(2.2)'$ and $(2.3)'$ implies that the functional given by (2.7) is nonincreasing on $[0, T^*[$, for all positive constants β, γ and M satisfying (2.8) and (2.9).

Theorem 2.2 is completely proved. ▮

Corollary 3. *Suppose that the function $f(r, s)$ is continuously differentiable, non-negative on Σ and satisfying conditions (1.5) and (1.6) .Then all solutions of (1.1)-(1.3) with initial data in Σ are global in time and uniformly bounded on* $(0, +\infty) \times \Omega$.

Proof. Let's take $\sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho}$, as it has been mentioned in the beginning of section 1.3; it suffices to derive an uniform estimate of $\left\|F(w, z)\right\|_p$ on $[0, T^*[$ for some $p > n/2$. Since, for u and v in Σ, $w \geq 0$ and $z \geq 0$, and as $w + z = 2\sqrt{b}v$ with w uniformly bounded on $[0, T^*[\times \Omega$ by M, then (1.6) is equivalent to

$$\lim_{s\to+\infty} \left[\frac{\log\left(1 + F(r, s)\right)}{s}\right] = 0, \text{ for all } r \geq 0.$$

As F is continuous on $\mathbb{R}^+ \times \mathbb{R}^+$, then

$$\lim_{s\to+\infty} \left[\frac{\log\left(1 + F(r, s)\right)}{s}\right] = 0,$$

uniformly for $r \in [0, M]$ and we can choose positive constants α and C such that:

$$1 + F(r, s) \leq Ce^{\alpha s}, \quad \text{for all } s \geq 0 \text{ and for all } r \in [0, M], \tag{2.10}$$

and

$$\alpha < \frac{2\lambda\left(a^2 - bc\right)}{n\mu bc \left\|w_0\right\|_\infty},$$

then we can choose $p > n/2$ such that

$$p\alpha < \frac{\lambda\left(a^2 - bc\right)}{\mu bc \left\|w_0\right\|_\infty}. \tag{2.11}$$

Set $\beta = p\alpha$, hence

$$(2.12) \qquad \beta \left\| w_0 \right\|_\infty < \frac{\lambda \left(a^2 - bc\right)}{\mu bc},$$

thus we can choose γ and M such that (2.8) and (2.9) are satisfied. Using Theorem 2.2 we get,

$$e^{\beta z(t,.)} = \left(e^{\alpha z(t,.)}\right)^p \in \mathbb{L}^\infty \left(\left[0, T^*\right[; \mathbb{L}^1 (\Omega)\right),$$

therefore

$$e^{\alpha z(t,.)} \in \mathbb{L}^\infty \left(\left[0, T^*\right[; \mathbb{L}^p (\Omega)\right),$$

and from (2.10) we deduce that

$$f(w(t,.), z(t,.)) \in \mathbb{L}^\infty \left(\left[0, T^*\right[; \mathbb{L}^p (\Omega)\right), \qquad \text{for some } p > n/2.$$

By the preliminary remarks, we conclude that the solution is global and uniformly bounded on $[0, +\infty[\times \Omega$.

For the case $\sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}$, the same reasoning with w and z given by $(2.6)'$ and using the limit (1.6) we deduce the same result. ∎

3. Remarks and comments

Remark 1. *In the case when* $\sqrt{\frac{b}{c}} = \dfrac{\sigma}{\rho}$ *and initial data given in* Σ *(definied in the case when* $\sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}$ *) we have global existence of solutions of problem (1.1)-(1.4) with no condition on the constants or on the growth of the function* f *to part its positivity and* $f(r, \sqrt{\frac{c}{b}}r) = 0$, *for all* $r \geq 0$. *To verify this, it suffices to apply the maximum principle directly to equations* $(2.2)' - (2.3)'$.

Remark 2. *By application of the comparison's principle to equation (2.3), blowing-up in finite time can occur in the case where* $\sqrt{\frac{b}{c}} = \dfrac{\sigma}{\rho}$, *especially when the reaction term satifies an inequality of the form:*

$$|f(u,v)| \geq C_1 \left|u\right|^{\alpha_1} + C_2 \left|v\right|^{\alpha_2},$$

where C_1, C_2, α_1 *and* α_2 *are positive constants such that*

$$C_1^2 + C_2^2 \neq 0, \ \alpha_1 > 1 \text{ and } \alpha_2 > 1.$$

Remark 3. *One showed the global existence for functions* $f(u,v)$ *of polynomial growth (condition 1.6), but our results remain valid for functions of exponential growth (but small) while replacing the condition 1.6 by:*

$$\begin{cases} \lim\limits_{s \to +\infty} \left[\frac{log(1+f(r,s))}{s}\right] < \dfrac{\lambda\left(a^2 - bc\right)}{2n\mu bc \left\|w_0\right\|_\infty}, \text{ for any } r \geq 0, \text{ when } \sqrt{\frac{b}{c}} < \dfrac{\sigma}{\rho}, \\ \qquad \text{and} \\ \lim\limits_{r \to +\infty} \left[\frac{log(1+f(r,s))}{r}\right] < \dfrac{\lambda\left(a^2 - bc\right)}{2n\mu bc \left\|w_0\right\|_\infty}, \text{ for any } s \geq 0, \text{ when } \sqrt{\frac{b}{c}} > \dfrac{\sigma}{\rho}. \end{cases}$$

Remark 4. *If $bc = 0$, we have global existence for any exponential growt.*

References

[1] N. Alikakos, L^p-Bounds of Solutions of Reaction-Diffusion Equations.Comm. P. D. E. 4 (1979). 827-828.

[2] E. L. Cussler, Multicomponant Diffusion, Chemical Engineering Monographs 3. Elsevier Scientific Publihing Compagny, Amesterdam, 1976.

[3] A.Friedman, Partial Differential Equations of Parabolic Type. Prentice Hall Englewood Chiffs. N. J. 1964.

[4] P. L. Garcia-Ybarra and P. Clavin, Cross transport effects in premixed flames. Progress in Astronautics and Aeronautics, Vol. 76, The American Institute of Aeronautics and Astronautics, New York, 463-481 (1981).

[5] S. R. De Groot and P. Mazur, Non-Equilibrium Thermodynamics. Dover Publications Inc., New York (1984).

[6] A. Haraux and A.Youkana, On a Result of K.Masuda Concerning Reaction-Diffusion Equations. Tôhoku. Math. J.40 (1988), 159-163.

[7] D.Henry, Geometric Theory of Semilinear Parabolic Equations. LectureNotes in Mathematics 840, Springer-Verlag,New-York, 1984.

[8] S. L. Hollis, R. H. Martin $_{JR}$ and M. Pierre, Global Existence and Boundlessness in Reaction-Diffusion Systems. Siam. J. Math. Anal, Vol 18, $n^0 3$, May 1987.

[9] J. Jorne, The diffusion Lotka-Volterra oscillationg system. J. Theor. Biol., 65, 133-139 (1977).

[10] M. Kirane, Global Bounds and Asymptotics of Reaction-Diffusion Equations. J. Math. An and Appl 138; (1989) 328-342.

[11] M. Kirane and S. Kouachi, Global Solutions to a System of Strongly Coupled Reaction-Diffusion Equations. Nonlinear Analysis Theory, Methods and Applications, Vol 126, $n^0 8$, (1996). USA.

[12] M. Kirane and S. Kouachi, Asymptitic Behavior for a System Describing Epidemics with Migration and Spatial Spread of Infection. Dynamical Systems and Applications, Vol 12; number 1, (1993), 121-130.

[13] J. S. Kirkaldy, Diffusion in multicomponant metallic systems. Canadian J. of Physics, 35, 435-440 (1957).

[14] S. Kouachi and A. Youkana, Global existence for a class of reaction-diffusion systems. Bulletin of the Polish Academy of Science (ref. 7171).

[15] A. I. Lee and J. M. Hill, On the general linear coupled system for diffusion in media with two diffusivities. J. Math. Anal. Appl., 89, 530-557 (1982).

[16] K. Masuda, On the Global Existence and Asymptotic Behaviour of Solutions of Reaction-Diffusion Equations. Hokkaido. Math. J. 12 (1983), 360-370.

[17] A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations. Applied Math. Sciences 44, Springer-Verlag, New york (1983).

[18] J. Savchik, B. Changs and H. Rabitz, Application of moments to the general linear multicomponant reaction-diffusion equations. J. Phys. Chem. 37, 1990-1997 (1983).

[19] J. A. Smoller, Shock waves and reaction-diffusion equations. Springer-Verlag, New York (1983).

Département de Mathématiques, Centre univ de Tébessa, 12002, Algérie.
E-mail address: kouachi.said@caramail.com
Current address: Cité Djabel Anwal, B. P. 24, Tébessa 12002, Algérie.