Congruence Subgroups of $\mathrm{PSL}(2, \mathbb{Z})$ of Genus Less than or Equal to 24

C. J. Cummins and S. Pauli

CONTENTS

In this paper, we report the computation and tabulation, using MAGMA, of all congruence subgroups of $PSL(2, \mathbb{Z})$ of genus less than or equal to 24. We include full tables of the congruence groups of genus 0, 1, 2, and 3 and a summary of the remaining cases.

1. INTRODUCTION

The group $\overline{\Gamma} = \mathrm{PSL}(2, \mathbb{Z}) = \mathrm{SL}(2, \mathbb{Z})/\{\pm 1\}$ acts on the extended upper half plane $\mathcal{H}^* = \mathcal{H} \cup \mathbb{Q} \cup \infty$ by fractional linear transformations. The genus of a subgroup G of $\overline{\Gamma}$ is the genus of the corresponding surface $\mathcal{H}^*/G$. The principal congruence subgroup of level N, $\overline{\Gamma}(N)$, is the image in $\mathrm{PSL}(2, \mathbb{Z})$ of the group

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{Z}) \,\middle|\, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}.$$

A subgroup of $\overline{\Gamma}$ which contains some principal congruence subgroup is called a *congruence subgroup*. The level of a congruence subgroup G is the smallest N such that $\overline{\Gamma}(N) \subset G$. The literature on congruence subgroups is vast, and the subject remains very active. Rademacher conjectured that there are only finitely many genus 0 congruence subgroups. This problem was studied by Knopp and Newman [Knopp and Newman 65], McQuillen [McQuillan 66a, McQuillan 66b], and Dennin [Dennin 71, Dennin 72, Dennin 74]. Stronger versions of the conjecture were proved by Thompson [Thompson 80] and Cox and Parry [Cox and Parry 84a, Cox and Parry 84b], which show that the number of congruence subgroups of any genus is finite.

Our aim in this paper is to extend the tabulation of Cox and Parry, who considered the genus zero case. This work is motivated by the current interest in congruence groups. In particular, a complete listing of all congruence groups of small genus for groups commensurable with $\mathrm{PSL}(2, \mathbb{Z})$ would be very useful for

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the study of the connections of modular functions with the finite simple groups (known as Moonshine [Conway and Norton 79, Borcherds 92]). We have computed a complete list of congruence groups up to genus 24, however, the results are too long to be contained in this article. The full tables are available online at http://www.math.tu-berlin.de/~pauli/congruence or http://www.mathstat.concordia.ca/faculty/cummins/congruence together with source code for the computation. In this paper, we provide tables containing a full list of the congruence groups up to genus 3 together with other data. A summary of the other cases is contained in Theorem 2.8.

2. THE CALCULATIONS

Thompson's results [Thompson 80] apply to any group commensurable with $\overline{\Gamma}$ and to any genus, however, they do not give an explicit bound on the level or index of the subgroups. The results of Cox and Parry give the bounds:

Proposition 2.1. (Cox and Parry.) *If G is a congruence subgroup of genus g and level ℓ, then*

$$\ell \leq \begin{cases} 168 & \text{if } g = 0 \\ 12g + \frac{1}{2}(13\sqrt{48g+121}) + 145) & \text{if } g \geq 1. \end{cases}$$

Proposition 2.2. (Cox and Parry.) *If G is a congruence subgroup of genus g and level ℓ and if p is a prime dividing ℓ, then $p \leq 12g + 13$.*

Using analytic methods derived to study the Selberg eigenvalue problem, Zograf [Zograf 91] gave a bound on the index of a congruence group:

Proposition 2.3. (Zograf.) *If G is a congruence subgroup of index m and genus g, then*

$$m < 128(g + 1).$$

Zograf's bound for the eigenvalue has been improved by Kim and Sarnak [Kim and Sarnak 02]. Using their bound, we obtain $m < 101(g+1)$ as a new bound for the index of congruence subgroups of genus g.

Cox and Parry used Propositions 2.1 and 2.2 as the basis for a calculation of the genus 0 congruence subgroups. Proposition 2.3 also, in principle, reduces the problem to a finite calculation—although a very large one.

We have used these three propositions to calculate all the congruence subgroups of the modular group of genus 0 to genus 24 using the computer algebra system MAGMA [Bosma et al. 97]. In order to find congruence subgroups of a given level ℓ, we recursively compute maximal subgroups in the group $\Gamma/\Gamma(\ell)$ in a convenient permutation representation. The subgroups of $\overline{\Gamma}/\overline{\Gamma}(\ell)$ correspond to those subgroups of $\Gamma/\Gamma(\ell)$ that contain -1. The algorithm used for the maximal subgroup computation in MAGMA is described in [Cannon et al. 01].

Let H be a subgroup of $\Gamma/\Gamma(\ell)$. Then the corresponding subgroup of Γ can be easily computed using the generators of $\Gamma(\ell)$ (see [Coste and Gannon 99] for instance) and the preimages of the generators of H as words in the generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

of Γ. The level of H is defined to be the level of this preimage in Γ. Thus, it is clear that the level of H is less than or equal to ℓ. We denote the subgroup of Γ corresponding to H by $\widehat{H}$.

Propositions 2.1 and 2.2 tell us which levels we have to consider to find all congruence subgroups of a given genus. To make the calculation more efficient, we also use the inequality of Proposition 2.3 and the fact that subgroups of genus g can only have subgroups of genus greater than or equal to g as additional criteria for terminating the search under a subgroup.

When the level is a power of a prime p, we can apply the following lemma:

Lemma 2.4. (Cox and Parry.) *For two positive integers ℓ and ℓ' with $\ell' \mid \ell$, let $\tau_{\ell'}$ denote the natural map*

$$\tau_{\ell'} : \Gamma/\Gamma(\ell) \longrightarrow \Gamma/\Gamma(\ell').$$

Let H be a subgroup of $\Gamma/\Gamma(\ell)$.

(i) Suppose that H has level ℓ and p is a prime with $p \mid \ell$ such that $\tau_p H = \Gamma/\Gamma(p)$. Then $p \leq 5$.

(ii) Suppose that $\ell = p^m$ and let $2 \leq k \leq m$. If $\tau_{p^k}(H)$ has level less than p^k, then the level of H is less than p^k.

We can obtain all congruence subgroups of level p^k with $k \geq 3$ by first computing all congruence subgroups of level p^{k-1}, which is done in $\Gamma/\Gamma(p^{k-1})$, and then computing the subgroups of level p^k under those. In other words, except for a possible gap at level p, congruence subgroups of prime power level only occur in chains with levels $1, p, p^2, p^3, \ldots, p^m$.

Algorithm 2.5.

Input: A genus g, a prime number p, an integer k, and a congruence subgroup $\widehat{G}$ of level p^n of Γ, $n \leq k$.
Output: A list L of all subgroups of $\widehat{G}$ of genus up to g containing $\Gamma(p^k)$

$L \leftarrow \{\,\}$
If $n = 0$ and $p \leq 5$ then [Lemma 2.4 (i)]
  Compute the set S of all maximal subgroups of $\Gamma/\Gamma(p^2)$.
Else if $n = 0$ and $p > 5$ then
  Compute the set S of all maximal subgroups of $\Gamma/\Gamma(p)$.
Else
  Compute the set S of all maximal subgroups of the group generated by the generators of G and $\Gamma(p^n)$ in $\Gamma/\Gamma(p^{n+1})$.
For all groups $H \in S$ do
  If genus $\widehat{H}$ is less than or equal to g then
    $L \leftarrow L \cup \{\widehat{H}\}$
    Let p^m be the level of $\widehat{H}$.
    If $m < k$ and $[\Gamma : \widehat{H}] \leq 64(g+1)$ then [Proposition 2.3]
      Compute the set N of all subgroups of $\widehat{H}$ genus g containing $\Gamma(p^{m+1})$ using Algorithm 2.5.
      $L \leftarrow L \cup N$
Return L.

In the general case, Lemma 2.4 (i) only allows us to do the first subgroup computation in $\Gamma/\Gamma(p)$ for some $p \geq 7$ dividing the level ℓ. All other computations have to take place in $\Gamma/\Gamma(\ell)$

Algorithm 2.6.

Input: A genus g, a level ℓ, and a congruence subgroup $\widehat{G}$ of Γ
Output: If 2, 3, or 5 divide ℓ, then all subgroups of $\widehat{G}$ of genus up to g containing $\Gamma(\ell)$ are returned. Otherwise, let q be the largest prime divisor of ℓ, then all subgroups of $\widehat{G}$ of genus up to g, which contain $\Gamma(\ell)$ and which have level divisible by q, are returned.

$L \leftarrow \{\,\}$
If $\widehat{G} = \Gamma$ and all primes dividing ℓ are greater than 5, then
  Let q the largest prime dividing ℓ.
  Compute the set S of all proper maximal subgroups of $\Gamma/\Gamma(q)$.
Else
  Compute the set S of all maximal subgroups of the group generated by the generators of G and $\Gamma(m)$ in $\Gamma/\Gamma(\ell)$, where m is the level of $\widehat{G}$.
For all groups $H \in S$:
  If genus $\widehat{H}$ is less than or equal to g, then
    $L \leftarrow L \cup \{\widehat{H}\}$
    If $[\Gamma : \widehat{H}] \leq 64(g+1)$, then [Proposition 2.3]
      Compute the set N of all subgroups of $\widehat{H}$ of genus up to g containing $\Gamma(\ell)$ using Algorithm 2.6.
      $L \leftarrow L \cup N$
Return L.

We should comment on the rather convoluted condition on the output of Algorithm 2.6. If the smallest prime factor of ℓ is at least 7, then only groups with level divisible by the largest prime factor of ℓ are returned. So, for example, if $\hat{G} = \Gamma$ and $\ell = 7 \times 11$, then only groups with levels divisible by 11 are returned. Thus, groups of level 7 would not be computed. The crucial point is that we wish to compute the full list of congruence subgroups of given genus and to do so we apply Algorithm 2.6 for all maximal levels (as computed by Algorithm 2.7). There will be a maximal level whose largest prime divisor is 7 and the groups of level 7 will be obtained in this part of the calculation.

To be more precise, given a positive integer B, and a set of primes $P = \{p_1, p_2, \ldots, p_k\}$ which satisfy $p_1 < p_2 < \cdots < p_k \leq B$, we define $M(B,P)$ to be the set of positive integers x such that:

(i) $x \leq B$,

(ii) the prime divisors of x are in P, and

(iii) the only positive multiple of x bounded by B is x.

In other words, $M(B,P)$ is the set of positive integers which are maximal with respect to the properties of being bounded by B and having prime divisors in P. Then $M(B, \{p_1, \ldots, p_{k-1}\}) \subset M(B,P)$, since if $x \in M(B, \{p_1, \ldots, p_{k-1}\})$ and x is not in $M(B,P)$, then there must be some $y > 1$ such that $yx \in M(B,P)$. But y cannot be divisible by any of the primes $p_1, \ldots, p_{k-1}$ since this would yield an integer larger than x, but still bounded by B. So y must be a power of p_k. But this gives a contradiction since it would imply $x < p_1 x < yx \leq b$. Thus, there is an element of $M(B,P)$ whose largest prime divisor is p_{k-1} and, iterating this construction, we have that for any $p \in P$, there is an element of $M(B,P)$ whose largest prime divisor is p.

	All Subgroups					Torsion-Free Subgroups				
g		PSL	PGL	ℓ	I		PSL	PGL	ℓ	I
0	1116	132	121	48	72	254	33	33	32	60
1	2801	187	163	52	108	459	48	48	36	108
2	4107	177	145	78	108	672	49	49	64	108
3	6513	284	241	96	168	1809	108	105	72	168
4	7257	261	215	108	180	1665	87	86	81	180
5	9386	303	256	126	192	3028	133	125	75	192
6	10416	230	175	126	192	1780	55	45	121	180
7	18191	480	388	156	216	6216	213	191	128	216
8	13726	277	212	169	220	2671	83	76	96	156
9	21014	469	403	154	288	6711	208	203	128	288
10	15622	304	235	168	324	4483	133	120	118	324
11	27466	489	381	198	288	8450	195	179	147	240
12	18095	269	198	210	330	4978	93	70	142	300
13	33241	664	549	231	384	12447	343	303	162	384
14	22871	268	178	252	300	4581	72	53	167	192
15	40880	596	485	240	384	16743	289	263	179	288
16	30809	410	294	243	364	8607	143	123	243	360
17	54794	819	667	289	480	17453	351	317	242	480
18	24935	273	191	264	384	4819	71	60	214	288
19	60648	812	647	273	504	24287	411	375	256	504
20	31137	308	203	286	408	9396	122	85	239	300
21	66841	888	729	308	480	27542	504	450	256	480
22	36135	365	284	361	486	11206	152	132	263	432
23	59450	686	537	338	504	22798	312	271	274	384
24	42289	336	212	336	546	6903	78	51	284	336

TABLE 1.

Thus, in order to determine all congruence subgroups up to a given genus, we compute a list of all possible maximal levels before calling Algorithms 2.5 and 2.6. Both Algorithms 2.5 and 2.6 can be sped up by checking whether a group (or one of its conjugates) is known already before computing its subgroups. We can also discard any subgroups not containing -1 since G and $< -1, G >$ have the same image in $\overline{\Gamma}$.

Algorithm 2.7.

Input: A genus g
Output: A list L of all congruence subgroups of Γ of genus up to g

Let M be the set of integers with

$\ell \in M$ then

$$\ell \leq \begin{cases} 168 & \text{if} \quad g = 0 \\ 12g + \frac{1}{2}(13\sqrt{48g+121}) + 145) & \\ & \text{if} \quad g \geq 1, \end{cases}$$

[Proposition 2.1]

$l \in M$, p prime with $p \mid l$
then $p \leq 12g + 13$, and [Proposition 2.2]
$\ell_1 \in M$ and $\ell_2 \in M$ then $\ell_1 \not|\ell_2$ and $\ell_2 \not|\ell_1$.

For all levels $\ell \in M$ with $\ell = p^k$ for some prime number p and some integer k:

Compute the set L of all congruence subgroups of Γ of genus up to g containing $\Gamma(p^k)$ using Algorithm 2.5.

For the remaining levels $\ell \in M$:

Add to L all congruence subgroups of Γ of genus g containing $\Gamma(\ell)$ as returned by Algorithm 2.6 (see the comment above concerning the output of Algorithm 2.6).

Return L

The results of the calculations for genus up to 3 are contained in Tables 2, 3 and 4, which are described in more detail below. A summary of the full results is as follows:

Theorem 2.8. *For genus up to 24, Table 1 contains: the total number of congruence subgroups of* $\mathrm{PSL}(2,\mathbb{Z})$*, the number of congruence subgroups up to conjugacy in* $\mathrm{PSL}(2,\mathbb{Z})$*, the number of congruence subgroups up to conjugacy in* $\mathrm{PGL}(2,\mathbb{Z})$*, the maximum level* ℓ *and the maximum index* I*. The same information for torsion-free congruence subgroups is also given.*

3. THE TABLES

3.1 Table 2

Table 2 uses the notation $(\text{level})(\text{label})^{(\text{genus})}$ to name the subgroups. So, for example, $1A^0$ is the name of $\mathrm{PSL}(2,\mathbb{Z})$. The additional data are I the index, Z the number of conjugates under outer automorphisms, L the number of $\mathrm{PSL}(2,\mathbb{Z})$ conjugates, c_2 the number of classes of elements of order 2, and c_3 the number of classes of elements of order 3 and the cusp widths written in partition notation.

The column labeled *Gal* gives the lengths of orbits under conjugation by the group

$$D = \left\{ \pm \begin{pmatrix} 1 & 0 \\ 0 & x \end{pmatrix} \middle| \, x \in (Z/m\mathbb{Z})^* \right\}$$

acting on the conjugates of the image of G in $\mathrm{PGL}(2,\mathbb{Z}/m\mathbb{Z})$. This is also written in partition notation. So, for example, for $3C^0$, the partition $1^1 2^1$ means one of the conjugates is fixed and the other two form an orbit of length 2. This data gives information on the degree of the field generated by the q-coefficients of a "minimal" field of automorphic functions of G (see [Shimura 71, Section 6, page 154]).

The final column of Table 2 gives a list of the minimal supergroups G of the group H. That is, all subgroups G of $\mathrm{PSL}(2,\mathbb{Z})$ such that H is a maximal proper subgroup of G (up to $PGL(2,\mathbb{Z})$ conjugacy).

3.2 Table 3

In most cases, the classes of groups in Table 2 are uniquely determined by the data we give. The exceptions are listed in Table 3 together with explicit generators in $\mathrm{PSL}(2,\mathbb{Z}/m\mathbb{Z})$ of the image of a conjugate of G. The more extensive online tables include subgroups of genus up to 24 and this extra information does differentiate these groups.

We note that a MAGMA computation shows that the seven pairs of groups $16K, L^1$, $32C, D^1$, $24G, H^2$, $25A, B^2$, $25C, D^2$, $56A, B^3$, $56C, D^3$ are precisely those groups from Table 2 which are not $\mathrm{PGL}(2,\mathbb{Z})$ conjugates, but whose images in $\mathrm{PGL}(2,\mathbb{Z}/m\mathbb{Z})$ are conjugate. They are also precisely the groups for which the partition of the *Gal* column in Table 2 is not a partition of ZL. In each case, it is a partition of $2ZL$.

It is perhaps worth noting that the other groups in Table 3 also appear to be paired, which suggests the existence of an additional symmetry of order 2.

3.3 Table 4

In Table 4, we list, for convenience, standard names of some of the groups in Table 2.

4. COMMENTS

The number of conjugacy classes of genus zero subgroups for each level were given in [Cox and Parry 84a] and more details of this extensive hand calculation are in [Cox and Parry 84b]. We note that our totals differ at levels 7, 10, and 24.[1]

We also mention agreement between our tables and the results of: Newman [Newman 64, Newman 65], who classified the normal congruence subgroups of genus 1 (which are $6A^1$, $6C^1$, $6D^1$, and $6F^1$ in our notation); Sebbar [Sebbar 01], who classified the torsion-free genus zero congruence groups (which are the groups with genus zero and $c_2 = c_3 = 0$), and also Petersson [Petersson 71], who classified all cycloidal congruence subgroups, which are the groups for which the cusp partition has only one part—some groups in his classification have genus greater than 3 and so are not contained in these tables.

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[1] Specifically, we find $7F^0$ ($\tilde{D}_8$ in the notation of Cox and Parry) has one rather than two $\mathrm{PGL}(2,\mathbb{Z})$ conjugates. At level 10, we find 10 classes of subgroups rather than 11 and at level 24, we find an extra class of subgroups of index 48.

	I	Z	L	c2	c3	*Cusps*	*Gal*	*Super*
$1A^0$	1	1	1	1	1	1^1	1^1	
$2A^0$	2	1	1	0	2	2^1	1^1	$1A^0$
$2B^0$	3	1	3	1	0	$1^1 2^1$	1^3	$1A^0$
$2C^0$	6	1	1	0	0	2^3	1^1	$2A^0 2B^0$
$3A^0$	3	1	1	3	0	3^1	1^1	$1A^0$
$3B^0$	4	1	4	0	1	$1^1 3^1$	$1^2 2^1$	$1A^0$
$3C^0$	6	1	3	2	0	3^2	$1^1 2^1$	$3A^0$
$3D^0$	12	1	1	0	0	3^4	1^1	$3B^0 3C^0$
$4A^0$	4	1	4	2	1	4^1	2^2	$1A^0$
$4B^0$	6	1	3	0	0	$1^2 4^1$	1^3	$2B^0$
$4C^0$	6	1	3	2	0	$2^1 4^1$	1^3	$2B^0$
$4D^0$	8	1	4	0	2	4^2	2^2	$2A^0 4A^0$
$4E^0$	12	1	3	0	0	$2^2 4^2$	1^3	$2C^0 4B^0 4C^0$
$4F^0$	12	1	6	2	0	4^3	$1^2 2^2$	$4A^0 4C^0$
$4G^0$	24	1	1	0	0	4^6	1^1	$4D^0 4E^0 4F^0$
$5A^0$	5	1	5	1	2	5^1	$1^1 4^1$	$1A^0$
$5B^0$	6	1	6	2	0	$1^1 5^1$	$1^2 4^1$	$1A^0$
$5C^0$	10	1	10	2	1	5^2	$2^1 4^2$	$1A^0$
$5D^0$	12	1	6	0	0	$1^2 5^2$	$1^2 4^1$	$5B^0$
$5E^0$	15	1	5	3	0	5^3	$1^1 4^1$	$5A^0$
$5F^0$	20	1	10	0	2	5^4	$2^1 4^2$	$5A^0 5C^0$
$5G^0$	30	1	15	2	0	5^6	$1^1 2^1 4^3$	$5B^0 5C^0 5E^0$
$5H^0$	60	1	1	0	0	5^{12}	1^1	$5D^0 5F^0 5G^0$
$6A^0$	6	1	2	0	3	6^1	2^1	$2A^0$
$6B^0$	6	1	3	4	0	6^1	$1^1 2^1$	$3A^0$
$6C^0$	8	1	4	0	2	$2^1 6^1$	$1^2 2^1$	$2A^0 3B^0$
$6D^0$	9	1	3	3	0	$3^1 6^1$	1^3	$2B^0 3A^0$
$6E^0$	12	1	3	4	0	6^2	$1^1 2^1$	$3C^0 6B^0$
$6F^0$	12	1	12	0	0	$1^1 2^1 3^1 6^1$	$1^6 2^3$	$2B^0 3B^0$
$6G^0$	18	1	9	2	0	$3^2 6^2$	$1^3 2^3$	$3C^0 6D^0$
$6H^0$	18	1	9	4	0	6^3	$1^3 2^3$	$6B^0 6D^0$
$6I^0$	24	1	4	0	0	$2^3 6^3$	$1^2 2^1$	$2C^0 6C^0 6F^0$
$6J^0$	24	1	8	0	3	6^4	2^4	$6A^0 6C^0$
$6K^0$	36	1	3	0	0	$3^4 6^4$	1^3	$3D^0 6F^0 6G^0$
$6L^0$	36	1	9	4	0	6^6	$1^3 2^3$	$6E^0 6G^0 6H^0$
$7A^0$	7	2	7	3	1	7^1	$2^1 6^2$	$1A^0$
$7B^0$	8	1	8	0	2	$1^1 7^1$	$1^2 6^1$	$1A^0$
$7C^0$	14	2	7	2	2	7^2	$2^1 6^2$	$7A^0$
$7D^0$	21	1	21	5	0	7^3	$3^1 6^3$	$7A^0$
$7E^0$	24	1	8	0	0	$1^3 7^3$	$1^2 6^1$	$7B^0$
$7F^0$	28	1	28	4	1	7^4	$1^1 3^1 6^4$	$7A^0$
$7G^0$	42	2	7	6	0	7^6	$2^1 6^2$	$7C^0 7D^0$
$8A^0$	8	2	4	2	2	8^1	4^2	$4A^0$
$8B^0$	12	1	3	4	0	$4^1 8^1$	1^3	$4C^0$
$8C^0$	12	1	6	0	0	$1^2 2^1 8^1$	$1^4 2^1$	$4B^0$
$8D^0$	12	1	6	2	0	$2^2 8^1$	$1^2 2^2$	$4C^0$
$8E^0$	16	1	4	0	4	8^2	2^2	$4D^0 8A^0$
$8F^0$	16	1	16	4	1	8^2	4^4	$4A^0$
$8G^0$	24	1	3	0	0	$2^4 8^2$	1^3	$4E^0 8C^0 8D^0$
$8H^0$	24	1	6	4	0	$4^2 8^2$	$1^2 2^2$	$4F^0 8B^0 8D^0$
$8I^0$	24	1	12	0	0	$1^2 2^1 4^1 8^2$	$1^4 2^2 4^1$	$8C^0$
$8J^0$	24	1	12	0	0	$2^2 4^3 8^1$	$1^8 2^2$	$4E^0$
$8K^0$	24	1	12	2	0	$4^4 8^1$	$2^2 4^2$	$4F^0$
$8L^0$	24	1	12	4	0	$4^2 8^2$	$1^4 2^2 4^1$	$8B^0$
$8M^0$	32	2	16	4	2	8^4	4^8	$8A^0 8F^0$
$8N^0$	48	1	3	0	0	$4^8 8^2$	1^3	$4G^0 8J^0 8K^0$
$8O^0$	48	1	6	0	0	$2^4 4^2 8^4$	$1^4 2^1$	$8G^0 8I^0 8J^0$
$8P^0$	48	1	12	4	0	$4^4 8^4$	2^6	$8H^0 8L^0$
$9A^0$	9	1	9	5	0	9^1	$1^1 2^1 6^1$	$3A^0$
$9B^0$	12	1	4	0	0	$1^3 9^1$	$1^2 2^1$	$3B^0$
$9C^0$	12	1	4	0	3	$3^1 9^1$	$1^2 2^1$	$3B^0$
$9D^0$	18	1	3	6	0	9^2	$1^1 2^1$	$3C^0 9A^0$
$9E^0$	18	1	18	2	0	$3^3 9^1$	$1^1 2^1 3^1 6^2$	$3C^0$
$9F^0$	27	1	27	3	3	9^3	$3^1 6^4$	$1A^0$
$9G^0$	27	1	27	7	0	9^3	$3^1 6^4$	$9A^0$
$9H^0$	36	1	6	0	0	$3^6 9^2$	$1^2 2^2$	$3D^0 9E^0$
$9I^0$	36	1	12	0	0	$1^3 3^2 9^3$	$1^2 2^2 6^1$	$9B^0$
$9J^0$	36	1	12	0	3	$3^3 9^3$	$1^2 2^2 6^1$	$9C^0$
$10A^0$	10	1	5	0	4	10^1	$1^1 4^1$	$2A^0 5A^0$
$10B^0$	12	1	6	4	0	$2^1 10^1$	$1^2 4^1$	$5B^0$
$10C^0$	18	1	18	2	0	$1^1 2^1 5^1 10^1$	$1^6 4^3$	$2B^0 5B^0$
$10D^0$	20	1	10	4	2	10^2	$2^1 4^2$	$5C^0$
$10E^0$	30	1	10	0	6	10^3	$2^1 4^2$	$10A^0$
$10F^0$	36	1	18	0	0	$1^2 2^2 5^2 10^2$	$1^6 4^3$	$5D^0 10C^0$
$10G^0$	36	1	18	4	0	$2^3 10^3$	$1^6 4^3$	$10B^0 10C^0$
$11A^0$	11	2	11	3	2	11^1	$2^1 10^2$	$1A^0$
$12A^0$	12	1	4	6	0	12^1	2^2	$3A^0 4A^0$
$12B^0$	16	1	4	0	4	$4^1 12^1$	$1^2 2^1$	$6C^0$
$12C^0$	18	1	3	6	0	$6^1 12^1$	1^3	$4C^0 6D^0$
$12D^0$	18	1	9	4	0	$3^2 12^1$	$1^3 2^3$	$6D^0$
$12E^0$	24	1	12	0	0	$1^2 3^2 4^1 12^1$	$1^6 2^3$	$4B^0 6F^0$
$12F^0$	24	1	12	8	0	12^2	$2^2 4^2$	$6B^0 12A^0$
$12G^0$	36	1	9	4	0	$3^4 12^2$	$1^3 2^3$	$6G^0 12D^0$
$12H^0$	36	1	9	8	0	$6^2 12^2$	$1^3 2^3$	$6H^0 12C^0 12D^0$
$12I^0$	48	1	12	0	0	$2^4 4^1 6^4 12^1$	$1^6 2^3$	$6I^0$
$12J^0$	48	1	24	0	0	$1^2 2^1 3^2 4^2 6^1 12^2$	$1^8 2^6 4^1$	$12E^0$
$13A^0$	14	1	14	2	2	$1^1 13^1$	$1^2 12^1$	$1A^0$
$13B^0$	28	1	14	0	4	$1^2 13^2$	$1^2 12^1$	$13A^0$
$13C^0$	42	1	14	6	0	$1^3 13^3$	$1^2 12^1$	$13A^0$
$14A^0$	14	2	7	4	2	14^1	$2^1 6^2$	$7A^0$
$14B^0$	16	1	8	0	4	$2^1 14^1$	$1^2 6^1$	$2A^0 7B^0$
$14C^0$	48	1	16	0	6	$2^3 14^3$	$1^4 6^2$	$14B^0$
$15A^0$	15	2	5	3	3	15^1	$2^1 8^1$	$5A^0$
$15B^0$	18	1	6	6	0	$3^1 15^1$	$1^2 4^1$	$3A^0 5B^0$
$15C^0$	36	1	18	8	0	$3^2 15^2$	$1^2 2^2 4^1 8^1$	$15B^0$
$16A^0$	16	2	8	2	4	16^1	8^2	$8A^0$
$16B^0$	24	1	3	8	0	$8^1 16^1$	1^3	$8B^0$
$16C^0$	24	1	6	0	0	$1^4 4^1 16^1$	$1^4 2^1$	$8C^0$
$16D^0$	24	1	12	0	0	$1^2 2^3 16^1$	$1^4 2^2 4^1$	$8C^0$
$16E^0$	24	1	12	2	0	$2^4 16^1$	$1^2 2^1 4^2$	$8D^0$
$16F^0$	32	1	4	0	8	16^2	2^2	$8E^0 16A^0$
$16G^0$	48	1	3	0	0	$2^8 16^2$	1^3	$8G^0 16D^0 16E^0$
$16H^0$	48	1	12	0	0	$1^4 2^2 4^2 16^2$	$1^4 2^2 4^1$	$8I^0 16C^0 16D^0$
$18A^0$	18	2	9	8	0	18^1	$2^3 6^2$	$6B^0 9A^0$
$18B^0$	24	1	4	0	6	$6^1 18^1$	$1^2 2^1$	$6C^0 9C^0$
$18C^0$	24	1	8	0	3	$2^3 18^1$	2^4	$6C^0$
$18D^0$	36	1	3	12	0	18^2	$1^1 2^1$	$6E^0 9D^0 18A^0$
$18E^0$	36	1	12	0	0	$1^3 2^3 9^1 18^1$	$1^6 2^3$	$6F^0 9B^0$
$20A^0$	36	1	18	4	0	$1^2 4^1 5^2 20^1$	$1^6 4^3$	$10C^0$
$21A^0$	21	2	7	9	0	21^1	$2^1 6^2$	$3A^0 7A^0$
$24A^0$	36	1	3	12	0	$12^1 24^1$	1^3	$8B^0 12C^0$
$24B^0$	48	1	12	0	0	$1^4 3^4 8^1 24^1$	$1^6 2^3$	$12E^0$
$25A^0$	30	1	30	2	0	$1^5 25^1$	$1^2 4^2 20^1$	$5B^0$
$25B^0$	60	1	6	0	0	$1^{10} 25^2$	$1^2 4^1$	$5D^0 25A^0$
$26A^0$	28	1	14	4	4	$2^1 26^1$	$1^2 12^1$	$13A^0$

TABLE 2. Congruence subgroups of genus 0, 1, 2, and 3. The notation is explained in the text.

	I	Z	L	$c2$	$c3$	$Cusps$	Gal	$Super$
$27A^0$	36	1	12	0	0	$1^{6}3^{1}27^{1}$	$1^{2}2^{2}6^{1}$	$9B^0$
$28A^0$	32	1	8	0	8	$4^{1}28^{1}$	$1^{2}6^{1}$	$14B^0$
$30A^0$	36	1	6	12	0	$6^{1}30^{1}$	$1^{2}4^{1}$	$10B^015B^0$
$32A^0$	48	1	6	0	0	$1^{8}8^{1}32^{1}$	$1^{4}2^{1}$	$16C^0$
$36A^0$	48	1	4	0	12	$12^{1}36^{1}$	$1^{2}2^{1}$	$12B^018B^0$
$48A^0$	72	1	3	24	0	$24^{1}48^{1}$	1^{3}	$16B^024A^0$
$6A^1$	6	1	1	0	0	6^{1}	1^{1}	$2A^03A^0$
$6B^1$	12	1	3	0	0	6^{2}	$1^{1}2^{1}$	$3C^06B^06A^1$
$6C^1$	18	1	1	0	0	6^{3}	1^{1}	$2C^06A^06D^06A^1$
$6D^1$	24	1	1	0	0	6^{4}	1^{1}	$3D^06C^06E^06B^1$
$6E^1$	36	1	3	0	0	6^{6}	$1^{1}2^{1}$	$6G^06H^06B^16C^1$
$6F^1$	72	1	1	0	0	6^{12}	1^{1}	$6I^06J^06K^06L^06D^16E^1$
$7A^1$	42	1	21	2	0	7^{6}	$3^{1}6^{3}$	$7D^0$
$7B^1$	56	1	28	0	2	7^{8}	$1^{1}3^{1}6^{4}$	$7B^07C^07F^0$
$7C^1$	84	1	21	4	0	7^{12}	$3^{1}6^{3}$	$7F^07G^07A^1$
$8A^1$	12	1	3	0	0	$4^{1}8^{1}$	1^{3}	$4C^0$
$8B^1$	24	1	3	0	0	$4^{2}8^{2}$	1^{3}	$4E^08B^08A^1$
$8C^1$	24	1	6	0	0	$4^{2}8^{2}$	$1^{2}2^{2}$	$4F^08D^08A^1$
$8D^1$	24	1	12	2	0	8^{3}	$2^{2}4^{2}$	$4F^08A^0$
$8E^1$	32	1	16	0	2	8^{4}	4^{4}	$4D^08F^0$
$8F^1$	48	1	3	0	0	$4^{4}8^{4}$	1^{3}	$4G^08G^08H^08B^18C^1$
$8G^1$	48	1	6	0	0	$4^{4}8^{4}$	$1^{4}2^{1}$	$8J^08L^08B^1$
$8H^1$	48	1	12	4	0	8^{6}	$2^{2}4^{2}$	$8H^08K^08D^1$
$8I^1$	48	1	12	4	0	8^{6}	$1^{2}2^{1}4^{2}$	$8F^08H^0$
$8J^1$	64	1	16	0	4	8^{8}	4^{4}	$8E^08M^08E^1$
$8K^1$	96	1	3	0	0	$4^{8}8^{8}$	1^{3}	$8N^08O^08P^08F^18G^1$
$9A^1$	12	1	4	0	0	$3^{1}9^{1}$	$1^{2}2^{1}$	$3B^0$
$9B^1$	18	2	9	2	0	9^{2}	$2^{3}6^{2}$	$3C^09A^0$
$9C^1$	36	1	4	0	0	$3^{3}9^{3}$	$1^{2}2^{1}$	$3D^09B^09C^09A^1$
$9D^1$	36	1	12	0	0	$3^{3}9^{3}$	$1^{2}2^{2}6^{1}$	$9A^1$
$9E^1$	54	1	18	6	0	9^{6}	$1^{1}2^{1}3^{1}6^{2}$	$9D^09E^0$
$9F^1$	54	2	9	6	0	9^{6}	$2^{3}6^{2}$	$9D^09G^09B^1$
$9G^1$	81	1	27	9	0	9^{9}	$3^{1}6^{4}$	$9F^09G^0$
$9H^1$	108	1	4	0	0	$3^{9}9^{9}$	$1^{2}2^{1}$	$9H^09I^09J^09C^19D^1$
$10A^1$	12	1	6	0	0	$2^{1}10^{1}$	$1^{2}4^{1}$	$2A^05B^0$
$10B^1$	15	1	15	1	0	$5^{1}10^{1}$	$1^{3}4^{3}$	$2B^05A^0$
$10C^1$	20	1	10	0	2	10^{2}	$2^{1}4^{2}$	$2A^05C^0$
$10D^1$	24	1	6	0	0	$2^{2}10^{2}$	$1^{2}4^{1}$	$5D^010B^010A^1$
$10E^1$	30	1	15	4	0	10^{3}	$1^{1}2^{1}4^{3}$	$5E^0$
$10F^1$	30	1	30	2	0	$5^{2}10^{2}$	$2^{3}4^{6}$	$2B^05C^0$
$10G^1$	36	1	6	0	0	$2^{3}10^{3}$	$1^{2}4^{1}$	$2C^010C^010A^1$
$10H^1$	40	1	10	0	4	10^{4}	$2^{1}4^{2}$	$5F^010A^010D^010C^1$
$10I^1$	45	1	15	3	0	$5^{3}10^{3}$	$1^{3}4^{3}$	$5E^010B^1$
$10J^1$	60	1	60	4	3	10^{6}	4^{15}	$10D^0$
$10K^1$	72	1	6	0	0	$2^{6}10^{6}$	$1^{2}4^{1}$	$10F^010G^010D^110G^1$
$11A^1$	12	1	12	0	0	$1^{1}11^{1}$	$1^{2}10^{1}$	$1A^0$
$11B^1$	55	1	55	3	4	11^{5}	$5^{1}10^{5}$	$11A^0$
$11C^1$	55	1	55	7	1	11^{5}	$5^{1}10^{5}$	$1A^0$
$11D^1$	60	1	12	0	0	$1^{5}11^{5}$	$1^{2}10^{1}$	$11A^1$
$12A^1$	16	1	16	0	1	$4^{1}12^{1}$	$2^{4}4^{2}$	$3B^04A^0$
$12B^1$	18	1	3	0	0	$3^{2}12^{1}$	1^{3}	$4B^06D^0$
$12C^1$	18	1	9	2	0	$6^{1}12^{1}$	$1^{3}2^{3}$	$6D^0$
$12D^1$	24	1	6	4	0	12^{2}	$2^{1}4^{1}$	$6E^0$
$12E^1$	24	1	8	0	3	12^{2}	4^{2}	$4D^06A^0$
$12F^1$	24	1	12	0	0	$2^{1}4^{1}6^{1}12^{1}$	$1^{6}2^{3}$	$4C^06F^0$
$12G^1$	24	1	12	4	0	12^{2}	$2^{2}4^{2}$	$3C^012A^0$
$12H^1$	24	1	24	0	3	12^{2}	4^{6}	$6A^0$
$12I^1$	32	1	16	0	2	$4^{2}12^{2}$	$2^{4}4^{2}$	$4D^06C^012A^1$
$12J^1$	36	1	6	6	0	12^{3}	$1^{2}2^{2}$	$4F^012A^012C^0$
$12K^1$	36	1	9	0	0	$3^{4}12^{2}$	$1^{3}2^{3}$	$6G^012D^012B^1$
$12L^1$	36	1	9	4	0	$6^{2}12^{2}$	$1^{3}2^{3}$	$6G^012C^012C^1$
$12M^1$	36	1	18	6	0	12^{3}	$1^{2}2^{4}4^{2}$	$12C^0$
$12N^1$	36	2	9	4	0	$6^{2}12^{2}$	2^{9}	$6H^012D^012C^1$
$12O^1$	48	1	8	0	6	12^{4}	2^{4}	$6J^012B^0$
$12P^1$	48	1	12	0	0	$2^{2}4^{2}6^{2}12^{2}$	$1^{6}2^{3}$	$4E^06I^012E^012F^1$
$12Q^1$	48	1	12	8	0	12^{4}	$2^{2}4^{2}$	$6E^012F^012G^1$
$12R^1$	64	1	16	0	4	$4^{4}12^{4}$	$2^{4}4^{2}$	$12B^012I^1$
$12S^1$	72	1	3	0	0	$3^{8}12^{4}$	1^{3}	$6K^012E^012G^012K^1$
$12T^1$	72	1	9	8	0	$6^{4}12^{4}$	$1^{3}2^{3}$	$6L^012G^012H^012L^112N^1$
$12U^1$	72	1	18	4	0	$6^{8}12^{2}$	$2^{3}4^{3}$	$6L^0$
$12V^1$	96	1	12	0	0	$2^{4}4^{4}6^{4}12^{4}$	$1^{6}2^{3}$	$12I^012J^012P^1$
$14A^1$	14	2	7	0	2	14^{1}	$2^{1}6^{2}$	$2A^07A^0$
$14B^1$	21	2	21	3	0	$7^{1}14^{1}$	$2^{3}6^{6}$	$2B^07A^0$
$14C^1$	24	1	24	0	0	$1^{1}2^{1}7^{1}14^{1}$	$1^{6}6^{3}$	$2B^07B^0$
$14D^1$	28	2	7	0	4	14^{2}	$2^{1}6^{2}$	$7C^014A^014A^1$
$14E^1$	42	1	21	8	0	14^{3}	$3^{1}6^{3}$	$7D^0$
$14F^1$	42	2	42	4	3	14^{3}	6^{14}	$14A^0$
$14G^1$	56	1	28	8	2	14^{4}	$1^{1}3^{1}6^{4}$	$7F^014A^0$
$14H^1$	72	1	24	0	0	$1^{3}2^{3}7^{3}14^{3}$	$1^{6}6^{3}$	$7E^014C^1$
$15A^1$	15	1	5	3	0	15^{1}	$1^{1}4^{1}$	$3A^05A^0$
$15B^1$	20	1	20	0	2	$5^{1}15^{1}$	$1^{2}2^{1}4^{2}8^{1}$	$3B^05A^0$
$15C^1$	24	1	24	0	0	$1^{1}3^{1}5^{1}15^{1}$	$1^{4}2^{2}4^{2}8^{1}$	$3B^05B^0$
$15D^1$	30	1	10	6	0	15^{2}	$2^{1}4^{2}$	$3A^05C^0$
$15E^1$	36	1	18	4	0	$3^{2}15^{2}$	$1^{2}2^{2}4^{1}8^{1}$	$3C^015B^0$
$15F^1$	45	1	5	9	0	15^{3}	$1^{1}4^{1}$	$5E^015A^015A^1$
$15G^1$	48	1	24	0	0	$1^{2}3^{2}5^{2}15^{2}$	$1^{4}2^{2}4^{2}8^{1}$	$5D^015C^1$
$15H^1$	72	1	18	8	0	$3^{4}15^{4}$	$1^{2}2^{2}4^{1}8^{1}$	$15C^015E^1$
$15I^1$	96	1	24	0	0	$1^{4}3^{4}5^{4}15^{4}$	$1^{4}2^{2}4^{2}8^{1}$	$15G^1$
$16A^1$	24	1	6	0	0	$2^{2}4^{1}16^{1}$	$1^{4}2^{1}$	$8C^0$
$16B^1$	24	1	6	4	0	$8^{1}16^{1}$	$1^{2}2^{2}$	$8B^0$
$16C^1$	24	1	12	2	0	$4^{2}16^{1}$	$1^{2}2^{1}4^{2}$	$8D^0$
$16D^1$	24	1	12	4	0	$8^{1}16^{1}$	$1^{4}2^{2}4^{1}$	$8B^0$
$16E^1$	48	1	6	0	0	$2^{4}4^{2}16^{2}$	$1^{4}2^{1}$	$8G^016C^016A^1$
$16F^1$	48	1	6	8	0	$8^{2}16^{2}$	$1^{2}2^{2}$	$8H^016B^016B^1$
$16G^1$	48	1	12	0	0	$2^{4}4^{2}16^{2}$	$1^{4}2^{2}4^{1}$	$8I^016D^016A^1$
$16H^1$	48	1	12	4	0	$4^{4}16^{2}$	$2^{2}4^{2}$	$8H^0$
$16I^1$	48	1	12	4	0	$4^{4}16^{2}$	$1^{2}2^{1}4^{2}$	$8H^016E^016C^1$
$16J^1$	48	1	12	8	0	$8^{2}16^{2}$	$1^{4}2^{2}4^{1}$	$8L^016B^016D^1$
$16K^1$	48	1	24	2	0	$4^{4}8^{2}16^{1}$	$4^{4}8^{4}$	$8K^0$
$16L^1$	48	1	24	2	0	$4^{4}8^{2}16^{1}$	$4^{4}8^{4}$	$8K^0$
$16M^1$	96	1	6	0	0	$2^{8}4^{4}16^{4}$	$1^{4}2^{1}$	$8O^016G^016H^016E^116G^1$
$17A^1$	18	1	18	2	0	$1^{1}17^{1}$	$1^{2}16^{1}$	$1A^0$
$17B^1$	36	1	18	4	0	$1^{2}17^{2}$	$1^{2}16^{1}$	$17A^1$
$17C^1$	72	1	18	8	0	$1^{4}17^{4}$	$1^{2}16^{1}$	$17B^1$
$18A^1$	18	1	9	4	0	18^{1}	$1^{1}2^{1}6^{1}$	$6B^09A^0$
$18B^1$	18	1	18	4	0	18^{1}	$2^{3}6^{2}$	$6B^0$
$18C^1$	24	1	4	0	0	$2^{3}18^{1}$	$1^{2}2^{1}$	$6C^09B^0$
$18D^1$	24	1	8	0	3	$6^{1}18^{1}$	2^{4}	$6C^0$
$18E^1$	27	1	27	5	0	$9^{1}18^{1}$	$1^{3}2^{3}6^{3}$	$6D^09A^0$
$18F^1$	36	1	18	4	0	$6^{3}18^{1}$	$1^{1}2^{1}3^{1}6^{2}$	$6E^09E^0$
$18G^1$	36	1	36	4	0	$6^{3}18^{1}$	$2^{3}3^{2}6^{4}$	$6E^0$
$18H^1$	54	1	27	12	0	18^{3}	$3^{1}6^{4}$	$9G^018A^0$
$18I^1$	54	1	54	2	0	$3^{3}6^{3}9^{1}18^{1}$	$1^{3}2^{3}3^{3}6^{6}$	$6G^09E^0$
$18J^1$	72	1	4	0	0	$2^{9}18^{3}$	$1^{2}2^{1}$	$6I^018C^018E^018C^1$
$18K^1$	72	1	8	0	9	$6^{3}18^{3}$	2^{4}	$6J^018B^018C^018D^1$
$19A^1$	20	1	20	0	2	$1^{1}19^{1}$	$1^{2}18^{1}$	$1A^0$
$19B^1$	60	1	20	0	6	$1^{3}19^{3}$	$1^{2}18^{1}$	$19A^1$

TABLE 2 (continued).

	I	Z	L	$c2$	$c3$	$Cusps$	Gal	$Super$
$20A^1$	20	1	20	2	2	20^1	$2^2 8^2$	$4A^0 5A^0$
$20B^1$	24	1	24	4	0	$4^1 20^1$	$2^4 8^2$	$4A^0 5B^0$
$20C^1$	24	2	6	4	0	$4^1 20^1$	$2^2 8^1$	$10B^0$
$20D^1$	36	1	18	0	0	$1^2 4^1 5^2 20^1$	$1^6 4^3$	$4B^0 10C^0$
$20E^1$	36	1	18	4	0	$2^1 4^1 10^1 20^1$	$1^6 4^3$	$4C^0 10C^0$
$20F^1$	40	2	10	4	4	20^2	$4^1 8^2$	$10D^0$
$20G^1$	48	1	24	8	0	$4^2 20^2$	$2^4 8^2$	$10B^0 20B^1$
$20H^1$	72	1	18	0	0	$1^4 4^2 5^4 20^2$	$1^6 4^3$	$10F^0 20A^0 20D^1$
$20I^1$	72	1	18	8	0	$2^2 4^2 10^2 20^2$	$1^6 4^3$	$10G^0 20A^0 20E^1$
$20J^1$	72	2	18	4	0	$2^4 4^1 10^4 20^1$	$2^6 8^3$	$10G^0$
$21A^1$	24	2	8	0	3	$3^1 21^1$	$2^2 12^1$	$7B^0$
$21B^1$	32	1	32	0	2	$1^1 3^1 7^1 21^1$	$1^4 2^2 6^2 12^1$	$3B^0 7B^0$
$21C^1$	42	2	14	6	3	21^2	$4^1 12^2$	$7C^0$
$21D^1$	42	2	21	10	0	21^2	$2^1 4^1 6^2 12^2$	$21A^0$
$21E^1$	63	1	21	15	0	21^3	$3^1 6^3$	$7D^0 21A^0$
$21F^1$	64	1	32	0	4	$1^2 3^2 7^2 21^2$	$1^4 2^2 6^2 12^1$	$21B^1$
$22A^1$	22	2	11	0	4	22^1	$2^1 10^2$	$2A^0 11A^0$
$24A^1$	24	2	4	6	0	24^1	4^2	$8A^0 12A^0$
$24B^1$	24	2	12	6	0	24^1	$4^2 8^2$	$12A^0$
$24C^1$	36	1	6	6	0	$6^2 24^1$	$1^2 2^2$	$8D^0 12C^0$
$24D^1$	36	1	9	8	0	$12^1 24^1$	$1^3 2^3$	$12C^0$
$24E^1$	36	1	18	6	0	$6^2 24^1$	$1^2 2^4 4^2$	$12C^0$
$24F^1$	48	1	16	12	0	24^2	4^4	$8F^0 12A^0$
$24G^1$	48	1	24	0	0	$1^2 2^1 3^2 6^1 8^1 24^1$	$1^8 2^6 4^1$	$8C^0 12E^0$
$24H^1$	72	1	9	16	0	$12^2 24^2$	$1^3 2^3$	$12H^0 24A^0 24D^1$
$24I^1$	72	1	18	4	0	$3^8 24^2$	$2^3 4^3$	$12G^0$
$24J^1$	96	1	24	0	0	$1^4 2^2 3^4 6^2 8^2 24^2$	$1^8 2^6 4^1$	$12J^0 24B^0 24G^1$
$26A^1$	28	1	14	0	4	$2^1 26^1$	$1^2 12^1$	$2A^0 13A^0$
$26B^1$	56	1	14	0	8	$2^2 26^2$	$1^2 12^1$	$13B^0 26A^0 26A^1$
$27A^1$	36	1	12	0	0	$1^3 3^2 27^1$	$1^2 2^2 6^1$	$9B^0$
$27B^1$	36	1	12	0	6	$9^1 27^1$	$1^2 2^2 6^1$	$9C^0$
$27C^1$	108	1	12	0	0	$1^9 3^6 27^3$	$1^2 2^2 6^1$	$9I^0 27A^0 27A^1$
$28A^1$	28	2	28	6	1	28^1	$4^2 12^4$	$4A^0 7A^0$
$30A^1$	30	1	10	0	6	30^1	$2^1 8^1$	$6A^0 10A^0$
$30B^1$	30	2	5	0	6	30^1	$2^1 8^1$	$10A^0 15A^0$
$30C^1$	36	1	18	8	0	$6^1 30^1$	$1^2 2^2 4^1 8^1$	$6B^0 15B^0$
$30D^1$	72	1	18	16	0	$6^2 30^2$	$1^2 2^2 4^1 8^1$	$15C^0 30A^0 30C^1$
$32A^1$	48	1	12	0	0	$1^4 2^2 8^1 32^1$	$1^4 2^2 4^1$	$16C^0$
$32B^1$	48	1	12	12	0	$16^1 32^1$	2^6	$16B^0$
$32C^1$	48	1	24	2	0	$2^4 4^2 32^1$	$2^2 4^3 8^4$	$16E^0$
$32D^1$	48	1	24	2	0	$2^4 4^2 32^1$	$2^2 4^3 8^4$	$16E^0$
$32E^1$	96	1	12	0	0	$1^8 2^4 8^2 32^2$	$1^4 2^2 4^1$	$16H^0 32A^0 32A^1$
$33A^1$	33	2	11	9	0	33^1	$2^1 10^2$	$3A^0 11A^0$
$36A^1$	36	1	36	10	0	36^1	$2^2 4^2 12^2$	$9A^0 12A^0$
$36B^1$	48	1	8	0	6	$4^3 36^1$	2^4	$12B^0 18C^0$
$36C^1$	72	1	12	0	0	$1^6 4^3 9^2 36^1$	$1^6 2^3$	$12E^0 18E^0$
$39A^1$	42	2	14	6	3	$3^1 39^1$	$2^2 24^1$	$13A^0$
$40A^1$	72	2	18	4	0	$1^4 5^4 8^1 40^1$	$2^6 8^3$	$20A^0$
$42A^1$	42	2	7	12	0	42^1	$2^1 6^2$	$14A^0 21A^0$
$42B^1$	42	2	21	12	0	42^1	$2^1 4^1 6^2 12^2$	$6B^0 21A^0$
$49A^1$	56	1	56	0	2	$1^7 49^1$	$1^2 6^2 42^1$	$7B^0$
$52A^1$	56	2	14	4	8	$4^1 52^1$	$2^2 24^1$	$26A^0$

	I	Z	L	$c2$	$c3$	$Cusps$	Gal	$Super$
$8A^2$	48	1	1	0	0	8^6	1^1	$4G^0 8E^0 8D^1$
$8B^2$	48	1	12	0	0	8^6	$2^2 4^2$	$8K^0 8C^1 8D^1$
$8C^2$	96	1	24	4	0	8^{12}	$2^2 4^5$	$8M^0 8P^0 8H^1 8I^1$
$9A^2$	36	1	3	0	0	9^4	$1^1 2^1$	$3D^0 9D^0 9B^1$
$9B^2$	54	1	27	2	0	9^6	$3^1 6^4$	$9E^0 9G^0 9B^1$
$10A^2$	30	1	5	0	0	10^3	$1^1 4^1$	$5E^0 10A^0$
$10B^2$	30	1	5	0	0	10^3	$1^1 4^1$	$2C^0 10A^0 10B^1$
$10C^2$	60	1	15	4	0	10^6	$1^1 2^1 4^3$	$5G^0 10B^0 10D^0 10E^1$
$10D^2$	60	1	30	0	0	$5^4 10^4$	$2^3 4^6$	$5F^0 10B^1 10F^1$
$10E^2$	60	1	30	4	0	10^6	$2^3 4^6$	$10D^0 10F^1$
$10F^2$	90	1	45	2	0	$5^6 10^6$	$1^3 2^3 4^9$	$5G^0 10C^0 10F^1 10I^1$
$11A^2$	66	1	66	6	0	11^6	$1^1 5^1 10^6$	$11A^0$
$12A^2$	24	1	4	0	0	12^2	2^2	$4D^0 12A^0 6A^1$
$12B^2$	36	1	3	0	0	$6^2 12^2$	1^3	$4E^0 12C^0 6C^1 12B^1$
$12C^2$	36	1	9	0	0	$6^2 12^2$	$1^3 2^3$	$6G^0 12C^1$
$12D^2$	36	1	9	0	0	$6^2 12^2$	$1^3 2^3$	$6H^0 12B^1 12C^1$
$12E^2$	36	1	9	0	0	$6^2 12^2$	$1^3 2^3$	$12D^0 6C^1 12C^1$
$12F^2$	48	1	4	0	0	$4^3 12^3$	$1^2 2^1$	$6I^0 12B^0$
$12G^2$	48	1	24	0	0	$4^3 12^3$	$1^4 2^6 4^2$	$4F^0 12A^1 12F^1$
$12H^2$	72	1	18	8	0	12^6	$2^5 4^2$	$12H^0 12M^1$
$12I^2$	72	1	18	8	0	12^6	$1^2 2^4 4^2$	$12F^0 12H^0 12J^1 12M^1$
$13A^2$	84	1	14	0	0	$1^6 13^6$	$1^2 12^1$	$13B^0 13C^0$
$14A^2$	42	2	21	2	0	$7^2 14^2$	$2^3 6^6$	$7C^0 14B^1$
$14B^2$	42	2	21	4	0	14^3	6^7	$7D^0 14A^0$
$14C^2$	42	2	21	4	0	14^3	$2^3 6^6$	$14A^0 14B^1$
$14D^2$	48	1	8	0	0	$2^3 14^3$	$1^2 6^1$	$7E^0 14B^0$
$14E^2$	48	1	8	0	0	$2^3 14^3$	$1^2 6^1$	$2C^0 14B^0 14C^1$
$14F^2$	63	1	63	5	0	$7^3 14^3$	$3^3 6^9$	$7D^0 14B^1$
$15A^2$	30	1	15	2	0	15^2	$1^1 2^1 4^1 8^1$	$3C^0 15A^1$
$15B^2$	36	1	6	0	0	$3^2 15^2$	$1^2 4^1$	$5D^0 15B^0$
$15C^2$	40	1	40	0	1	$5^2 15^2$	$2^2 4^5 8^2$	$3B^0 5C^0$
$15D^2$	60	1	30	8	0	15^4	$2^1 4^3 8^2$	$15D^1$
$15E^2$	60	2	60	4	3	15^4	8^{15}	$15A^0$
$16A^2$	24	1	3	0	0	$8^1 16^1$	1^3	$8B^0$
$16B^2$	24	1	12	0	0	$8^1 16^1$	$1^4 2^2 4^1$	$8A^1$
$16C^2$	48	1	3	0	0	$4^4 16^2$	1^3	$8G^0 16C^1$
$16D^2$	48	1	12	0	0	$4^4 16^2$	$1^2 2^1 4^2$	$16E^0 8C^1 16C^1$
$16E^2$	48	1	12	4	0	$8^2 16^2$	2^6	$8H^0 16D^1$
$16F^2$	48	2	12	4	0	$8^2 16^2$	$2^8 4^2$	$8L^0 16B^1 16D^1$
$16G^2$	64	1	64	8	1	16^4	8^8	$8F^0$
$16H^2$	64	2	32	4	4	16^4	8^8	$8M^0 16A^0$
$16I^2$	96	1	12	0	0	$4^8 8^4 16^2$	$1^4 2^4$	$8N^0 16K^1 16L^1$
$16J^2$	96	1	24	0	0	$1^4 2^2 4^2 8^2 16^4$	$1^4 2^2 4^2 8^1$	$16H^0$
$16K^2$	96	1	24	0	0	$2^4 4^2 8^6 16^2$	$1^8 2^4 4^2$	$8O^0$
$16L^2$	96	1	24	4	0	$4^8 16^4$	$2^2 4^5$	$8P^0 16H^1 16I^1$
$18A^2$	18	1	9	0	0	18^1	$1^1 2^1 6^1$	$9A^0 6A^1$
$18B^2$	24	1	4	0	0	$6^1 18^1$	$1^2 2^1$	$6C^0 9A^1$
$18C^2$	24	1	8	0	0	$6^1 18^1$	2^4	$6C^0$
$18D^2$	36	1	12	0	0	$3^1 6^1 9^1 18^1$	$1^6 2^3$	$6F^0 9C^0$
$18E^2$	36	1	12	0	0	$3^1 6^1 9^1 18^1$	$1^6 2^3$	$6F^0 9A^1$
$18F^2$	36	1	18	0	0	$6^3 18^1$	$1^1 2^1 3^1 6^2$	$9E^0 6B^1$
$18G^2$	36	2	9	4	0	18^2	$2^3 6^2$	$6E^0 18A^0 9B^1 18A^1$
$18H^2$	36	2	18	4	0	18^2	$2^6 6^4$	$6E^0 18B^1$
$18I^2$	54	1	9	6	0	$9^2 18^2$	$1^3 2^3$	$6G^0 9D^0 18E^1$
$18J^2$	54	1	27	0	6	18^3	$3^1 6^4$	$2A^0 9F^0$
$18K^2$	54	2	27	8	0	18^3	6^9	$9G^0 18A^0 18A^1$
$18L^2$	54	2	27	8	0	18^3	$2^9 6^6$	$6H^0 18A^0 18E^1$
$18M^2$	54	2	54	8	0	18^3	6^{18}	$18A^0 18B^1$

TABLE 2 (continued).

	I	Z	L	$c2$	$c3$	$Cusps$	Gal	$Super$
$18N^2$	72	1	12	0	6	$6^3 18^3$	$1^2 2^2 6^1$	$9J^0 18B^0$
$18O^2$	72	1	24	0	6	$6^3 18^3$	$1^4 2^4 6^2$	$18B^0$
$18P^2$	108	1	18	0	0	$3^6 6^6 9^2 18^2$	$1^6 2^6$	$6K^0 9H^0 18I^1$
$18Q^2$	108	1	36	0	0	$1^3 2^3 3^2 6^2 9^3 18^3$	$1^6 2^6 6^3$	$9I^0 18E^0$
$19A^2$	57	2	57	5	3	19^3	$6^1 18^6$	$1A^0$
$20A^2$	30	1	15	0	0	$5^2 20^1$	$1^3 4^3$	$4B^0 10B^1$
$20B^2$	30	1	15	2	0	$10^1 20^1$	$1^3 4^3$	$4C^0 10B^1$
$20C^2$	36	1	18	0	0	$2^1 4^1 10^1 20^1$	$1^6 4^3$	$10C^0$
$20D^2$	40	1	20	0	4	20^2	$2^2 8^2$	$4D^0 10A^0 20A^1$
$20E^2$	40	1	40	4	1	20^2	$4^2 8^4$	$4A^0 5C^0$
$20F^2$	60	1	30	4	0	$5^4 20^2$	$2^3 4^6$	$10F^1$
$21A^2$	24	1	8	0	0	$3^1 21^1$	$1^2 6^1$	$3A^0 7B^0$
$21B^2$	28	2	28	0	1	$7^1 21^1$	$2^2 4^1 6^4 12^2$	$3B^0 7A^0$
$21C^2$	42	2	7	6	0	21^2	$2^1 6^2$	$7C^0 21A^0$
$21D^2$	42	2	21	6	0	21^2	$2^1 4^1 6^2 12^2$	$3C^0 21A^0$
$22A^2$	24	1	12	0	0	$2^1 22^1$	$1^2 10^1$	$2A^0 11A^1$
$22B^2$	33	2	33	3	0	$11^1 22^1$	$2^3 10^6$	$2B^0 11A^0$
$22C^2$	36	1	36	0	0	$1^1 2^1 11^1 22^1$	$1^6 10^3$	$2B^0 11A^1$
$23A^2$	24	1	24	0	0	$1^1 23^1$	$1^2 22^1$	$1A^0$
$24A^2$	32	2	16	0	2	$8^1 24^1$	$4^4 8^2$	$8A^0 12A^1$
$24B^2$	36	1	6	0	0	$3^2 6^1 24^1$	$1^4 2^1$	$8C^0 12B^1$
$24C^2$	36	1	9	4	0	$12^1 24^1$	$1^3 2^3$	$12C^0$
$24D^2$	36	1	18	0	0	$3^2 6^1 24^1$	$1^4 2^5 4^1$	$12B^1$
$24E^2$	48	1	8	0	6	24^2	4^2	$8E^0 12E^1$
$24F^2$	48	1	12	0	0	$2^2 6^2 8^1 24^1$	$1^6 2^3$	$12E^0$
$24G^2$	48	1	12	8	0	24^2	$4^2 8^2$	$12F^0 24B^1$
$24H^2$	48	1	12	8	0	24^2	$4^2 8^2$	$12F^0 24B^1$
$24I^2$	48	1	24	0	0	$2^2 6^2 8^1 24^1$	$1^4 2^6 4^2$	$8D^0 12F^1$
$24J^2$	48	1	24	0	6	24^2	4^6	$12H^1$
$24K^2$	48	2	12	8	0	24^2	$4^2 8^2$	$12F^0 24A^1 24B^1$
$24L^2$	72	1	6	12	0	$12^2 24^2$	$1^2 2^2$	$8H^0 24A^0 12J^1 24C^1$
$24M^2$	72	1	12	12	0	$12^2 24^2$	$1^4 2^2 4^1$	$8L^0 24A^0$
$24N^2$	72	1	18	8	0	$6^4 24^2$	$2^5 4^2$	$12H^0 24E^1$
$24O^2$	72	1	18	8	0	$6^4 24^2$	$1^2 2^4 4^2$	$12H^0 24C^1 24E^1$
$24P^2$	72	1	18	12	0	$12^2 24^2$	$1^2 2^4 4^2$	$24A^0 12M^1 24E^1$
$24Q^2$	72	1	36	12	0	$12^2 24^2$	$1^4 2^6 4^3 8^1$	$24A^0$
$25A^2$	30	1	30	2	0	$5^1 25^1$	$2^2 4^4 20^2$	$5B^0$
$25B^2$	30	1	30	2	0	$5^1 25^1$	$2^2 4^4 20^2$	$5B^0$
$25C^2$	30	1	30	2	0	$5^1 25^1$	$2^2 4^4 20^2$	$5B^0$
$25D^2$	30	1	30	2	0	$5^1 25^1$	$2^2 4^4 20^2$	$5B^0$
$25E^2$	50	1	50	2	5	25^2	$2^1 4^2 20^2$	$5C^0$
$25F^2$	75	1	75	7	0	$5^5 25^2$	$2^1 4^2 5^1 20^3$	$5E^0$
$26A^2$	42	1	42	2	0	$1^1 2^1 13^1 26^1$	$1^6 12^3$	$2B^0 13A^0$
$26B^2$	84	1	14	12	0	$2^3 26^3$	$1^2 12^1$	$13C^0 26A^0$
$27A^2$	36	1	12	0	0	$3^3 27^1$	$1^2 2^2 6^1$	$9B^0$
$27B^2$	36	1	12	0	3	$9^1 27^1$	$1^2 2^2 6^1$	$9C^0$
$28A^2$	32	1	32	0	2	$4^1 28^1$	$2^4 12^2$	$4A^0 7B^0$
$28B^2$	42	2	21	4	0	$7^2 28^1$	$2^3 6^6$	$14B^1$
$28C^2$	42	2	21	6	0	$14^1 28^1$	$2^3 6^6$	$4C^0 14B^1$
$28D^2$	48	1	24	0	0	$1^2 4^1 7^2 28^1$	$1^6 6^3$	$4B^0 14C^1$
$28E^2$	56	2	28	8	2	28^2	$4^2 12^4$	$14A^0 28A^1$
$28F^2$	96	1	16	0	12	$4^3 28^3$	$1^4 6^2$	$14C^0 28A^0$
$29A^2$	30	1	30	2	0	$1^1 29^1$	$1^2 28^1$	$1A^0$
$30A^2$	30	1	15	4	0	30^1	$1^1 2^1 4^1 8^1$	$6B^0 15A^1$
$30B^2$	30	2	10	0	3	30^1	$4^1 8^2$	$10A^0$
$30C^2$	36	1	18	4	0	$6^1 30^1$	$1^2 2^2 4^1 8^1$	$15B^0$
$30D^2$	40	1	20	0	4	$10^1 30^1$	$1^2 2^1 4^2 8^1$	$6C^0 10A^0 15B^1$
$30E^2$	54	1	18	6	0	$3^1 6^1 15^1 30^1$	$1^6 4^3$	$6D^0 10C^0 15B^0$
$30F^2$	60	1	10	12	0	30^2	$2^1 4^2$	$10D^0 15D^1$
$31A^2$	32	1	32	0	2	$1^1 31^1$	$1^2 30^1$	$1A^0$
$32A^2$	48	1	6	0	0	$2^4 8^1 32^1$	$1^4 2^1$	$16C^0$
$32B^2$	96	1	12	0	0	$2^8 4^4 32^2$	2^6	$16G^0 32C^1 32D^1$
$32C^2$	96	1	24	0	0	$1^4 2^2 4^6 32^2$	$1^4 2^2 4^2 8^1$	$16H^0$
$35A^2$	35	2	35	3	2	35^1	$2^1 6^2 8^1 24^2$	$5A^0 7A^0$
$35B^2$	40	1	40	0	4	$5^1 35^1$	$1^2 4^2 6^1 24^1$	$5A^0 7B^0$
$35C^2$	42	2	42	6	0	$7^1 35^1$	$2^2 6^4 8^1 24^2$	$5B^0 7A^0$
$36A^2$	48	1	8	0	6	$12^1 36^1$	2^4	$12B^0 18D^1$
$36B^2$	54	1	27	10	0	$18^1 36^1$	$1^3 2^3 6^3$	$12C^0 18E^1$
$36C^2$	54	2	27	8	0	$9^2 36^1$	$2^9 6^6$	$12D^0 18E^1$
$36D^2$	72	2	36	16	0	36^2	$4^6 12^4$	$12F^0 18A^0 36A^1$
$37A^2$	38	1	38	2	2	$1^1 37^1$	$1^2 36^1$	$1A^0$
$38A^2$	40	1	20	0	4	$2^1 38^1$	$1^2 18^1$	$2A^0 19A^1$
$39A^2$	42	1	14	6	0	$3^1 39^1$	$1^2 12^1$	$3A^0 13A^0$
$40A^2$	40	2	20	2	4	40^1	$4^2 16^2$	$8A^0 20A^1$
$42A^2$	42	2	21	8	0	42^1	$2^1 4^1 6^2 12^2$	$21A^0$
$42B^2$	48	1	16	0	6	$6^1 42^1$	$2^2 12^1$	$6A^0 14B^0$
$42C^2$	48	2	8	0	6	$6^1 42^1$	$2^2 12^1$	$14B^0 21A^1$
$44A^2$	44	2	44	6	2	44^1	$4^2 20^4$	$4A^0 11A^0$
$45A^2$	54	1	54	10	0	$9^1 45^1$	$1^2 2^2 4^1 6^2 8^1 24^1$	$9A^0 15B^0$
$48A^2$	72	1	9	16	0	$24^1 48^1$	$1^3 2^3$	$24A^0$
$50A^2$	60	1	30	4	0	$2^5 50^1$	$1^2 4^2 20^1$	$10B^0 25A^0$
$50B^2$	90	1	90	2	0	$1^5 2^5 25^1 50^1$	$1^6 4^6 20^3$	$10C^0 25A^0$
$54A^2$	72	1	12	0	12	$18^1 54^1$	$1^2 2^2 6^1$	$18B^0 27B^1$
$54B^2$	108	1	36	0	0	$1^6 2^6 3^1 6^1 27^1 54^1$	$1^6 2^6 6^3$	$18E^0 27A^0$
$63A^2$	63	2	63	15	0	63^1	$2^1 4^1 6^2 12^3 36^2$	$9A^0 21A^0$
$64A^2$	96	1	24	0	0	$1^8 2^4 16^1 64^1$	$2^8 8^1$	$32A^0$
$78A^2$	84	2	14	12	6	$6^1 78^1$	$2^2 24^1$	$26A^0 39A^1$
$7A^3$	168	1	1	0	0	7^{24}	1^1	$7E^0 7B^1 7C^1$
$8A^3$	96	1	1	0	0	8^{12}	1^1	$8E^1 8F^1 8I^1$
$8B^3$	96	1	3	0	0	8^{12}	1^3	$8N^0 8F^1 8H^1 8A^2 8B^2$
$9A^3$	108	1	36	0	3	9^{12}	$3^2 6^5$	$9F^0 9J^0$
$10A^3$	60	1	10	0	0	10^6	$2^1 4^2$	$2C^0 10C^1 10F^1$
$10B^3$	60	1	15	0	0	10^6	$1^1 2^1 4^3$	$5G^0 10A^1 10C^1 10E^1 10A^2$
$10C^3$	90	1	45	4	0	10^9	$1^3 2^3 4^9$	$10E^1 10I^1$
$10D^3$	120	1	20	0	6	10^{12}	4^5	$10E^0 10H^1 10J^1$
$11A^3$	110	2	55	6	2	11^{10}	10^{11}	$11A^0 11C^1$
$12A^3$	48	1	1	0	0	12^4	1^1	$12B^0 6D^1 12D^1$
$12B^3$	48	1	4	0	0	12^4	2^2	$3D^0 12A^1 12G^1$
$12C^3$	48	1	12	0	0	12^4	$2^2 4^2$	$12F^0 6B^1 12G^1 12A^2$
$12D^3$	72	1	3	0	0	$6^4 12^4$	1^3	$6K^0 12F^1 12L^1 12C^2$
$12E^3$	72	1	9	0	0	$6^4 12^4$	$1^3 2^3$	$6L^0 12K^1 12N^1 12C^2 12D^2$

TABLE 2 (continued).

	I	Z	L	$c2$	$c3$	$Cusps$	Gal	$Super$
$12F^3$	72	1	9	0	0	$6^4 12^4$	$1^3 2^3$	$12G^0 6E^1 12N^1 12C^2 12E^2$
$12G^3$	72	1	9	0	0	$6^4 12^4$	$1^3 2^3$	$12H^0 6E^1 12K^1 12L^1 12B^2 12D^2 12E^2$
$12H^3$	72	1	18	4	0	12^6	$2^3 4^3$	$6L^0 12D^1$
$12I^3$	72	1	18	4	0	12^6	$1^2 2^4 4^2$	$12L^1 12M^1$
$12J^3$	72	1	18	4	0	12^6	$1^2 2^4 4^2$	$12G^1 12J^1 12L^1 12M^1$
$12K^3$	96	1	4	0	0	$4^6 12^6$	$1^2 2^1$	$4G^0 12I^1 12P^1 12G^2$
$12L^3$	96	1	12	0	0	$4^6 12^6$	$1^6 2^3$	$12I^0 12P^1 12F^2$
$12M^3$	96	1	24	8	0	12^8	4^6	$12D^1 12Q^1$
$12N^3$	144	1	3	0	0	$6^{16} 12^4$	1^3	$12I^0 6F^1 12U^1$
$12O^3$	144	1	6	0	0	$3^8 6^4 12^8$	$1^4 2^1$	$12J^0 12S^1$
$12P^3$	144	1	18	8	0	$6^8 12^8$	$2^7 4^1$	$12T^1$
$13A^3$	78	1	78	6	0	13^6	$6^1 12^6$	$1A^0$
$13B^3$	91	1	91	3	4	13^7	$3^1 4^1 12^7$	$1A^0$
$13C^3$	91	1	91	7	1	13^7	$1^1 6^1 12^7$	$1A^0$
$14A^3$	42	1	21	0	0	14^3	$3^1 6^3$	$7D^0 14A^1$
$14B^3$	42	2	7	0	0	14^3	$2^1 6^2$	$2C^0 14A^1 14B^1$
$14C^3$	56	1	28	0	2	14^4	$1^1 3^1 6^4$	$7F^0 14A^1$
$14D^3$	84	1	84	4	0	$7^4 14^4$	$1^3 3^3 6^{12}$	$7F^0 14B^1$
$14E^3$	84	2	14	0	6	14^6	$2^2 6^4$	$14D^1 14F^1$
$14F^3$	84	2	21	8	0	14^6	6^7	$7G^0 14E^1 14B^2$
$15A^3$	60	1	20	0	0	$5^3 15^3$	$1^2 2^1 4^2 8^1$	$5E^0 15B^1$
$15B^3$	60	1	20	0	3	15^4	$4^1 8^2$	$5F^0 15A^0$
$15C^3$	60	1	30	4	0	15^4	$2^1 4^3 8^2$	$3C^0 15D^1$
$15D^3$	60	2	20	0	3	15^4	$2^2 4^1 8^4$	$15A^0 15B^1$
$15E^3$	72	1	6	0	0	$3^4 15^4$	$1^2 4^1$	$3D^0 15C^1 15E^1$
$15F^3$	72	1	18	0	0	$3^4 15^4$	$1^2 2^2 4^1 8^1$	$15C^0 15E^1 15B^2$
$15G^3$	80	1	40	0	2	$5^4 15^4$	$2^2 4^5 8^2$	$5F^0 15B^1 15C^2$
$15H^3$	90	1	45	10	0	15^6	$1^1 2^2 4^4 8^3$	$15F^1$
$15I^3$	144	1	36	8	0	$3^8 15^8$	$2^6 8^3$	$15H^1$
$16A^3$	48	1	3	0	0	$8^2 16^2$	1^3	$8B^1 16B^1$
$16B^3$	48	1	3	0	0	$8^2 16^2$	1^3	$16B^0 8B^1 16A^2$
$16C^3$	48	1	6	0	0	$8^2 16^2$	$1^2 2^2$	$8H^0 16B^1 16A^2$
$16D^3$	48	1	6	0	0	$8^2 16^2$	$1^4 2^1$	$8B^1 16D^1 16B^2$
$16E^3$	48	1	12	0	0	$8^2 16^2$	2^6	$8C^1 16B^2$
$16F^3$	48	1	12	0	0	$8^2 16^2$	$1^4 2^2 4^1$	$8L^0 16D^1 16A^2$
$16G^3$	48	2	24	2	0	16^3	8^6	$16A^0 8D^1$
$16H^3$	96	1	3	0	0	$4^8 16^4$	1^3	$8F^1 16E^1 16H^1$
$16I^3$	96	1	3	0	0	$4^8 16^4$	1^3	$16G^0 8F^1 16I^1 16C^2 16D^2$
$16J^3$	96	1	6	0	0	$4^8 16^4$	$1^4 2^1$	$8O^0 16E^1 16C^2$
$16K^3$	96	1	12	8	0	$8^4 16^4$	$2^2 4^2$	$8H^1 16F^1 16H^1$
$16L^3$	96	1	12	8	0	$8^4 16^4$	$1^2 2^1 4^2$	$8I^1 16F^1 16I^1$
$16M^3$	96	1	12	8	0	$8^4 16^4$	2^6	$8P^0 16F^1 16J^1 16E^2 16F^2$
$16N^3$	96	1	24	0	0	$2^4 4^2 8^2 16^4$	$1^4 2^2 4^2 8^1$	$16G^1$
$16O^3$	96	1	24	0	0	$4^4 8^6 16^2$	$1^8 2^4 4^2$	$8G^1$
$16P^3$	96	1	24	4	0	$8^8 16^2$	$4^2 8^2$	$8H^1$
$16Q^3$	96	1	24	4	0	$8^8 16^2$	$4^2 8^2$	$8H^1 16K^1 16L^1$
$16R^3$	96	1	24	8	0	$8^4 16^4$	$2^8 8^1$	$16J^1$
$16S^3$	96	1	24	8	0	$8^4 16^4$	$2^8 8^1$	$16J^1$
$18A^3$	36	1	3	0	0	18^2	$1^1 2^1$	$9D^0 6B^1 18A^1 18A^2$
$18B^3$	36	1	6	0	0	18^2	2^3	$6B^1 18B^1$
$18C^3$	36	2	9	0	0	18^2	$2^3 6^2$	$18A^0 6B^1 9B^1 18A^2$
$18D^3$	54	1	27	4	0	18^3	$1^3 2^3 6^3$	$6H^0 18A^1 18B^1 18E^1$
$18E^3$	54	2	27	2	0	$9^2 18^2$	$2^9 6^6$	$6G^0 9B^1 18E^1$
$18F^3$	72	1	6	0	0	$6^6 18^2$	$1^2 2^2$	$9H^0 6D^1 18F^1 18F^2$
$18G^3$	72	1	12	0	0	$2^3 6^2 18^3$	$1^2 2^2 6^1$	$9I^0 18C^1$
$18H^3$	72	1	12	0	0	$6^6 18^2$	$1^2 2^5$	$6D^1 18G^1$
$18I^3$	72	1	24	0	0	$2^3 6^2 18^3$	$1^4 2^4 6^2$	$18C^1$
$18J^3$	81	1	81	7	0	$9^3 18^3$	$3^3 6^{12}$	$9G^0 18E^1$
$18K^3$	108	1	54	4	0	$6^9 18^3$	$1^3 2^3 3^3 6^6$	$6L^0 18F^1 18G^1 18I^1$
$20A^3$	48	1	6	0	0	$4^2 20^2$	$1^2 4^1$	$10D^1 20C^1$
$20B^3$	48	1	24	0	0	$4^2 20^2$	$2^4 8^2$	$5D^0 20B^1$
$20C^3$	48	1	24	0	0	$4^2 20^2$	$2^4 8^2$	$4D^0 10A^1 20B^1$
$20D^3$	60	1	20	6	0	20^3	$2^2 8^2$	$5E^0 20A^1$
$20E^3$	60	1	30	0	0	$5^4 20^2$	$2^3 4^6$	$4B^0 10F^1$
$20F^3$	60	1	30	4	0	$10^2 20^2$	$2^3 4^6$	$4C^0 10F^1$
$20G^3$	72	1	18	0	0	$2^2 4^2 10^2 20^2$	$1^6 4^3$	$10F^0 20E^1 20C^2$
$20H^3$	72	1	18	0	0	$2^2 4^2 10^2 20^2$	$1^6 4^3$	$10G^0 20D^1 20C^2$
$20I^3$	72	1	18	0	0	$2^2 4^2 10^2 20^2$	$1^6 4^3$	$20A^0 10G^1 20C^2$
$20J^3$	72	1	18	0	0	$2^2 4^2 10^2 20^2$	$1^6 4^3$	$4E^0 10G^1 20D^1 20E^1$
$20K^3$	72	1	36	4	0	$4^3 20^3$	$1^4 2^4 4^2 8^2$	$20E^1$
$20L^3$	72	1	36	4	0	$4^3 20^3$	$1^4 2^4 4^2 8^2$	$4F^0 20B^1 20E^1$
$20M^3$	72	2	18	4	0	$4^3 20^3$	$2^6 8^3$	$10G^0 20C^1$
$20N^3$	80	1	10	0	8	20^4	$2^1 4^2$	$10H^1 20F^1$
$20O^3$	80	1	40	8	2	20^4	$4^2 8^4$	$10D^0 20E^2$
$20P^3$	90	1	45	4	0	$5^6 20^3$	$1^3 2^3 4^9$	$10I^1$
$20Q^3$	96	2	24	8	0	$4^4 20^4$	$2^8 8^4$	$20C^1 20G^1$
$20R^3$	144	1	18	0	0	$2^8 4^2 10^8 20^2$	$1^6 4^3$	$10K^1 20J^1$
$20S^3$	144	1	36	0	0	$1^4 2^2 4^4 5^4 10^2 20^4$	$1^8 2^2 4^4 8^1$	$20H^1$
$20T^3$	144	1	36	8	0	$2^4 4^4 10^4 20^4$	$1^8 2^2 4^4 8^1$	$20I^1$
$21A^3$	48	1	24	0	0	$3^2 21^2$	$1^2 2^2 6^1 12^1$	$3C^0 21A^2$
$21B^3$	56	2	28	0	2	$7^2 21^2$	$2^2 4^1 6^4 12^2$	$7C^0 21B^2$
$21C^3$	84	1	28	12	0	21^4	$1^1 3^1 6^4$	$7F^0 21A^0$
$21D^3$	96	1	32	0	0	$1^3 3^3 7^3 21^3$	$1^4 2^2 6^2 12^1$	$7E^0 21B^1$
$24A^3$	36	1	3	0	0	$12^1 24^1$	1^3	$12C^0 8A^1$
$24B^3$	48	1	12	0	0	$4^1 8^1 12^1 24^1$	$1^6 2^3$	$8B^0 12F^1$
$24C^3$	48	1	12	0	0	$4^1 8^1 12^1 24^1$	$1^6 2^3$	$8A^1 12F^1$
$24D^3$	48	2	12	4	0	24^2	$4^2 8^2$	$12G^1 24B^1$
$24E^3$	48	2	12	4	0	24^2	$4^2 8^2$	$12G^1 24A^1 24B^1$
$24F^3$	64	1	16	0	4	$8^2 24^2$	$2^4 4^2$	$12I^1$
$24G^3$	64	1	16	0	4	$8^2 24^2$	$2^4 4^2$	$8E^0 12I^1 24A^2$
$24H^3$	72	1	9	8	0	$12^2 24^2$	$1^3 2^3$	$12L^1 24D^1$
$24I^3$	72	1	9	8	0	$12^2 24^2$	$1^3 2^3$	$24A^0 12L^1 24C^2$
$24J^3$	72	1	12	6	0	$12^4 24^1$	$2^2 4^2$	$8K^0 12J^1$
$24K^3$	72	1	18	0	0	$3^4 6^2 24^2$	$1^4 2^5 4^1$	$12K^1 24D^2$
$24L^3$	72	1	18	0	0	$3^4 6^2 24^2$	$1^4 2^5 4^1$	$12K^1 24B^2 24D^2$
$24M^3$	72	1	18	4	0	$6^4 24^2$	$2^3 4^3$	$12G^0$
$24N^3$	72	1	18	4	0	$6^4 24^2$	$1^2 2^4 4^2$	$12L^1 24E^1$
$24O^3$	72	1	18	4	0	$6^4 24^2$	$1^2 2^4 4^2$	$12L^1 24C^1 24E^1$
$24P^3$	72	1	18	8	0	$12^2 24^2$	$1^2 2^4 4^2$	$12J^1 24D^1 24E^1$
$24Q^3$	72	1	18	8	0	$12^2 24^2$	$1^2 2^4 4^2$	$12M^1 24C^1 24D^1$
$24R^3$	72	1	36	6	0	$12^4 24^1$	$2^2 4^4 8^2$	$12J^1$
$24S^3$	72	2	9	8	0	$12^2 24^2$	2^9	$12H^0 24D^1 24C^2$
$24T^3$	72	2	18	8	0	$12^2 24^2$	$2^6 4^6$	$12M^1 24D^1 24E^1$
$24U^3$	96	1	12	0	0	$2^4 6^4 8^2 24^2$	$1^6 2^3$	$24B^0 12P^1 24F^2$
$24V^3$	96	1	12	0	0	$2^4 6^4 8^2 24^2$	$1^6 2^3$	$8G^0 12P^1 24G^1 24I^2$
$24W^3$	96	1	24	0	0	$2^4 6^4 8^2 24^2$	$1^8 2^6 4^1$	$12J^0 24G^1 24F^2$
$24X^3$	96	1	48	0	0	$1^2 2^1 3^2 4^1 6^1 8^2 12^1 24^2$	$1^8 2^8 4^4 8^1$	$24G^1$
$24Y^3$	96	1	48	0	0	$1^2 2^1 3^2 4^1 6^1 8^2 12^1 24^2$	$1^8 2^8 4^4 8^1$	$8I^0 24G^1$
$24Z^3$	96	1	48	0	0	$2^2 4^3 6^2 8^1 12^3 24^1$	$1^{16} 2^{12} 4^2$	$12P^1$
$24AA^3$	96	1	48	0	0	$2^2 4^3 6^2 8^1 12^3 24^1$	$1^{16} 2^{12} 4^2$	$8J^0 12P^1$
$24AB^3$	96	1	48	16	0	24^4	$4^4 8^4$	$12F^0 24F^1$
$24AC^3$	144	1	3	0	0	$3^{16} 24^4$	1^3	$24B^0 12S^1 24I^1$
$26A^3$	84	1	28	0	6	$2^3 26^3$	$1^4 12^2$	$26A^1$
$27A^3$	36	1	12	0	0	$9^1 27^1$	$1^2 2^2 6^1$	$9C^0$
$27B^3$	81	2	81	13	0	27^3	$6^9 18^6$	$9G^0$
$27C^3$	108	1	36	0	3	$3^9 27^3$	$3^2 6^5$	$9J^0$
$28A^3$	42	2	21	0	0	$7^2 28^1$	$2^3 6^6$	$4B^0 14B^1$
$28B^3$	42	2	21	2	0	$14^1 28^1$	$2^3 6^6$	$14B^1$
$28C^3$	48	1	24	0	0	$2^1 4^1 14^1 28^1$	$1^6 6^3$	$4C^0 14C^1$
$28D^3$	56	2	28	4	2	28^2	$4^2 12^4$	$7C^0 28A^1$
$28E^3$	64	1	32	0	4	$4^2 28^2$	$2^4 12^2$	$4D^0 14B^0 28A^2$
$30A^3$	30	1	5	0	0	30^1	$1^1 4^1$	$10A^0 6A^1 15A^1$
$30B^3$	36	1	6	0	0	$6^1 30^1$	$1^2 4^1$	$15B^0 6A^1 10A^1$
$30C^3$	36	1	12	0	0	$6^1 30^1$	$2^2 8^1$	$6A^0 10A^1$

TABLE 2 (continued).

	I	Z	L	c2	c3	*Cusps*	*Gal*	*Super*
$30D^3$	45	1	15	3	0	$15^1 30^1$	$1^3 4^3$	$6D^0 10B^1 15A^1$
$30E^3$	45	2	15	3	0	$15^1 30^1$	$2^3 8^3$	$15A^0 10B^1$
$30F^3$	48	1	24	0	0	$2^1 6^1 10^1 30^1$	$1^4 2^2 4^2 8^1$	$10B^0 15C^1$
$30G^3$	48	1	24	0	0	$2^1 6^1 10^1 30^1$	$1^4 2^2 4^2 8^1$	$6C^0 10A^1 15C^1$
$30H^3$	60	1	30	8	0	30^2	$2^1 4^3 8^2$	$6B^0 15D^1$
$30I^3$	72	1	18	8	0	$6^2 30^2$	$1^2 2^2 4^1 8^1$	$6E^0 15E^1 30C^1$
$30J^3$	72	1	18	8	0	$6^2 30^2$	$1^2 2^2 4^1 8^1$	$30A^0 15E^1 30C^2$
$30K^3$	72	1	72	0	0	$1^1 2^1 3^1 5^1 6^1 10^1 15^1 30^1$	$1^{12} 2^6 4^6 8^3$	$6F^0 10C^0 15C^1$
$30L^3$	72	2	18	8	0	$6^2 30^2$	$2^6 8^3$	$15C^0 30C^1 30C^2$
$32A^3$	48	1	6	0	0	$4^2 8^1 32^1$	$1^4 2^1$	$16A^1$
$32B^3$	48	1	6	0	0	$4^2 8^1 32^1$	$1^4 2^1$	$16A^1$
$32C^3$	48	1	12	0	0	$4^2 8^1 32^1$	$1^4 2^2 4^1$	$16A^1$
$32D^3$	48	1	12	4	0	$16^1 32^1$	2^6	$16B^0$
$32E^3$	48	1	24	4	0	$16^1 32^1$	$2^8 8^1$	$16D^1$
$32F^3$	48	1	24	4	0	$16^1 32^1$	$2^8 8^1$	$16D^1$
$32G^3$	48	1	24	4	0	$16^1 32^1$	$2^8 8^1$	$16D^1$
$32H^3$	48	1	24	4	0	$16^1 32^1$	$2^8 8^1$	$16D^1$
$32I^3$	48	2	24	2	0	$8^2 32^1$	$2^2 4^3 8^4$	$16C^1$
$32J^3$	96	1	6	0	0	$2^8 8^2 32^2$	$1^4 2^1$	$16E^1 32A^1$
$32K^3$	96	1	6	0	0	$2^8 8^2 32^2$	$1^4 2^1$	$32A^0 16E^1 32A^2$
$32L^3$	96	1	6	16	0	$16^2 32^2$	$1^2 2^2$	$16F^1 32B^1$
$32M^3$	96	1	12	0	0	$2^8 8^2 32^2$	$1^4 2^2 4^1$	$16H^0 32A^1 32A^2$
$32N^3$	96	1	12	16	0	$16^2 32^2$	$1^4 2^2 4^1$	$16J^1 32B^1$
$32O^3$	96	1	24	0	0	$2^4 4^6 32^2$	$1^4 2^2 4^2 8^1$	$16G^1$
$32P^3$	96	1	24	4	0	$4^8 32^2$	$1^2 2^1 4^1 8^2$	$16I^1$
$32Q^3$	96	1	24	4	0	$4^8 32^2$	$2^4 8^2$	$16I^1 32C^1 32D^1$
$33A^3$	36	1	12	0	0	$3^1 33^1$	$1^2 10^1$	$3A^0 11A^1$
$33B^3$	44	2	44	0	2	$11^1 33^1$	$2^2 4^1 10^4 20^2$	$3B^0 11A^0$
$33C^3$	48	1	48	0	0	$1^1 3^1 11^1 33^1$	$1^4 2^2 10^2 20^1$	$3B^0 11A^1$
$34A^3$	36	1	18	0	0	$2^1 34^1$	$1^2 16^1$	$17A^1$
$34B^3$	36	1	18	0	0	$2^1 34^1$	$1^2 16^1$	$2A^0 17A^1$
$34C^3$	54	1	54	2	0	$1^1 2^1 17^1 34^1$	$1^6 16^3$	$2B^0 17A^1$
$35A^3$	48	1	48	0	0	$1^1 5^1 7^1 35^1$	$1^4 4^2 6^2 24^1$	$5B^0 7B^0$
$36A^3$	48	1	4	0	0	$4^3 36^1$	$1^2 2^1$	$12B^0 18C^1$
$36B^3$	48	1	16	0	0	$4^3 36^1$	$2^4 4^2$	$9B^0 12A^1$
$36C^3$	48	1	16	0	3	$12^1 36^1$	$2^4 4^2$	$9C^0 12A^1$
$36D^3$	54	1	27	4	0	$9^2 36^1$	$1^3 2^3 6^3$	$12D^0 18E^1$
$36E^3$	54	1	27	6	0	$18^1 36^1$	$1^3 2^3 6^3$	$12C^1 18E^1$
$36F^3$	72	1	6	12	0	36^2	$2^1 4^1$	$18D^0 12D^1$
$36G^3$	72	1	12	0	0	$2^3 4^3 18^1 36^1$	$1^6 2^3$	$18E^0 12F^1$
$36H^3$	72	1	12	12	0	36^2	$2^2 4^2$	$9D^0 12G^1 36A^1$
$36I^3$	108	1	54	4	0	$3^6 9^2 12^3 36^1$	$1^3 2^3 3^3 6^6$	$12G^0 18I^1$
$36J^3$	144	1	12	0	0	$2^{12} 4^3 18^4 36^1$	$1^6 2^3$	$12I^0 18J^1$
$36K^3$	144	1	24	0	0	$1^6 2^3 4^6 9^2 18^1 36^2$	$1^8 2^6 4^1$	$12J^0 36C^1$
$39A^3$	56	1	56	0	2	$1^1 3^1 13^1 39^1$	$1^4 2^2 12^2 24^1$	$3B^0 13A^0$
$40A^3$	48	2	24	4	0	$8^1 40^1$	$4^4 16^2$	$20B^1$
$40B^3$	48	2	24	4	0	$8^1 40^1$	$4^4 16^2$	$8A^0 20B^1$
$40C^3$	72	1	18	8	0	$4^1 8^1 20^1 40^1$	$1^6 4^3$	$20E^1$
$40D^3$	72	1	18	8	0	$4^1 8^1 20^1 40^1$	$1^6 4^3$	$8B^0 20E^1$
$40E^3$	72	1	36	0	0	$1^2 2^1 5^2 8^1 10^1 40^1$	$1^8 2^2 4^4 8^1$	$20D^1$
$40F^3$	72	1	36	0	0	$1^2 2^1 5^2 8^1 10^1 40^1$	$1^8 2^2 4^4 8^1$	$8C^0 20D^1$
$40G^3$	72	1	36	4	0	$2^2 8^1 10^2 40^1$	$1^4 2^4 4^2 8^2$	$20E^1$
$40H^3$	72	1	36	4	0	$2^2 8^1 10^2 40^1$	$1^4 2^4 4^2 8^2$	$8D^0 20E^1$
$40I^3$	72	2	18	4	0	$2^2 8^1 10^2 40^1$	$2^6 8^3$	$20A^0$
$40J^3$	144	1	18	0	0	$1^8 5^8 8^2 40^2$	$1^6 4^3$	$20H^1 40A^1$
$41A^3$	42	1	42	2	0	$1^1 41^1$	$1^2 40^1$	$1A^0$
$42A^3$	42	2	14	0	3	42^1	$4^1 12^2$	$6A^0 14A^1$
$42B^3$	48	2	16	0	3	$6^1 42^1$	$2^4 12^2$	$14B^0$
$42C^3$	63	2	21	9	0	$21^1 42^1$	$2^3 6^6$	$6D^0 21A^0 14B^1$

	I	Z	L	c2	c3	*Cusps*	*Gal*	*Super*
$42D^3$	64	1	32	0	4	$2^1 6^1 14^1 42^1$	$1^4 2^2 6^2 12^1$	$21B^1$
$42E^3$	64	1	32	0	4	$2^1 6^1 14^1 42^1$	$1^4 2^2 6^2 12^1$	$6C^0 14B^0 21B^1$
$42F^3$	84	2	21	16	0	42^2	$2^1 4^1 6^2 12^2$	$21D^1 42A^1 42B^1$
$43A^3$	44	1	44	0	2	$1^1 43^1$	$1^2 42^1$	$1A^0$
$45A^3$	45	1	45	5	0	45^1	$1^1 2^1 4^1 6^1 8^1 24^1$	$9A^0 15A^1$
$45B^3$	60	1	20	0	6	$15^1 45^1$	$1^2 2^1 4^2 8^1$	$9C^0 15B^1$
$45C^3$	60	2	20	0	3	$5^3 45^1$	$2^2 4^1 8^4$	$15B^1$
$45D^3$	72	1	24	0	0	$1^3 5^3 9^1 45^1$	$1^4 2^2 4^2 8^1$	$9B^0 15C^1$
$48A^3$	48	2	8	6	0	48^1	8^2	$16A^0 24A^1$
$48B^3$	48	2	24	6	0	48^1	$8^2 16^2$	$24A^1$
$48C^3$	72	1	6	12	0	$24^1 48^1$	$1^2 2^2$	$24A^0 16B^1$
$48D^3$	72	1	12	6	0	$6^4 48^1$	$1^2 2^1 4^2$	$16E^0 24C^1$
$48E^3$	72	1	12	12	0	$24^1 48^1$	$1^4 2^2 4^1$	$24A^0 16D^1$
$48F^3$	72	1	18	12	0	$24^1 48^1$	$1^2 2^4 4^2$	$24A^0$
$48G^3$	72	1	36	6	0	$6^4 48^1$	$1^2 2^3 4^3 8^2$	$24C^1$
$48H^3$	72	1	36	12	0	$24^1 48^1$	$1^4 2^6 4^3 8^1$	$24A^0$
$48I^3$	96	1	24	0	0	$1^4 3^4 4^1 12^1 16^1 48^1$	$1^8 2^6 4^1$	$24G^1$
$48J^3$	96	1	24	0	0	$1^4 3^4 4^1 12^1 16^1 48^1$	$1^8 2^6 4^1$	$16C^0 24G^1$
$48K^3$	96	1	48	0	0	$1^2 2^3 3^2 6^3 16^1 48^1$	$1^8 2^8 4^4 8^1$	$24G^1$
$48L^3$	96	1	48	0	0	$1^2 2^3 3^2 6^3 16^1 48^1$	$1^8 2^8 4^4 8^1$	$16D^0 24G^1$
$48M^3$	144	1	9	32	0	$24^2 48^2$	$1^3 2^3$	$48A^0 24H^1 48A^2$
$49A^3$	168	1	8	0	0	$1^{21} 49^3$	$1^2 6^1$	$7E^0 49A^1$
$50A^3$	60	1	30	0	0	$2^5 50^1$	$1^2 4^2 20^1$	$25A^0 10A^1$
$51A^3$	54	1	18	6	0	$3^1 51^1$	$1^2 16^1$	$3A^0 17A^1$
$52A^3$	56	1	56	4	2	$4^1 52^1$	$2^4 24^2$	$4A^0 13A^0$
$52B^3$	112	1	14	0	16	$4^2 52^2$	$1^2 12^1$	$26B^1 52A^1$
$54A^3$	72	1	12	0	0	$2^6 6^1 54^1$	$1^2 2^2 6^1$	$27A^0 18C^1$
$54B^3$	72	1	24	0	0	$2^6 6^1 54^1$	$2^6 6^2$	$18C^1$
$54C^3$	72	1	24	0	9	$18^1 54^1$	$2^6 6^2$	$18B^0$
$55A^3$	55	2	55	3	4	55^1	$2^1 8^1 10^2 40^2$	$5A^0 11A^0$
$56A^3$	56	2	28	6	2	56^1	$8^2 24^4$	$28A^1$
$56B^3$	56	2	28	6	2	56^1	$8^2 24^4$	$28A^1$
$56C^3$	56	2	28	6	2	56^1	$8^2 24^4$	$8A^0 28A^1$
$56D^3$	56	2	28	6	2	56^1	$8^2 24^4$	$8A^0 28A^1$
$60A^3$	60	2	20	6	3	60^1	$4^2 16^2$	$15A^0 20A^1$
$60B^3$	72	1	24	12	0	$12^1 60^1$	$2^4 8^2$	$12A^0 15B^0 20B^1$
$60C^3$	72	2	6	12	0	$12^1 60^1$	$2^2 8^1$	$30A^0 20C^1$
$60D^3$	72	2	18	12	0	$12^1 60^1$	$2^2 4^2 8^1 16^1$	$30A^0$
$64A^3$	96	1	12	0	0	$1^8 4^2 16^1 64^1$	$1^4 2^2 4^1$	$32A^1$
$64B^3$	96	1	12	0	0	$1^8 4^2 16^1 64^1$	$1^4 2^2 4^1$	$32A^1$
$66A^3$	66	2	33	12	0	66^1	$2^1 4^1 10^2 20^2$	$6B^0 33A^1$
$72A^3$	144	1	12	0	0	$1^{12} 8^3 9^4 72^1$	$1^6 2^3$	$24B^0 36C^1$
$84A^3$	84	2	28	18	0	84^1	$4^2 12^4$	$12A^0 21A^0 28A^1$
$96A^3$	144	1	12	36	0	$48^1 96^1$	2^6	$48A^0 32B^1$

TABLE 2 (continued).

	MatrixGenerators
$16K^1$	[[11, 2, 5, 1], [3, 8, 12, 11], [9, 0, 8, 9], [15, 0, 0, 15], [3, 14, 5, 13], [5, 0, 12, 13]]
$16L^1$	[[11, 2, 5, 1], [3, 8, 12, 11], [9, 0, 8, 9], [7, 4, 0, 7], [15, 0, 0, 15], [5, 0, 12, 13]]
$32C^1$	[[1, 0, 16, 1], [1, 0, 8, 1], [15, 0, 16, 15], [29, 24, 8, 21], [25, 2, 23, 7], [1, 0, 4, 1], [17, 0, 0, 17], [1, 0, 2, 1], [9, 16, 16, 25]]
$32D^1$	[[1, 0, 16, 1], [1, 0, 8, 1], [15, 0, 16, 15], [29, 24, 8, 21], [25, 18, 25, 27], [25, 2, 23, 7], [1, 0, 4, 1], [17, 0, 0, 17], [9, 16, 16, 25]]
$24G^2$	[[7, 12, 12, 7], [13, 3, 3, 10], [0, 11, 13, 12], [13, 12, 0, 13], [16, 19, 13, 8], [13, 20, 20, 5], [17, 0, 0, 17], [7, 0, 0, 7], [13, 0, 12, 13]]
$24H^2$	[[7, 12, 12, 7], [1, 16, 4, 17], [13, 3, 3, 10], [0, 11, 13, 12], [13, 12, 0, 13], [5, 20, 20, 13], [17, 0, 0, 17], [7, 0, 0, 7], [13, 0, 12, 13]]
$25A^2$	[[14, 9, 24, 3], [1, 2, 24, 24], [16, 5, 10, 11], [24, 0, 0, 24], [11, 0, 10, 16]]
$25B^2$	[[1, 2, 24, 24], [16, 5, 10, 11], [0, 22, 17, 17], [24, 0, 0, 24], [11, 0, 10, 16]]
$25C^2$	[[1, 2, 24, 24], [16, 5, 10, 11], [24, 0, 0, 24], [12, 23, 3, 10], [11, 0, 10, 16]]
$25D^2$	[[9, 4, 4, 13], [1, 2, 24, 24], [16, 5, 10, 11], [24, 0, 0, 24], [11, 0, 10, 16]]
$16R^3$	[[9, 10, 11, 7], [9, 14, 9, 7], [15, 0, 0, 15], [1, 8, 4, 1], [1, 0, 8, 1]]
$16S^3$	[[9, 14, 9, 7], [15, 0, 0, 15], [1, 0, 8, 1], [1, 8, 4, 1], [3, 12, 14, 3]]
$32A^3$	[[1, 0, 16, 1], [17, 0, 16, 17], [7, 24, 9, 31], [1, 0, 24, 1], [25, 16, 26, 9], [1, 0, 12, 1], [17, 16, 6, 17], [31, 0, 28, 31], [29, 24, 31, 29]]
$32B^3$	[[1, 0, 16, 1], [19, 8, 27, 3], [17, 0, 16, 17], [9, 8, 25, 1], [1, 0, 24, 1], [25, 16, 26, 9], [1, 0, 12, 1], [17, 16, 6, 17], [31, 0, 28, 31]]
$32E^3$	[[25, 16, 24, 9], [1, 0, 16, 1], [17, 0, 16, 17], [17, 16, 8, 17], [25, 24, 4, 9], [27, 6, 5, 13], [11, 8, 12, 3], [5, 4, 30, 5], [31, 0, 0, 31]]
$32F^3$	[[25, 16, 24, 9], [1, 0, 16, 1], [31, 4, 6, 7], [17, 0, 16, 17], [17, 16, 8, 17], [25, 24, 4, 9], [27, 6, 5, 13], [11, 8, 12, 3], [31, 0, 0, 31]]
$32G^3$	[[1, 0, 16, 1], [25, 24, 4, 9], [25, 10, 19, 23], [31, 0, 0, 31], [17, 16, 8, 17], [31, 4, 6, 7], [17, 0, 16, 17], [25, 16, 24, 9], [11, 8, 4, 3]]
$32H^3$	[[25, 16, 24, 9], [1, 0, 16, 1], [3, 10, 15, 29], [31, 4, 14, 7], [17, 0, 16, 17], [17, 16, 8, 17], [25, 24, 4, 9], [11, 8, 4, 3], [31, 0, 0, 31]]
$56A^3$	[[25, 24, 20, 17], [43, 49, 14, 29], [36, 21, 7, 43], [1, 28, 28, 1], [33, 8, 16, 9], [49, 8, 48, 41], [15, 0, 0, 15], [17, 24, 48, 25], [29, 0, 28, 29], [41, 0, 0, 41]]
$56B^3$	[[17, 24, 48, 25], [36, 21, 7, 43], [49, 8, 48, 41], [41, 0, 0, 41], [1, 28, 28, 1], [15, 0, 0, 15], [29, 0, 28, 29], [11, 17, 6, 45], [43, 49, 42, 1], [33, 8, 16, 9]]
$56C^3$	[[17, 24, 48, 25], [36, 21, 7, 43], [49, 8, 48, 41], [41, 0, 0, 41], [25, 24, 48, 17], [1, 28, 28, 1], [15, 0, 0, 15], [29, 0, 28, 29], [43, 49, 42, 1], [33, 8, 16, 9]]
$56D^3$	[[17, 24, 48, 25], [36, 21, 7, 43], [49, 8, 48, 41], [41, 0, 0, 41], [25, 24, 48, 17], [1, 28, 28, 1], [15, 0, 0, 15], [43, 49, 14, 29], [29, 0, 28, 29], [33, 8, 16, 9]]
$64A^3$	[[1, 0, 50, 1], [17, 0, 28, 49], [33, 0, 32, 33], [1, 0, 36, 1], [1, 0, 56, 1], [1, 0, 48, 1], [1, 0, 32, 1], [41, 0, 46, 25], [63, 0, 2, 63], [29, 0, 15, 53], [1, 0, 57, 1]]
$64B^3$	[[1, 0, 50, 1], [17, 0, 28, 49], [33, 0, 32, 33], [1, 0, 36, 1], [1, 0, 56, 1], [1, 0, 48, 1], [1, 0, 32, 1], [41, 0, 46, 25], [63, 0, 2, 63], [61, 32, 15, 53], [1, 0, 57, 1]]

TABLE 3. Generators in $\mathrm{PSL}(2, \mathbb{Z}/m\mathbb{Z})$, where m is the level, for those groups which are not uniquely specified by the information in Table 2.

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$1A^0$ $\overline{\Gamma}$		$21B^1$ $\overline{\Gamma}_0(21)$	$37A^2$ $\overline{\Gamma}_0(37)$
	$12E^0$ $\overline{\Gamma}_0(12)$		
$2A^0$ $\overline{\Gamma}^2$	$12J^0$ $\overline{\Gamma}_1(12)$	$24G^1$ $\overline{\Gamma}_0(24)$	$50B^2$ $\overline{\Gamma}_0(50)$
$2B^0$ $\overline{\Gamma}_0(2)$ $\overline{\Gamma}_1(2)$			
$2C^0$ $\overline{\Gamma}(2)$	$13A^0$ $\overline{\Gamma}_0(13)$	$27A^1$ $\overline{\Gamma}_0(27)$	$7A^3$ $\overline{\Gamma}(7)$
$3A^0$ $\overline{\Gamma}^3$	$16C^0$ $\overline{\Gamma}_0(16)$	$32A^1$ $\overline{\Gamma}_0(32)$	$20S^3$ $\overline{\Gamma}_1(20)$
$3B^0$ $\overline{\Gamma}_0(3)$ $\overline{\Gamma}_1(3)$			
$3D^0$ $\overline{\Gamma}(3)$	$18E^0$ $\overline{\Gamma}_0(18)$	$36C^1$ $\overline{\Gamma}_0(36)$	$30K^3$ $\overline{\Gamma}_0(30)$
$4B^0$ $\overline{\Gamma}_0(4)$ $\overline{\Gamma}_1(4)$	$25A^0$ $\overline{\Gamma}_0(25)$	$49A^1$ $\overline{\Gamma}_0(49)$	$33C^3$ $\overline{\Gamma}_0(33)$
$4G^0$ $\overline{\Gamma}(4)$			
	$6A^1$ $\overline{\Gamma}'$	$13A^2$ $\overline{\Gamma}_1(13)$	$34C^3$ $\overline{\Gamma}_0(34)$
$5B^0$ $\overline{\Gamma}_0(5)$	$6F^1$ $\overline{\Gamma}(6)$		
$5D^0$ $\overline{\Gamma}_1(5)$		$16J^2$ $\overline{\Gamma}_1(16)$	$35A^3$ $\overline{\Gamma}_0(35)$
$5H^0$ $\overline{\Gamma}(5)$	$11A^1$ $\overline{\Gamma}_0(11)$		
	$11D^1$ $\overline{\Gamma}_1(11)$	$18Q^2$ $\overline{\Gamma}_1(18)$	$39A^3$ $\overline{\Gamma}_0(39)$
$6F^0$ $\overline{\Gamma}_0(6)$ $\overline{\Gamma}_1(6)$			
	$14C^1$ $\overline{\Gamma}_0(14)$	$22C^2$ $\overline{\Gamma}_0(22)$	$40F^3$ $\overline{\Gamma}_0(40)$
$7B^0$ $\overline{\Gamma}_0(7)$	$14H^1$ $\overline{\Gamma}_1(14)$		
$7E^0$ $\overline{\Gamma}_1(7)$		$23A^2$ $\overline{\Gamma}_0(23)$	$41A^3$ $\overline{\Gamma}_0(41)$
	$15C^1$ $\overline{\Gamma}_0(15)$		
$8C^0$ $\overline{\Gamma}_0(8)$	$15I^1$ $\overline{\Gamma}_1(15)$	$26A^2$ $\overline{\Gamma}_0(26)$	$43A^3$ $\overline{\Gamma}_0(43)$
$8I^0$ $\overline{\Gamma}_1(8)$			
	$17A^1$ $\overline{\Gamma}_0(17)$	$28D^2$ $\overline{\Gamma}_0(28)$	$45D^3$ $\overline{\Gamma}_0(45)$
$9B^0$ $\overline{\Gamma}_0(9)$			
$9I^0$ $\overline{\Gamma}_1(9)$	$19A^1$ $\overline{\Gamma}_0(19)$	$29A^2$ $\overline{\Gamma}_0(29)$	$48J^3$ $\overline{\Gamma}_0(48)$
$10C^0$ $\overline{\Gamma}_0(10)$	$20D^1$ $\overline{\Gamma}_0(20)$	$31A^2$ $\overline{\Gamma}_0(31)$	$64A^3$ $\overline{\Gamma}_0(64)$
$10F^0$ $\overline{\Gamma}_1(10)$			

TABLE 4. Standard names for some of the groups of Table 2.

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C. J. Cummins, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec, Canada (cummins@mathstat.concordia.ca)

S. Pauli, Technische Universität Berlin, Institut für Mathematik - MA 8-1, Strasse des 17. Juni 136, Berlin 10623, Germany (pauli@math.tu-berlin.de)

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