Gen. Math. Notes, Vol. 25, No. 2, December 2014, pp. 137-147
ISSN 2219-7184; Copyright © ICSRS Publication, 2014
www.i-csrs.org
Available free online at http://www.geman.in

Seasonal Evaluation of Hydro-Geochemical Parameters using Goal Programming with Multiple Nonlinear Regression

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(Received: 30-3-13 / Accepted: 14-10-14)

Abstract

Goal programming is regarded as a proven valuable mathematical programming form at a number of places. Numerous regression models can also be used to more accurately combine multiple criteria measures that can be used in GP model parameters. Those parameters can include the relative weighting and the goal constraint parameters. Multiple regression techniques were used to develop models relating to water chemical quality parameters to a set of independent chemical variables. This paper mainly focuses on predicting chemical quality parameters to a set of independent chemical variables in post and pre-monsoon seasons in the Todaraishing area of Tonk district, Rajasthan

(India) by using goal programming model to solve the multiple regression equation with an aid of MS-Excel and TORA computer software packages.

Keywords: *Goal Programming, Multiple Regression, Hydro-geochemical Parameters.*

1 Introduction

Linear regression is the oldest and most widely used predictive model in biological, behavioral and social sciences, used to describe possible relationships between variables. Linear regression finds relevance in a wide range of environmental science applications. The earliest form of regression was the method of least squares, which was published by Legendre [1805] and by Gauss [1809]. Multiple linear regression (MLR) analysis is a statistical tool for understanding the relationship between two or more variables. The main idea of a MLR analysis is to understand the relationship between several independent variables and a single dependent variable. A number of methods, for the estimation of the regression parameters, are available in the literature. These include methods of minimizing the sum of absolute residuals, minimizing the maximum of absolute residuals and minimizing the sum of squares of residuals [1985], where the last method of minimizing the sum of squares of residuals popularly known as least square method, is commonly used.

Goal programming has proven a valuable tool in support of decision making. The first publication using GP was the form of a “constrained” regression model, used by Charnes et. al. [1955]. There have been many books devoted to this topic over past years (Ijiri [1965]; Lee [1972]; Spronk [1981]; Ignizio [1986] and others). This tool often represents a substantial improvement in the modeling and analysis of multi-objective problems (Charnes and Cooper [1977]; Eiselt et al. [1987]; Ignizio [1978]). By minimizing deviation, the GP model can generate decision variable values that are equivalent to the beta values in some types of multiple regression models.

Regression analysis is extensively used in geochemistry. It is useful for interpreting commonly collected groundwater quality data and relating them to specific hydro-geological processes. In the present study, we found that the ground water is getting polluted by anthropogenically and naturally. In anthropogenic sources, water is polluted due to discharge of untreated or partially treated sewage water, industrial waste, improper urban garbage disposal, unsuitable agriculture practices, leach ate from land fills, poorly designed septic tanks, mining waste etc. (A. Ganesha [2008]). In the natural sources, water is polluted due to high concentration of Fluorspar (CaF2.3Ca3(Po4)2) and Fluoropatite (CaF2) in the ground stripe, found in the base of Aravali series. This strip expands from Panch Mahal (Gujarat) to Gurgoan (Haryana) which passes through Rajasthan. Thus, districts viz. Jalore, Shirohi, Pali, Bhilwara, Tonk, Ajmer and Jaipur are in high concentration zone of water quality parameters.

In recent years, an alternative approach based on statistical data, has been used to develop spatial distribution of pH (pondus Hydrogenii - acidity or basicity of a solution), electrical conductivity (EC) and total dissolved solid (TDS) (P. Trivedi, A. Bajpai, and S. Thareja [2009]; N.S. Bhandari and K. Nayal [2008]; S.P. Verma, K.G. Ojha [2007]; N. Janardhana Raju [2006]).

In order to study the seasonal quality variations of the ground water, in the Todaraishing area of Tonk district Rajasthan (India), 30 samples during the post monsoon and 30 samples during pre-monsoon season were collected.

Regression equation for dissolve solids (TDS) as a function of SEC and for (TDS) as a function of SEC and SiO2 were determined for the upper Gunjaneru river basin 12. Multiple linear regression were used to relate the given chemical quality parameters to a set of independent chemical variable.

This paper is mainly focuses on the most significant multiple linear regression with GP model for predicting EC, PH, TDS, ALKALINITY from various combinations of the known concentration of a set of independent chemical variables such as (Ca2+, Mg2+, Cl- , No3-, F-). Individual contributions of various independent chemical variables, in prediction of chemical quality parameters by using goal programming are shown in table 1& 2, for post and pre monsoon season respectively.

Background

This paper is an extension to the previous work “A goal programming model for the interaction effects in multiple non linear regressions”, where the research has proven the edge that goal programming model has posed over multiple non programming regression parameters. (S.C. Sharma, D.S. Hada, Umesh Gupta [2010]).

This paper attempts to solve the goal programming problem on Seasonal evaluation of hydro-chemical parameters by the employing simplex method by the application of TORA software.

TORA is an algorithm or a set of mathematical instructions in the area of operation research that can be executed in automated and tutorial mode (Hamdy A. Taha. [2008]). The software is a tutorial package for novice, as is menu driven and widows-base, which makes it user friendly in solving simplex problems.

The current market provides us with varied software’s - LINDO, LINGO, AMPL, etc. for goal programming but this software are not used in this paper due to their commercial viability. TORA software has been used for the ease of novice in obtaining simplex problem solutions.

Study Area

Tonk district is located in north eastern part of the state bordering Jaipur in north Swaimadhopur in east Bundi & Bhilwara in the south & Ajmer in the west. Tonk is known for its unity among Hindus and Muslims for which it is same time called as “Hindus Muslims Ekta Ka Maskan”. The history of Tonk is very old. The city was popularly called Nawabi Nagari “Tonk”. The Tonk is also known as the “Lucknow of Rajasthan” due to its elegance. Tonk is admired among tourist for its magnificent mosques, mansion and havelis.

Material and Methods

In the present study, 30 ground water samples have been collected in clean and dry - one liter plastic cans. The cans were flushed with distilled water and sealed before been taken to the field. Before collection of the sample, cans have been rinsed with respective samples for two to three times and then sample was collected.

The samples have been numbered just after collection with a permanent marker and same number registered in the field note book with description regarding location. The samples are collected once during pre- monsoon period (April & May) and next, during post- monsoon period (October & November). The samples were analyzed immediately for pH, electrical conductivity (EC) and Total Dissolved Solids (TDS) as per standard methods (APHA [1992]; AWWA [1986]; A.M .Goon [1986]).

Reagents and Apparatus

The chemical used were AR grade and water was double distilled.

Data Collected From the Study Area

CHEMICAL QUALITY PARAMETER VALUES FROM THE STUDY AREA

DURING PRE-MONSOON PERIOD

S.No	pH	EC	TDS	ALKALINITY	Ca^{2+}	Mg^{2+}	Cl^-	No^3	F^-
1	7.3	5.9	4130	730	342	228	1610	22	5
2	7.39	5.8	4060	950	526	284	1160	100.4	3.1
3	8	0.87	609	350	168	112	70	808	1.4
4	7.6	1.47	1029	580	130	70	200	2604	5.6
5	8.1	2.13	1491	790	108	72	310	13.2	3.2
6	7.8	0.56	392	300	132	88	90	22	0.66
7	8.7	2.77	1940	660	435	235	550	26.4	2.5
8	7.1	8.85	6195	690	918	612	2450	132	2.5
9	7.8	6.2	4340	870	456	304	1150	44	3.5
10	7.5	6.97	4880	420	754	406	1650	22	2
11	7.7	3.82	2674	660	450	110	790	52.8	3.7
12	7.3	2.55	1785	780	377	203	420	8.8	1.5
13	7.8	3.32	2324	480	384	206	740	22	2
14	7.9	3.75	2625	780	399	171	770	220	7
15	7.5	1.81	1267	580	198	132	460	22	6.4
16	8.5	4.2	2940	650	810	540	1060	92.4	4.5
17	7.7	1.08	721	450	258	172	210	8.8	1.5
18	9.3	3.79	2653	940	396	264	510	44	5.4
19	8.6	2.83	1981	580	288	192	410	264	0.47
20	7.5	2.78	1946	690	408	272	490	132	0.5
21	8.89	3.75	2625	640	351	189	820	61.6	4.5
22	9.2	3.71	2597	580	340	160	1130		6
23	7.9	3.6	2520	440	695	375	600	220	1.3
24	9.04	2.87	2009	520	300	90	640	22	3.3
25	7.96	3.09	2163	490	371	159	610	22	2.2
26	8.37	5.49	3843	670	480	320	1300	22	4.8
27	9.24	1.93	1351	740	90	60	310	22	10.7
28	7.5	7.11	4977	630	630	340	2000	22	5
29	8.7	2.09	1463	510	130	70	370	4.4	4.47
30	8.1	0.46	322	290	120	80	60	4.4	1.28

CHEMICAL QUALITY PARAMETER VALUES FROM THE STUDY AREA

DURING POST-MONSOON PERIOD

S.No	pH	EC	TDS	ALKALINITY	Ca^{2+}	Mg^{2+}	Cl^{-}	No^{3}	F^{-}
1	7.2	4.22	2954	500	280	150	1060	8.8	2.17
2	7.07	3.47	2429	620	351	189	720	4.4	2.3
3	6.8	73	511	310	198	92	70	4.4	1.2
4	6.1	1.1	770	580	90	60	80	4.4	4
5	7.5	1.99	1393	660	108	72	300	4.4	2.8
6	6.9	0.4	280	250	66	44	50	4.4	0.5
7	7.4	1.71	1197	500	234	156	360	17.6	1.3
8	7	4.9	3480	430	980	420	1360	35.2	0.5
9	7.5	5.75	3521	750	348	232	750	35.4	3.1
10	7.26	5.7	4025	340	582	388	1470	8.8	1.7
11	7	3.04	2128	450	252	168	540	30.8	2.8
12	7	1.12	784	400	204	140	160	4.4	1.2
13	7.6	2.27	1589	390	210	140	500	4.4	1.7
14	7.6	2.5	1750	400	192	128	340	37.4	2.9
15	7.1	1.75	1225	470	149	81	310	17.6	4.5
16	7.1	2.28	1596	570	462	308	540	48.4	3.5
17	7.3	0.99	693	230	240	160	120	4.4	0.6
18	7.3	1.3	910	540	102	68	180	4.4	2.6
19	8	2.5	1897	450	240	160	320	17.6	0.3
20	7.4	1.85	1295	470	315	135	260	30.8	0.2
21	7.3	2.88	2016	520	217	133	820	4.4	2.3
22	8.72	3.26	2282	460	246	164	910	13.2	4.4
23	7.6	3.02	2114	260	714	306	130	4.4	0.9
24	7.4	1.16	812	480	104	56	530	8.8	0.3
25	7.53	0.68	476	310	180	120	100	4.4	0.84
26	7.44	0.99	693	330	168	112	180	4.4	0.76
27	7.9	1.86	1302	610	80	60	250	17.8	7.3
28	7.03	6.59	4613	400	455	245	1820	4.4	4.4
29	7.3	1.05	735	400	102	68	140	4.4	3.3
30	7.6	0.39	273	260	42	28	50	4.4	0.7

2 Multiple Regressions

The regression equation is:

$$y_{ir} = b_0 + b_1 X_i + b_2 X_i^{\,2} + e_i, \qquad i = 1, 2, \ldots, m.$$

Where, b_0, b_1 and b_2 are the parameters to be estimated (Alken, L. S. and West, S. G. [1991]).

e_i is the error component which is assumed to be normally and independently distributed with zero mean and constant variance.

The linear absolute residual method requires us to estimate the values of these unknown parameters so as to minimize $\sum_{i=1}^{m} |y_i - y_{ir}|$.

3 Linear Goal Programming Formulation

Let y_{iG} be the ith goal, d_i^+ be positive deviation from the ith goal and d_i^- be the negative deviation from the ith goal. Then the problem of minimizing $\sum_{i=1}^{m} |y_i - y_{ir}|$ may be reformulated (Bassey, U. N. and Effanga, E. O. [2008]) as:

Minimize $\sum_{i=1}^{m} (d_i^+ + d_i^-)$
Subject to:

$$a_0 + \sum_{i=1}^{m} \sum_{j=1}^{n} a_j X_{ij} = y_{iG} - d_i^+ + d_i^-$$

$d_i^+ \geq 0$, i = 1, 2, …, n
$d_i^- \geq 0$, i = 1, 2, …, n
a_0 , a_j are unrestricted, i = 0, 1, 2, …,m. and j = 0, 1, 2, …,n.

4 The Linear Goal Programming Method

Reformulating the problem into linear goal programming model:

Minimize $\sum_{i=1}^{32} (d_i^+ + d_i^-)$
Subject to:

$$a_0 + a_1 X_{i1} + a_2 X_{i2} + a_3 X_{i3} + a_4 X_{i4} + a_5 X_{i5} + a_6 X_{i6} + a_7 X_{i7} + a_8 X_{i8} + a_9 X_{i9} + a_{10} X_{i10} = y_{iEC} - d_i^+ + d_i^-$$
$$a_0 + a_1 X_{i1} + a_2 X_{i2} + a_3 X_{i3} + a_4 X_{i4} + a_5 X_{i5} + a_6 X_{i6} + a_7 X_{i7} + a_8 X_{i8} + a_9 X_{i9} + a_{10} X_{i10} = y_{iPH} - d_i^+ + d_i^-$$
$$a_0 + a_1 X_{i1} + a_2 X_{i2} + a_3 X_{i3} + a_4 X_{i4} + a_5 X_{i5} + a_6 X_{i6} + a_7 X_{i7} + a_8 X_{i8} + a_9 X_{i9} + a_{10} X_{i10} = y_{iTDS} - d_i^+ + d_i^-$$
$$a_0 + a_1 X_{i1} + a_2 X_{i2} + a_3 X_{i3} + a_4 X_{i4} + a_5 X_{i5} + a_6 X_{i6} + a_7 X_{i7} + a_8 X_{i8} + a_9 X_{i9} + a_{10} X_{i10} = y_{iALKA.} - d_i^+ + d_i^-$$

$d_i^+ \geq 0$, i = 1, 2, …, 32
$d_i^- \geq 0$, i = 1, 2, …, 32

and $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ are unrestricted.
i = 1, 2, …, 32.

Where $y_{iEC}, y_{iPH}, y_{iTDS}, y_{iALKALINETY}$, are dependent chemical quality parameters EC, PH, TDS, ALKALINITY and $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}$ are concentrations of a set of independent chemical variables such as Ca^{2+}, Mg^{2+}, Cl^- ,No^{3-}, F^- while X_{i1}^2, X_{i2}^2 X_{i3}^2, X_{i4}^2 , X_{i5}^2 are taken as X_{i6}, X_{i7}, X_{i8} X_{i9}, X_{i10} respectively to formulate the multiple regression problem into linear goal programming mode. $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ are individual contributions of various independent chemical variables used in the prediction of water quality.

Solution

The solution of the above formulated problem, through simplex method using TORA computer software package:

Table 1: Individual contributions of various independent chemical variables in prediction of chemical quality parameters during pre-monsoon by using goal programming

Dependent variable	Independent variable (Ca^{+2} , Mg^{+2} , Cl^- , No^{-3} , F^-)										
	a_o	**a_1**	**a_2**	**a_3**	**a_4**	**a_5**	**a_6**	**a_7**	**a_8**	**a_9**	**a_{10}**
EC	**0.2463**	**0.0022**	**0.0000**	**0.0029**	**0.0000**	**0.0012**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0048**
TDS	**398.4461**	**1.3126**	**0.0000**	**2.0105**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**1.8450**
PH	**7.79450**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0126**
ALKALINITY	**391.17263**	**0.0000**	**0.0966**	**0.0671**	**0.0000**	**30.1147**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 2: Individual contributions of various independent chemical variables in prediction of chemical quality parameters during post-monsoon by using goal programming

	Independent variable (Ca^{+2} , Mg^{+2} , Cl^- , No^{-3} , F^-)										
Dependent variable	a_o	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
EC	0.0317	0.0032	0.0000	0.0020	0.0076	0.1249	0.0000	0.0000	0.0000	0.0000	0.0000
TDS	10.7745	1.0198	0.8363	1.9578	10.5956	73.5212	0.0000	0.0000	0.0000	0.0000	0.0000
PH	7.2833	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ALKALINITY	293.48609	0.0000	0.0000	0.0000	3.4864	27.0468	0.0000	0.0000	0.0000	0.0000	1.0698

The above tables (1 and 2) consist of Individual contribution of various independent chemical variables in prediction of chemical quality parameters during pre and post monsoon period. These individual contributions ($a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$) are collectively used in the goal programming model to calculate or predict the water chemical quality.

Result and Conclusion

The above multiple regression equation is solved using the linear goal programming model (of goal programming technique), where y_{iG} is taken to be the estimate of the i^{th} response.

From the use of goal programming model with regression analysis, it is concluded that in pre and post monsoon season, we can find significant effect in predicting water chemical quality parameter to a set of independent chemical variable.

The goal programming technique provides the users with a better degree of estimates of the multiple regression parameters.

Acknowledgement

The authors acknowledge their gratitude towards Mr. Ashok Yadav, Department of Chemistry, Kautilya Institute of Technology and Engineering for providing data support to the paper.

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