Gen. Math. Notes, Vol. 9, No. 2, April 2012, pp.1-18
ISSN 2219-7184; Copyright ©ICSRS Publication, 2012
www.i-csrs.org
Available free online at http://www.geman.in

Contra-e-Continuous Functions

M.K. Ghosh[1] and C.K. Basu[2]

[1]Department of Mathematics, Dumkal College,
Basantapur-742406, Murshidabad, West Bengal, India
E-mail: manabghosh@gmail.com
[2]Department of Mathematics, West Bengal State University,
Berunanpukuria, Malikapur, Barasat, Kolkata-700126,
24 Pgs. (N), West Bengal, India
E-mail: ckbasu1962@yahoo.com

(Received: 25-10-11/ Accepted: 5-4-12)

Abstract

In 2008, E. Ekici [13] introduced a new class of generalized open sets in a topological space called e-open sets. M. Ozkoc and G Aslim [26], in their recent paper (Bull. Korean Math. Soc. 47 (2010), No. 5, pp. 1025-1036), initiated two strong forms of e-open sets for the study of strongly-θ-e-continuous functions. J. Dontchev [8] introduced the notion of contra continuous functions. In this paper, by means of e-open sets, we introduce and investigate certain ramifications of contra continuous and allied functions, namely, contra-e-continuous, almost-e-continuous, almost weakly-e-continuous and almost contra-e-continuous functions along with their several properties, characterizations and mutual relationships. Further, we introduce new types of graphs, called e-closed, contra-e-closed and strongly contra-e-closed graphs via e-open sets. Several characterizations and properties of such notions are investigated.

Keywords: *almost contra-e-continuous, contra-e-closed graphs, contra-e-continuous, e-connected, e-open.*

1 Introduction

In recent literature, we find many topologists have focused their research in the direction of investigating different types of generalized continuity. One of the outcomes of their research leads to the initiation of different orientations of contra-continuous functions. The notion of contra continuity was first investigated by Dontchev [8]. Subsequently, Jafari and Noiri [16, 17] exhibited contra-α-continuous and contra-pre-continuous functions. Contra δ-precontinuous functions [14] was obtained by Ekici and Noiri. A good number of researchers have also initiated different types of contra continuous-like functions, some of which are found in the papers [5, 9, 12, 21, 24, 25].

In [13], Ekici obtained a new class of sets in a topological space, known as e-open sets. Very recently M. Ozkoc and G Aslim [26], in their recent paper (Bull. Korean Math. Soc. 47 (2010), No. 5, pp. 1025-1036), used such e-open sets for the study of strongly-θ-e-continuous functions. Here, in this paper also, attempt has been made to employ this notion of e-open sets to introduce and investigate a new variation of contra continuous functions, called contra-e-continuous functions. In section 3 we introduce and study fundamental properties of contra-e-continuous functions, almost-e-continuous etc.; and using such functions we characterize e-connectedness. Section 4 is devoted to the investigation of almost contra-e-continuous functions. Section 5 concerns to the notions of e-closed, contra-e-closed and strongly contra e-closed graphs.

2 Preliminaries

In this paper, spaces X and Y always represent topological spaces (X, τ) and (Y, σ) respectively on which no separation axioms are assumed unless otherwise stated. For a subset A of a space X, $cl(A)$ and $int(A)$ denote the closure and the interior of A respectively.

A subset A of a space (X, τ) is called regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A subset A of a space X is said to be δ-open [32] if for each $x \in A$ there exists a regular open set G such that $x \in G \subset A$. A point $x \in X$ is called a δ-cluster point [32] of A if $A \cap int(cl(U)) \neq \emptyset$ for each open set U of X containing x. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by δ-$cl(A)$. If δ-$cl(A) = A$, then A is said to be δ-closed. The set $\{x : x \in U \subset A$ for some regular open set U of $X\}$ is called δ-interior of A and is denoted by δ-$int(A)$.

A subset A of a space (X, τ) is called e-open [13] (resp. α-open [23], β-open [1] or semi-preopen [2], b-open [3] or sp-open [10] or γ-open [4], preopen [20], semiopen [19], δ-preopen [28], δ-semiopen [27]) if $A \subset cl(\delta\text{-}int(A)) \cup int(\delta\text{-}cl(A))$ (resp. $A \subset int(cl(int(A)))$, $A \subset cl(int(cl(A)))$, $A \subset int(cl(A)) \cup cl(int(A))$, $A \subset int(cl(A))$, $A \subset cl(int(A))$, $A \subset int(\delta\text{-}cl(A))$, $A \subset cl(\delta\text{-}int(A))$). The complement of an e-open (resp. δ-semiopen, δ-preopen) set is called an e-closed (resp. δ-semiclosed, δ-preclosed) set. The intersection of all e-closed (resp. δ-semiclosed, δ-preclosed) sets containing a set A in a topological space X is called the e-closure [13] (resp. δ-semiclosure [27], δ-preclosure [28]) of A and it is denoted by $e\text{-}cl(A)$ (resp. $\delta\text{-}scl(A)$, $\delta\text{-}pcl(A)$). The union of all e-open (resp. δ-semiopen, δ-preopen) sets contained in a set A in a topological space X is called the e-interior (resp. δ-semiinterior [27], δ-preinterior [28]) of A and it is denoted by $e\text{-}int(A)$ (resp. $\delta\text{-}sint(A)$, $\delta\text{-}pint(A)$). A subset A of a topological space X is e-regular [26] if it is e-open and e-closed. The family of all e-open (resp. e-closed, e-regular) sets in X will be denoted by $eO(X)$ (resp. $eC(X)$, $eR(X)$). The family of all e-open (resp. e-closed, e-regular) sets which contain x in X will be denoted by $eO(X, x)$ (resp. $eC(X, x)$, $eR(X, x)$).

A function $f : (X, \tau) \to (Y, \sigma)$ is called contra continuous [8] (resp. contra α-continuous [16], contra-precontinuous [17], contra δ-precontinuous [14]) if inverse image of each open set of Y is closed (resp. α-closed, preclosed, δ-preclosed) in X. A function $f : (X, \tau) \to (Y, \sigma)$ is called e-continuous [13] if $f^{-1}(V)$ is e-open in X for each $V \in \sigma$. A topological space (X, τ) is called Urysohn [29] if, for each $x, y \in X$ with $x \neq y$, there exist open sets P and Q of X containing x and y respectively such that $cl(P) \cap cl(Q) = \emptyset$. A topological space X is called S-closed [18] (resp. countably S-closed [7], S-Lindeloff [11]) if every regular closed (resp. countably regular closed, regular closed) cover of X has a finite (resp. finite, countable) subcover. A topological space X is said to be nearly compact [29] (resp. nearly countably compact [29], nearly Lindeloff [29]) if every regular open (resp. countable regular open, regular open) cover of X has a finite (resp. finite, a countably) subcover. A topological space (X, τ) is ultra Hausdorff [31] if for each pair of distinct points x and y of X there exist closed sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. A topological space (X, τ) is said to be weakly Hausdorff [30] if each element of X is the intersection of regular closed sets of X.

Lemma 2.1 *[13, 27] The following hold for a subset A of a space X:*
(i) $\delta\text{-}pcl(A) = A \cup cl(\delta\text{-}int(A))$ and $\delta\text{-}pint(A) = A \cap int(\delta\text{-}cl(A))$;
(ii) $\delta\text{-}sint(A) = A \cap cl(\delta\text{-}int(A))$ and $\delta\text{-}scl(A) = A \cup int(\delta\text{-}cl(A))$;
(iii) A is e-open if and only if $A = \delta\text{-}pint(A) \cup \delta\text{-}sint(A)$;
(iv) $e\text{-}cl(A) = \delta\text{-}pcl(A) \cap \delta\text{-}scl(A) = A \cup [cl(\delta\text{-}int(A)) \cap int(\delta\text{-}cl(A))]$;
(v) $e\text{-}int(A) = \delta\text{-}pint(A) \cup \delta\text{-}sint(A) = A \cap [int(\delta\text{-}cl(A)) \cup cl(\delta\text{-}int(A))]$;
(vi) A is δ-preopen if and only if $A \subset \delta\text{-}pint(\delta\text{-}pcl(A))$;
(vii) A is e-open if and only if $A \subset \delta\text{-}pcl(\delta\text{-}pint(A))$.

Lemma 2.2 *[13] In a topological space (X, τ)*
(i) The union of any family of e-open sets is an e-open set;
(ii) The intersection of any family of e-closed sets is an e-closed set.

3 Contra-e-Continuous Functions

Definition 3.1 *A function $f : X \to Y$ is called contra-e-continuous if $f^{-1}(V)$ is e-closed in X for every open set V of Y.*

If $f : X \to Y$ is contra-e-continuous at each point of X, we call f is contra-e-continuous on X.

Definition 3.2 *For a topological space (X, τ) and $A \subset X$,*
(a) Intersection of all open sets of X containing A is called kernel of A and is denoted by $ker(A)$.
(b) Intersection of all e-closed sets of X containing A is called e-closure of A [13] and is denoted by $e\text{-}cl(A)$.
(c) Union of all e-open sets of X contained in A is called e-interior of A [13] and is denoted by $e\text{-}int(A)$.

Lemma 3.3 *[15] The following properties holds for subsets A, B of a topological space (X, τ):*
(a) $x \in ker(A)$ iff $A \cap F \neq \emptyset$ for any closed set F of X containing x.
(b) $A \subset ker(A)$ and $A = ker(A)$ if A is open in X.
(c) If $A \subset B$, then $ker(A) \subset ker(B)$.

Lemma 3.4 *The following properties holds for a subset A of a topological space (X, τ):*
(i) $e\text{-}int(A) = X - e\text{-}cl(X - A)$;
(ii) $x \in e\text{-}cl(A)$ iff $A \cap U \neq \emptyset$ for each $U \in eO(X, x)$;
(iii) A is e-open iff $A = e\text{-}int(A)$;
(iv) A is e-closed iff $A = e\text{-}cl(A)$.

Theorem 3.5 *For a function $f : X \to Y$ the following conditions are equivalent:*
(a) f is contra-e-continuous;
(b) for each closed subset F of Y, $f^{-1}(F)$ is e-open in X;
(c) for each $x \in X$ and each closed subset F of Y containing $f(x)$, there exist $U \in eO(X, x)$ such that $f(U) \subset F$;
(d) $f(e\text{-}cl(A)) \subset ker(f(A))$ for every subset A of X;
(e) $e\text{-}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$ for every subset B of Y.

Proof. (a) $\Rightarrow$ (c) : Let $x \in X$ and F be any closed set of Y containing $f(x)$. Using (a), we have $f^{-1}(Y - F) = X - f^{-1}(F)$ is e-closed in X and so $f^{-1}(F)$ is e-open in X. Taking $U = f^{-1}(F)$, we get $x \in U$ and $f(U) \subset F$.
(c) $\Rightarrow$ (b): Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exist an e-open subset U_x containing x such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$ – which is e-open in X.

(b) $\Rightarrow$ (a) : Let U be any open set of Y. Then since $(Y - U)$ is closed in Y, by (b), $f^{-1}(Y - U) = X - f^{-1}(U)$ is e-open in X. Therefore $f^{-1}(U)$ is e-closed in X.
(b) $\Rightarrow$ (d): Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 3.3, there exist closed set F of Y containing y such that $f(A) \cap F = \emptyset$. This implies that $A \cap f^{-1}(F) = \emptyset$ and so $e\text{-}cl(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(e\text{-}cl(A)) \cap F = \emptyset$ and $y \notin f(e\text{-}cl(A))$. Hence, $f(e\text{-}cl(A)) \subset ker(f(A))$.
(d) $\Rightarrow$ (e): Let B be any subset of Y. Using (d) and Lemma 3.3 we have $f(e\text{-}cl(f^{-1}(B)) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and so $e\text{-}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$.
(e) $\Rightarrow$ (a): Let V be any open subset of Y. Then from Lemma 3.3 and (d) we have $e\text{-}cl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and hence $e\text{-}cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is e-closed in X.

Remark 3.6 *The following diagram holds for a function $f : X \to Y$:*

contra continuous $\Rightarrow$ contra α-continuous $\Rightarrow$ contra-precontinuous $\Rightarrow$ contra δ-precontinuous $\Rightarrow$ contra-e-continuous

None of these implications is reversible as shown from the following examples:

Example 3.7 *Let $X = Y = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ be a topology on X and $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}\}$ be a topology on Y. Define a function $f : (X, \tau) \to (Y, \sigma)$ as $f(x) = x$. Then f is contra-e-continuous but not contra δ-precontinuous, as $\{b, c\}$ is closed in Y but $f^{-1}(\{b, c\}) = \{b, c\}$ is not δ-preopen in X.*

The other examples are shown in the related paper.

The following lemma can be verified easily:

Lemma 3.8 *The following conditions are equivalent for a function $f : X \to Y$:*
(a) f is e-continuous;
(b) for each $x \in X$ and for each open set V of Y containing $f(x)$, there exist $U \in eO(X, x)$ such that $f(U) \subset V$.

Theorem 3.9 *If a function $f : X \to Y$ is contra-e-continuous and Y is regular, then f is e-continuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. Since Y is regular, there exist an open set G in Y containing $f(x)$ such that $cl(G) \subset V$. Again, since f is contra-e-continuous, so by Theorem 3.5, there exists $U \in eO(X, x)$ such that $f(U) \subset cl(G)$. Then $f(U) \subset cl(G) \subset V$. Hence f is e-continuous.

Definition 3.10 *A topological space X is called e-space (resp. locally e-indiscrete) if every e-open set is open (resp. closed).*

Theorem 3.11 *A contra-e-continuous function $f : (X, \tau) \to (Y, \sigma)$ is continuous when X is locally e-indiscrete.*

Proof. Straightforward.

Recall that a function $f : X \to Y$ is preclosed [6] if the image of every closed subset of X is preclosed in Y. A space X is called locally indiscrete [22] if every open set is closed.

Theorem 3.12 *Let $f : X \to Y$ be a surjective preclosed contra-e-continuous function. If X is an e-space, then Y is locally indiscrete.*

Proof. Let V be an open set of Y. Since f is contra-e-continuous, $f^{-1}(V)$ is e-closed in X. Let $f^{-1}(V) = U$. Then, since X is an e-space, U is closed in X. Again, since f is preclosed, $f(U) = V$ is preclosed in Y. So, we get $cl(V) = int(cl(V)) \subset V$. This shows that V is closed and hence Y is locally indiscrete.

Definition 3.13 *A function $f : X \to Y$ is called almost-e-continuous if, for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in eO(X, x)$ such that $f(U) \subset e\text{-}int(cl(V))$.*

Definition 3.14 *A function $f : X \to Y$ is said to be*
(a) pre-e-open if image of each e-open set of X is an e-open set of Y.
(b) e-irresolute if preimage of an e-open subset of Y is an e-open subset of X.

Theorem 3.15 *If a function $f : X \to Y$ is pre-e-open, contra-e-continuous then it is almost-e-continuous.*

Proof. Let $x \in X$ and V be an open set containing $f(x)$. Since f is contra-e-continuous, then by Theorem 3.5, there exists $U \in eO(X, x)$ such that $f(U) \subset cl(V)$. Again, since f is pre-e-open, $f(U)$ is e-open in Y. Therefore, $f(U) = e\text{-}int(f(U))$ and hence $f(U) \subset e\text{-}int(cl(f(U)) \subset e\text{-}int(cl(V))$. So f is almost-e-continuous.

Definition 3.16 *The e-frontier of a subset A of a space X, denoted by $e\text{-}Fr(A)$, is defined as $e\text{-}Fr(A) = e\text{-}cl(A) \cap e\text{-}cl(X - A) = e\text{-}cl(A) - e\text{-}int(A)$.*

Theorem 3.17 *The set of all points x of X at which $f : X \to Y$ is not contra-e-continuous is identical with the union of e-frontier of the inverse images of closed sets of Y containing $f(x)$.*

Proof. Necessity : Let f be not contra-e-continuous at a point x of X. Then by Theorem 3.5, there exists a closed set F of Y containing $f(x)$ such that $f(U) \cap (Y - F) \neq \emptyset$ for every $U \in eO(X, x)$, which implies $U \cap f^{-1}(Y - F) \neq \emptyset$. Therefore, $x \in e\text{-}cl(f^{-1}(Y - F)) = e\text{-}cl(X - f^{-1}(F))$. Again, since $x \in f^{-1}(F)$, we get $x \in e\text{-}cl(f^{-1}(F))$ and so $x \in e\text{-}Fr(f^{-1}(F))$.

Sufficiency : Suppose that $x \in e\text{-}Fr(f^{-1}(F))$ for some closed set F of Y containing $f(x)$ and f is contra-e-continuous at x. Then there exists $U \in eO(X, x)$ such that $f(U) \subset F$. Therefore $x \in U \subset f^{-1}(F)$ and hence $x \in e\text{-}int(f^{-1}(F)) \subset X - e\text{-}Fr(f^{-1}(F))$ — which is a contradiction. So f is not contra-e-continuous at x.

Definition 3.18 *A function $f : X \to Y$ is called almost weakly-e-continuous if, for each $x \in X$ and for each open set V of Y containing $f(x)$, there exist $U \in eO(X, x)$ such that $f(U) \subset cl(V)$.*

Theorem 3.19 *If a function $f : X \to Y$ is contra-e-continuous then f is almost weakly-e-continuous.*

Proof. For any open set V of Y, $cl(V)$ is closed in Y. Since f is contra-e-continuous, $f^{-1}(cl(V))$ is e-open set in X. We take $U = f^{-1}(cl(V))$, then $f(U) \subset cl(V)$. Hence f is almost weakly-e-continuous.

Theorem 3.20 *For two functions $f : X \to Y$ and $g : Y \to Z$ the following properties holds:*
(i) If f is contra-e-continuous function and g is a continuous function, then $g \circ f$ is contra-e-continuous.
(ii) If f is e-irresolute and g is contra-e-continuous, then $g \circ f$ is contra-e-continuous.

Proof. (i) For $x \in X$, let W be any closed set of Z containing $g \circ f(x)$. Since g is continuous, $V = g^{-1}(W)$ is closed in Y. Also, since f is contra-e-continuous, there exists $U \in eO(X, x)$ such that $f(U) \subset V$. Therefore $g \circ f(U) \subset W$. Hence, $g \circ f$ is contra-e-continuous.
(ii) For $x \in X$, let W be any closed set of Z containing $g \circ f(x)$. Since g is contra-e-continuous, there exist $V \in eO(Y, f(x))$ such that $g(V) \subset W$. Again, since f is e-irresolute there exist $U \in eO(X, x)$ such that $f(U) \subset V$. This shows that $g \circ f(U) \subset W$. Hence, $g \circ f$ is contra-e-continuous.

Theorem 3.21 *Let $f : X \to Y$ be surjective e-irresolute and pre-e-open function and $g : Y \to Z$ be any function. Then $g \circ f : X \to Z$ is contra-e-continuous if and only if g is contra-e-continuous.*

Proof. The "if" part is easy to prove. To prove "only if" part, let $g \circ f : X \to Z$ is contra-e-continuous and let F be a closed subset of Z. Then $(g \circ f)^{-1}(F)$ is an e-open subset of X i.e. $f^{-1}(g^{-1}(F))$ is pre-e-open in X. Since f is e-open, $f(f^{-1}(g^{-1}(F)))$ is an e-open subset of Y and so $g^{-1}(F)$ is e-open in Y. Hence, g is contra-e-continuous.

Definition 3.22 *A topological space (X, τ) is said to be*
(a) e-normal if each pair of non-empty disjoint closed sets can be separated by disjoint e-open sets.
(b) ultranormal [31] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.23 *If $f : X \to Y$ is contra-e-continuous, closed injection and Y is ultranormal, then X is e-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is e-closed injection, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Again, since Y is ultranormal $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets P and Q (say) respectively. Therefore, $f(F_1) \subset P$ and $f(F_2) \subset Q$ i.e. $F_1 \subset f^{-1}(P)$ and $F_2 \subset f^{-1}(Q)$, where $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint e-open sets of X (since f is contra-e-continuous). This shows that X is e-normal.

Definition 3.24 *A topological space (X, τ) is called e-connected provided that X is not the union of two disjoint nonempty e-open sets of X.*

Theorem 3.25 *If $f : X \to Y$ is contra-e-continuous surjection, where X is e-connected and Y is any topological space, then Y is not a discrete space.*

Proof. If possible, suppose that Y is a discrete space. Let P be a proper nonempty open and closed subset of Y. Then $f^{-1}(P)$ is a proper nonempty e-open and e-closed subset of X, which contradicts to the fact that X is e-connected. Hence the theorem follows.

Theorem 3.26 *If $f : X \to Y$ is contra-e-continuous surjection and X is e-connected, then Y is connected.*

Proof. If possible, suppose that Y is not connected. Then there exist nonempty disjoint open sets P and Q such that $Y = P \cup Q$. So P and Q are clopen sets of Y. Since f is contra-e-continuous function, $f^{-1}(P)$ and $f^{-1}(Q)$ are e-open sets of X. Also $f^{-1}(P)$ and $f^{-1}(Q)$ are nonempty disjoint e-open sets of X and $X = f^{-1}(P) \cup f^{-1}(Q)$, which contradicts to the fact that X is e-connected. Hence Y is connected.

Theorem 3.27 *A space X is e-connected if and only if every contra-e-continuous function from X into any T_1 space Y is constant.*

Proof. Let X be e-connected. Now, since Y is a T_1 space, $\Omega = \{f^{-1}(y) : y \in Y\}$ is disjoint e-open partition of X. If $|\Omega| \geq 2$ (where $|\Omega|$ denotes the cardinality of Ω), then X is the union of two nonempty disjoint e-open sets. Since X is e-connected, we get $|\Omega| = 1$. Hence, f is constant.

Conversely, suppose that X is not e-connected and every contra-e-continuous function from X into any T_1 space Y is constant. Since X is not e-connected, there exist a non-empty proper e-open as well as e-closed set V (say) in X. We consider the space $Y = \{0, 1\}$ with the discrete topology σ. The function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(V) = \{0\}$ and $f(X - V) = \{1\}$ is obviously contra-e-continuous and which is non-constant – a contradiction. Hence X is e-connected.

Definition 3.28 *[26] A space X is said to be e-T_2 if for each pair of distinct points x, y in X there exist $U \in eO(X, x)$ and $V \in eO(X, y)$ such that $U \cap V = \emptyset$.*

Theorem 3.29 *Let X and Y be two topological spaces. If for each pair of distinct points x and y in X there exist a function f of X into Y such that $f(x) \neq f(y)$ where Y is an Urysohn space and f is contra-e-continuous function at x and y then X is e-T_2.*

Proof. Let $x, y \in X$ and $x \neq y$. Then by assumption, there exist a function $f : X \to Y$, such that $f(x) \neq f(y)$ where Y is Urysohn and f is contra-e-continuous at x and y. Now, since Y is Urysohn, there exist open sets U and V of Y containing $f(x)$ and $f(y)$ respectively, such that $cl(U) \cap cl(V) = \emptyset$. Also, f being contra-e-continuous at x and y there exist e-open sets P and Q containing x and y respectively such that $f(P) \subset cl(U)$ and $f(Q) \subset cl(V)$. Then $f(P) \cap f(Q) = \emptyset$ and so $P \cap Q = \emptyset$. Therefore X is e-T_2.

Corollary 3.30 *If $f : X \to Y$ is contra-e-continuous injection where Y is an Urysohn space, then X is e-T_2.*

Corollary 3.31 *If f is contra-e-continuous injection of a topological space X into a ultra Hausdorff space Y, then X is e-T_2.*

Proof. Let $x, y \in X$ where $x \neq y$. Then, since f is an injection and Y is ultra Hausdorff, $f(x) \neq f(y)$ and there exist disjoint closed sets U and V containing $f(x)$ and $f(y)$ respectively. Again, since f is contra-e-continuous, $f^{-1}(U) \in eO(X, x)$ and $f^{-1}(V) \in eO(X, y)$ with $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is e-T_2.

4 Almost Contra-e-Continuous Functions

Definition 4.1 *A function $f : X \to Y$ is said to be almost contra-e-continuous if $f^{-1}(V)$ is e-closed in X for every regular open set V of Y.*

Theorem 4.2 *The following are equivalent for a function $f : X \to Y$:*
(a) f is almost contra-e-continuous;
(b) $f^{-1}(F)$ is e-open in X for every regular closed set F of Y;
(c) for each $x \in X$ and each regular open set F of Y containing $f(x)$, there exist $U \in eO(X, x)$ such that $f(U) \subset F$;
(d) for each $x \in X$ and each regular open set V of Y non-containing $f(x)$, there exist an e-closed set K of X non-containing x such that $f^{-1}(V) \subset K$;

Proof. (a) $\Leftrightarrow$ (b) : Let F be any regular closed set of Y. Then $(Y - F)$ is regular open and therefore $f^{-1}(Y - F) = X - f^{-1}(F) \in eC(X)$. Hence, $f^{-1}(F) \in eO(X)$. The converse part is obvious.

(b) $\Rightarrow$ (c) : Let F be any regular closed set of Y containing $f(x)$. Then $f^{-1}(F) \in eO(X)$ and $x \in f^{-1}(F)$. Taking $U = f^{-1}(F)$ we get $f(U) \subset F$.

(c) $\Rightarrow$ (b) : Let F be any regular closed set of Y and $x \in f^{-1}(F)$. Then, there exist $U_x \in eO(X, x)$ such that $f(U_x) \subset F$ and so $U_x \subset f^{-1}(F)$. Also, we have $f^{-1}(F) \subset \cup_{x \in f^{-1}(F)} U_x$. Hence $f^{-1}(F) \in eO(X)$.

(c) $\Leftrightarrow$ (d) : Let V be any regular open set of Y non-containing $f(x)$. Then $(Y-V)$ is regular closed set of Y containing $f(x)$. Hence by (c), there exist $U \in eO(X, x)$ such that $f(U) \subset (Y - V)$. Hence, $U \subset f^{-1}(Y - V) \subset X - f^{-1}(V)$ and so $f^{-1}(V) \subset (X - U)$. Now, since $U \in eO(X)$, $(X - U)$ is e-closed set of X not containing x. The converse part is obvious.

Theorem 4.3 *If $f : X \to Y$ is almost contra-e-continuous, then f is almost weakly-e-continuous.*

Proof. For $x \in X$, let Q be any open set of Y containing $f(x)$. Then $cl(Q)$ is a regular closed set of Y containing $f(x)$. Then by Theorem 4.2, there exist $P \in eO(X, x)$ such that $f(P) \subset cl(Q)$. So f is almost weakly-e-continuous.

The following lemma can be easily verified.

Lemma 4.4 *A function $f : X \to Y$ is almost e-continuous, if and only if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exist $U \in eO(X, x)$ such that $f(U) \subset V$.*

We recall that a topological (X, τ) is said to be extremally disconnected if the closure of every open set of X is open in X.

Theorem 4.5 *Let $f : X \to Y$ be a function, where Y is extremally disconnected. Then f is almost contra-e-continuous if and only if it is almost e-continuous.*

Proof. Suppose $x \in X$ and V be an regular open set of Y containing $f(x)$. Then since Y is extremally disconnected, V is clopen and so it is regular closed. Then using Theorem 4.2, there exist $U \in eO(X, x)$ such that $f(U) \subset V$. Hence by Lemma 4.4, f is almost e-continuous.

Conversely, let f be almost e-continuous and W be any regular closed set of Y. Since Y is extremally disconnected, W is also regular open in Y. Therefore, $f^{-1}(W)$ is e-open in X. This shows that f is almost contra-e-continuous.

Theorem 4.6 *If $f : X \to Y$ is an almost contra-e-continuous injection and Y is weakly Hausdorff, then X is e-T_1.*

Proof. Since Y is weakly Hausdorff, for distinct points x, y of Y, there exist regular closed sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. Now, f being almost contra-e-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are e-open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$. This shows that X is e-T_1.

Corollary 4.7 *If $f : X \to Y$ is an contra-e-continuous injection and Y is weakly Hausdorff, then X is e-T_1.*

Theorem 4.8 *If $f : X \to Y$ is an almost contra-e-continuous surjection and X is e-connected, then Y is connected.*

Proof. If possible, suppose that Y is not connected. Then there exist disjoint non-empty open sets U and V of Y such that $Y = U \cup V$. Since U and V are clopen sets in Y, they are regular open sets of Y. Again, since f is almost contra-e-continuous surjection, $f^{-1}(U)$ and $f^{-1}(V)$ are e-open sets of X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not e-connected – a contradiction. Hence Y is connected.

Definition 4.9 *A topological space (X, τ) is said to be*
(a) e-compact if every e-open cover of X has a finite subcover.
(b) countably e-compact if every countable cover of X by e-open sets has a finite subcover.
(c) e-Lindeloff if every e-open cover of X has a countable subcover.

Theorem 4.10 *Let $f : X \to Y$ is an almost contra-e-continuous surjection. Then the following statements holds:*
(a) If X is e-compact, then Y is S-closed.
(b) If X is e-Lindeloff, then Y is S-Lindeloff.
(c) If X is countably e-compact, then Y is countably S-closed.

Proof. (a) : Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra-e-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a e-open cover of X. Again, since X is e-compact, there exist a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ and hence $Y = \cup\{V_\alpha : \alpha \in I_0\}$. Therefore, Y is S-closed.

Other proofs are being similar to (a) omitted.

Definition 4.11 *A topological space (X, τ) is said to be*
(a) e-closed compact if every e-closed cover of X has a finite subcover.
(b) countably e-closed compact if every countable cover of X by e-closed sets has a finite subcover.
(c) e-closed Lindeloff if every cover of X by e-closed sets has a countable subcover.

Theorem 4.12 *For an almost contra-e-continuous surjection $f : X \to Y$, the following statements holds :*
(a) If X is e-closed compact, then Y is nearly compact.
(b) If X is e-closed Lindeloff, then Y is nearly Lindeloff.
(c) If X is countably e-closed compact, then Y is nearly countable compact.

Proof. (a): Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra-e-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an e-closed cover of X. Again, since X is e-closed compact, there exist a finite subset I_0 of I such that $X = \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$. Hence Y is nearly compact.

Others proofs are being similar to (a) omitted.

5 Closed Graphs via e-Open Sets

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 5.1 *The graph $G(f)$ of a function $f : X \to Y$ is said to be e-closed (resp. contra-e-closed) if for each $(x, y) \in (X \times Y) - G(f)$, there exist an $U \in eO(X, x)$ and an open (resp. a closed) set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 5.2 *The graph $G(f)$ of a function $f : X \to Y$ is e-closed (resp. contra-e-closed) in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$ there exist $U \in eO(X, x)$ and an open set (resp. a closed set) V in Y containing y such that $f(U) \cap V = \emptyset$.*

Proof. We shall prove that $f(U) \cap V = \emptyset \Leftrightarrow (U \times V) \cap G(f) = \emptyset$. Let $(U \times V) \cap G(f) \neq \emptyset$. Then there exist $(x, y) \in (U \times V)$ and $(x, y) \in G(f)$. This implies that $x \in U$, $y \in V$ and $y = f(x) \in V$. Therefore, $f(U) \cap V \neq \emptyset$. Hence the result follows.

Theorem 5.3 *If a function $f : X \to Y$ is contra-e-continuous and Y is Urysohn, then $G(f)$ is contra-e-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is Urysohn, there exist open sets P, Q in Y such that $f(x) \in P$, $y \in Q$ and $cl(P) \cap cl(Q) = \emptyset$. Now, since f is contra-e-continuous, there exist $U \in eO(X, x)$ such that $f(U) \subset cl(P)$ which implies that $f(U) \cap cl(Q) = \emptyset$. Hence by above Lemma 5.2, $G(f)$ is contra-e-closed in $X \times Y$.

Theorem 5.4 *If $f : X \to Y$ is e-continuous and Y is T_1, then $G(f)$ is contra-e-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is T_1 there exist open set V of Y, such that $f(x) \in V$, $y \notin V$. Since f is e-continuous, there exist e-open set U of X containing x such that $f(U) \subset V$. Therefore $f(U) \cap (Y - V) = \emptyset$ and $Y - V$ is a closed set in Y containing y. Hence by Lemma 5.2, $G(f)$ is contra-e-closed in $X \times Y$.

Theorem 5.5 *If $f : X \to Y$ and $g : X \to Y$ are contra-e-continuous functions, where Y is Urysohn, then $D = \{x \in X : f(x) = g(x)\}$ is e-closed in X.*

Proof. Let $x \in (X - D)$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exist open sets U and V such that $f(x) \in U$ and $g(x) \in V$ with $cl(U) \cap cl(V) = \emptyset$. Again, since f and g are contra-e-continuous, then $f^{-1}(cl(U))$ and $f^{-1}(cl(V))$ are e-open sets in X. Let $P = f^{-1}(cl(U))$ and $Q = f^{-1}(cl(V))$, then P and Q are e-open sets of X containing x. Let $M = P \cap Q$, then M is e-open in X. Hence $f(M) \cap g(M) = f(P \cap Q) \cap g(P \cap Q) \subset f(P) \cap g(Q) = cl(U) \cap cl(V) = \emptyset$. Therefore, $D \cap M = \emptyset$ and hence $x \notin e\text{-}cl(D)$. Thus, D is e-closed in X.

Definition 5.6 *The graph $G(f)$ of a function $f : X \to Y$ is said to be strongly contra-e-closed if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in eO(X, x)$ and regular closed set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$*

Lemma 5.7 *The graph $G(f)$ of a function $f : X \to Y$ is strongly contra-e-closed in $X \times Y$ iff for each $(x, y) \in (X \times Y) - G(f)$ there exist $U \in eO(X, x)$ and regular closed set V in Y containing y such that $f(U) \cap V = \emptyset$.*

Theorem 5.8 *If a function $f : X \to Y$ is almost weakly-e-continuous and Y is Urysohn, then $G(f)$ is strongly contra-e-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is Urysohn there exist open sets P, Q in Y such that $f(x) \in P$, $y \in Q$ and $cl(P) \cap cl(Q) = \emptyset$. Now, since f is almost weakly-e-continuous, there exist $U \in eO(X, x)$ such that $f(U) \subset cl(P)$. This implies that $f(U) \cap cl(Q) = f(U) \cap cl(int(Q)) = \emptyset$, where $cl(int(Q))$ is regular closed in Y. Hence by above Lemma 5.7, $G(f)$ is strongly contra-e-closed in $X \times Y$.

Almost-e-continuous functions can be equivalently defined as:

A function $f : X \to Y$ is called almost e-continuous if $f^{-1}(V)$ is e-open in X for every regular open set V of Y.

Now we state an useful lemma:

Lemma 5.9 *A function $f : X \to Y$ is almost e-continuous if and only if for each $x \in X$ and each regular open set Q of Y containing $f(x)$, there exists $P \in eO(X, x)$ such that $f(P) \subset Q$.*

Theorem 5.10 *If $f : X \to Y$ is almost e-continuous and Y is T_2, then $G(f)$ is strongly contra-e-closed.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is T_2, there exist open sets P and Q containing y and $f(x)$, respectively, such that $P \cap Q = \emptyset$; which is equivalent to $cl(P) \cap int(cl(Q)) = \emptyset$. Again, since f is almost e-continuous and Q is regular open, by Lemma 5.9, there exists $S \in eO(X, x)$ such that $f(S) \subset Q \subset int(cl(Q))$. This implies that $f(S) \cap cl(P) = \emptyset$ and so by Lemma 5.7, $G(f)$ is strongly contra-e-closed.

Definition 5.11 *A subset A of a topological space X is called e-dense if $e\text{-}cl(A) = X$.*

Theorem 5.12 *Let $f : X \to Y$ and $g : X \to Y$ be any two functions. If Y is Urysohn, f, g are contra-e-continuous functions and $f = g$ on e-dense set $A \subset X$, then $f = g$ on X.*

Proof. Since f, g are contra-e-continuous and Y is Urysohn, using Theorem 5.5, $D = \{x \in X : f(x) = g(x)\}$ is e-closed in X. Also, we have $f = g$ on e-dense set $A \subset X$. Now, since $A \subset D$ and A is e-dense in X, we have $X = e\text{-}cl(A) \subset e\text{-}cl(D) = D$. Hence $f = g$ on X.

Theorem 5.13 *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ be the graph function of f. Then f is contra-e-continuous if g is contra-e-continuous.*

Proof. Let G be an open set of Y, then $X \times U$ is an open set of $X \times Y$. Since g is contra-e-continuous, we get $f^{-1}(G) = g^{-1}(X \times U)$ is an e-closed of X. Therefore f is contra-e-continuous.

Definition 5.14 *A filter base $\mathcal{F}$ on a topological space X is said to e-converge to a point $x \in X$ if for each $V \in eO(X, x)$, there exists $F \in \mathcal{F}$ such that $F \subset V$.*

Theorem 5.15 *Every function $\psi : X \to Y$, where Y is compact with e-closed graph is e-continuous.*

Proof. Let ψ be not e-continuous at $x \in X$. Then there exists an open set S in Y containing $\psi(x)$ such that $\psi(T) \not\subset S$ for every $T \in eO(X, x)$. It is obvious to verify that $\mathcal{G} = \{T \subset X : T \in eO(X, x)\}$ is a filterbase on X that e-converges to x. Now we shall show that $Y_{\mathcal{G}} = \{\psi(T) \cap (Y - S) : T \in eO(X, x)\}$ is a filterbase on Y. Here for every $T \in eO(X, x)$, $\psi(T) \not\subset S$, i.e. $\psi(T) \cap (Y - S) \neq \emptyset$. So $\emptyset \notin Y_{\mathcal{G}}$. Let $G, H \in Y_{\mathcal{G}}$. Then there are $T_1, T_2 \in \mathcal{G}$ such that $G = \psi(T_1) \cap (Y - S)$ and $H = \psi(T_2) \cap (Y - S)$. Since $\mathcal{G}$ is a filterbase, there exists a $T_3 \in \mathcal{G}$ such that $T_3 \subset T_1 \cap T_2$ and so $W = \psi(T_3) \cap (Y - S) \in Y_{\mathcal{G}}$ with $W \subset G \cap H$. It is clear that $G \in Y_{\mathcal{G}}$ and $G \subset H$ imply $H \in Y_{\mathcal{G}}$. Hence $Y_{\mathcal{G}}$ is a filterbase on Y. Since $Y - S$ is closed in compact space Y, S is itself compact. So, $Y_{\mathcal{G}}$ must adheres at some point $y \in Y - S$. Here $y \neq \psi(x)$ ensures that $(x, y) \notin G(\psi)$. Thus Lemma 5.2 gives us an $U \in eO(X, x)$ and a open set V in Y containing y such that $\psi(U) \cap V = \emptyset$, i.e. $(\psi(U) \cap (Y - S)) \cap V = \emptyset$ — a contradiction.

Theorem 5.16 *If a surjection $\psi : X \to Y$ possesses an e-closed graph, then Y is T_1.*

Proof. Let $p_1, p_2 \in Y$ with $p_1 \neq p_2$. Since ψ is a surjection, there exists an $x_1 \in X$ such that $\psi(x_1) = p_1$ and $\psi(x_1) \neq p_2$. Therefore $(x_1, p_2) \notin G(\psi)$ and so by Lemma 5.2, there exist $U_1 \in eO(X, x_1)$ and open set V_1 in Y containing p_2 such that $\psi(U_1) \cap V_1 = \emptyset$. Then $p_1 \in \psi(U_1)$ but $p_1 \notin V_1$. Similarly, there exists an $x_2 \in X$ such that $\psi(x_2) = p_2$ and $\psi(x_2) \neq p_1$. Therefore $(x_2, p_1) \notin G(\psi)$ and so by Lemma 5.2, there exist $U_2 \in eO(X, x_2)$ and open set V_2 in Y containing p_1 such that $\psi(U_2) \cap V_2 = \emptyset$. Then $p_2 \in \psi(U_2)$ but $p_2 \notin V_2$. Hence V_1 and V_2 are two open sets containing p_1 and p_2 respectively but $p_1 \notin V_1$ and $p_2 \notin V_2$. So Y is T_1.

Theorem 5.17 *If an open surjection $\psi : X \to Y$ possesses an e-closed graph, then Y is e-T_2.*

Proof. Let $p_1, p_2 \in Y$ with $p_1 \neq p_2$. Since ψ is a surjection, there exists an $x_1 \in X$ such that $\psi(x_1) = p_1$ and $\psi(x_1) \neq p_2$. Therefore $(x_1, p_2) \notin G(\psi)$ and so by Lemma 5.2, there exist $U_1 \in eO(X, x_1)$ and open set V in Y containing p_2 such that $\psi(U) \cap V = \emptyset$. Since ψ is e-open, $\psi(U)$ and V are disjoint e-open sets containing p_1 and p_2. So Y is e-T_2.

Acknowledgement: The first author gratefully acknowledges financial support for his research work from University Grants Commission, India vide Project No. PSW-154/11-12.

References

[1] M.E. Abd El-Monsef, S.N. El-Deeb and R.A. Mahmood, β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77-90.

[2] D. Andrijević, Semi-preopen sets, *Math. Vesnik*, 38(1986), 24-32.

[3] D. Andrijević, On b-open sets, *Math. Vesnik*, 48(1996), 59-64.

[4] A.A. Atik, A study of some types of mappings on topological spaces, *Msc. Thesis*, Tanta Univ., Egypt, (1997).

[5] M. Caldas and S. Jafari, Some properties of contra-β-continuous functions, *Mem. Fac. Sci. Kochi. Univ.*, 22(2001), 19-28.

[6] S.N. El-Deeb, I.A. Hasanein, A.S. Mashhour and T. Noiri, On p-regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie*, 27(1983), 311-315.

[7] K. Dlaska, N. Ergun and M. Ganster, Countably S-closed spaces, *Mathematica Slovaca*, 44(3) (1994), 337-348.

[8] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, *Internat. J. Math. Math. Sci.*, 19(2) (1966), 303-310.

[9] J. Dontchev and T. Noiri, Contra-semi-continuous functions, *Mathematica Pannonica*, 10(2) (1999), 159-168.

[10] J. Dontchev and M. Przemski, On various decompositions of continuous and some weakly continuous functions, *Acta. Math. Hungar.*, 71(1-2) (1966), 109-120.

[11] E. Ekici, Almost contra-precontinuous functions, *Bull. Malaysian Math. Soc.*, 27(1) (2004), 53-65.

[12] E. Ekici, On contra-πg-continuous functions, *Chaos, Solitons and Fractals*, 35(2008), 71-81.

[13] E. Ekici, On e-open sets, DP*-sets and DPE*-sets and decompositions of continuity, *Arabian Jour. Sci. Engg.*, 33(24) (2008), 269-282.

[14] E. Ekici and T. Noiri, Contra δ-precontinuous functions, *Bull. Cal. Math. Soc.*, 98(3) (2006), 275-284.

[15] S. Jafari and T. Noiri, Contra-super-continuous functions, *Annales Univ. Sci. Budapest* 42(1999), 27-34.

[16] S. Jafari and T. Noiri, Contra α-continuous functions between topological spaces, *Iranian. Int. J. Sci*, 2(2001), 153-167.

[17] S. Jafari and T. Noiri, On contra-precontinuous functions, *Bull. Malaysian Math. Soc.*, 25(2002), 115-128.

[18] J.E. Joseph and M.H. Kwack, On S-closed spaces, *Proceedings Amer. Math. Soc.*, 80(2) (1980), 341-348.

[19] N. Levine, Semiopen sets and semicontinuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[20] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt*, 53(1982), 47-53.

[21] A.A. Nasef, Some properties of contra-γ-continuous functions, *Chaos, Solitons and Fractals*, 24(2005), 471-477.

[22] T. Nieminen, On ultrapseudocompact spaces and related topics, *Ann. Acad. Sci. Fenn. Ser. A I Math.*, 3(1977), 185-205.

[23] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 15(1965), 961-970.

[24] A. Al-Omari and M.S. Md Noorani, Contra-ω-continuous and almost contra-ω-continuous, *Internat. Jour. Math. Math. Sci.*, Article ID 40469,(2007) 13 pages.

[25] A. Al-Omari and M.S. Md Noorani, Some properties of contra-b-continuous and almost contra-b-continuous functions, *European Jour. Pure. Appl. Math.*, 2(2) (2009), 213-220.

[26] M. Ozkoc and G. Aslim, On strongly-θ-e-continuous functions, *Bull. Korean Math. Soc.*, 47(5) (2010), 10251036.

[27] J.H. Park, B.Y. Lee and M.J. Son, On δ-semiopen sets in topological spaces, *J. Indian Acad. Math.*, 19(1) (1997), 59-67.

[28] S. Raychaudhuri and M.N. Mukherjee, On δ-almost continuity and δ-preopen sets, *Bull. Inst. Math. Acad. Sin.*, 21(1963), 357-366.

[29] M.K. Singal and A. Mathur, On nearly compact spaces, *Boll. Un. Mat. Ital.*, 2(1969), 702-710.

[30] T. Soundararajan, Weakly Hausdorff spaces and the cardinality of topological spaces in general topology and its relations to modern analysis and algebra, *III Proc. Conf. Kanpur, (1968)*, Academia, Prague, (1971), 301–306.

[31] R. Staum, The algebra of bounded continuous functions into a non-archimedean field, *Pacific J. Math.*, 50(1974), 169-185.

[32] N.V. Velicko, H-closed topological spaces, *Amer. Math. Soc. Trans.*, 78(2)(1968), 103-118.