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Contra $\nu g-$Continuity

S. Balasubramanian

Department of Mathematics,
Government Arts College (Autonomous), Karur-639 005(T.N.)
E-mail: mani55682@rediffmail.com

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Abstract

The object of the present paper is to study the basic properties of Contra $\nu g-$continuous functions.

Keywords: *$\nu g-$open sets, $\nu g-$continuity, $\nu g-$irresolute, $\nu g-$open map, $\nu g-$closed map, $\nu g-$homeomorphisms and Contra $\nu g-$continuity.*

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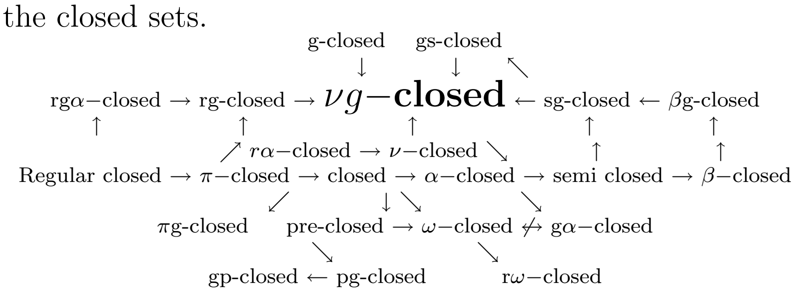
§1 Introduction

In 1996, Dontchev introduced contra-continuous functions. J. Dontchev and T. Noiri introduced Contra-semicontinuous functions in 1999. S. Jafari and T. Noiri defined Contra-super-continuous functions in 1999; Contra-$\alpha-$continuous functions in 2001 and contra-precontinuous functions in 2002. M. Caldas and S. Jafari studied Some Properties of Contra-$\beta-$Continuous Functions in 2001. T. Noiri and V. Popa studied unified theory of contra-continuity in 2002. A.A. Nasef studied some properties of contra-$\gamma-$continuous functions in 2005. M.K.R.S.V.Kumar introduced Contra-Pre-Semi-Continuous Functions in 2005. Ekici E., introduced and studied another form of contra-continuity in 2006. Jamal M. Mustafa introduced Contra Semi-I-Continuous functions in 2010. Recently S. Balasubramanian and P.A.S.Vyjayanthui defined and studied contra $\nu-$continuity in 2011. Inspired with these developments, I introduce a new class of functions called contra $\nu g-$continuous function. Moreover, we obtain basic properties, preservation theorem and relationship with other types of functions.

§2 Preliminaries

Definition 2.1. $A \subset X$ is called
(i) closed if its complement is open.
(ii) regular open[pre-open; semi-open; α-open; β-open] if $A = (\overline{A})^0[A \subseteq (\overline{A})^o$; $A \subseteq \overline{(A^o)}$; $A \subseteq (\overline{(A^o)})^o$; $A \subseteq \overline{((\overline{A})^o)}]$ and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = \overline{A^0}[\overline{(A^o)} \subseteq A$; $(\overline{A})^o \subseteq A$; $\overline{((\overline{A})^o)} \subseteq A$; $(\overline{(A^o)})^o \subseteq A]$
(iii) $\nu-$open[r$\alpha-$open] if there exists a regular open set O such that $O \subset A \subset \overline{O}[O \subset A \subset \alpha(\overline{O})]$
(iv) semi-$\theta-$open if it is the union of semi-regular sets and its complement is semi-$\theta-$closed.
(v) g-closed[resp: rg-closed] if $\overline{A} \subseteq U$ whenever $A \subseteq U$ and U is open[resp: r-open] in X.
(vi) sg-closed[resp: gs-closed] if $s(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is semi-open[resp: open] in X.
(vii) pg-closed[resp: gp-closed; gpr-closed] if $p(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is pre-open[resp: open; regular-open] in X.
(viii)αg-closed[resp: g$\alpha-$closed; rgα-closed] if $\alpha(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is open[resp: $\alpha-$open; r$\alpha-$open] in X.
(ix) νg-closed if $\nu(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is $\nu-$open in X.

Note 1: From definition 2.1 we have the following interrelations among the closed sets.



Definition 2.2: A function $f : X \to Y$ is called
(i) contra-[resp: contra-semi-; contra-pre-;contra-r-;contra-$r\alpha-$; contra-$\alpha-$; contra-$\beta-$; contra-$\omega-$; contra-pre-semi-; contra $\nu-$]continuuos if inverse image of every open set in Y is closed[resp: semi-closed; pre-closed; regular-closed; $r\alpha-$closed; $\alpha-$closed; $\beta-$closed; $\omega-$closed; pre-semi-closed; $\nu-$closed] in X.

§3 Contra νg-continuous maps:

Definition 3.1: A function $f : X \to Y$ is said to be contra $\nu g-$continuous if the inverse image of every open set is $\nu g-$closed.

Note 2: Here after we call contra $\nu g-$continuous function as c.νg.c function shortly.

Example 1: $X = Y = \{a, b, c\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f be identity function, then f is c.νg.c.

Example 2: $X = Y = \{a, b, c, d\}; \tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\} = \sigma$. Let f be identity function, then f is not c.νg.c.

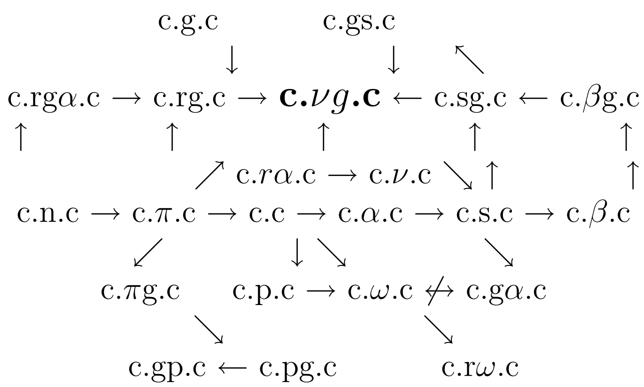
Example 3: $X = Y = \{a, b, c, d\} : \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, Y\}$. Let f be identity function, then f is c.νg.c; c.gpr.c; but not c.gr.c; c.rg.c; c.gs.c; c.sg.c; c.g.c; c.pg.c; c.gp.c; c.rpg.c.

Theorem 3.1:
(i) f is c.νg.c. iff $f^{-1}(A) \in \nu GO(X)$ whenever A is closed in Y.
(ii)Let f be c.rg.c. and r-open, and $A \in \nu GO(X)$ then $f(A) \in \nu GC(Y)$.

Remark 1: Above theorem is false if r-open is removed from the statement as shown by:

Example 4: Let $X = Y = \Re$ and f be defined as $f(x) = 1$ for all $x \in X$ then X is $\nu g-$open in X but $f(X)$ is not $\nu g-$closed in Y.

Remark 2: We have the following implication diagram for a function $f : (X, \tau) \to (Y, \sigma)$



Example 5: If f in Example 4 is defined as f(a) = b; f(b) = c; f(c) = a, then f is c.νg.c. but not c.g.c; c.rg.c; c.gr.c; c.rgα.c. and c.ν.c.

Example 6: Let $X = Y = \{a, b, c\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$. Let f be defined as f(a) = b; f(b) = c; f(c) = a, then

f is c.νg.c. but not c.sg.c.

Example 7: Let $X = Y = \{a, b, c\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let f be defined as f(a) = c; f(b) = a; f(c) = b. Then f is c.rg.c; c.νg..c. but not c.c; c.r.c; and c.ν..c.
under usual topology on $\Re$ both c.g.c and c.rg.c. are same.
under usual topology on $\Re$ both c.sg.c. and c.νg.c. are same.

Theorem 3.2: *(i) If f is $\nu g-$open and c.νg.c., then $f^{-1}(A) \in \nu GC(X)$ whenever $A \in \nu GO(Y)$.*
(ii) If f is an r-open and c.rg.c. mapping, then $f^{-1}(A) \in \nu GC(X)$ whenever $A \in \nu GO(Y)$.

Theorem 3.3: *Let $f_i : X_i \rightarrow Y_i$ be c.νg.c. for i = 1, 2. Let $f : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is c.νg.c.*
Proof: Let $U_1 \times U_2 \subset Y_1 \times Y_2$ where U_i be regular open in Y_i for i = 1, 2. Then $f^{-1}(U_1 \times U_2) = f_1^{-1}(U_1) \times f_2^{-1}(U_2)$. But $f_1^{-1}(U_1)$ and $f_2^{-1}(U_2)$ are $\nu g-$closed in X_1 and X_2 respectively and thus $f_1^{-1}(U_1) \times f_2^{-1}(U_2)$ is $\nu g-$closed in $X_1 \times X_2$. Now if U is any regular open set in $Y_1 \times Y_2$, then $f^{-1}(U) = f^{-1}(\cup U_i)$ where $U_i = U_1^i \times U_2^i$. Then $f^{-1}(U) = \cup f^{-1}(U_i)$ which is $\nu g-$closed, since $f^{-1}(U_i)$ is $\nu g-$closed by the above argument.

Theorem 3.4: *Let $h : X \rightarrow X_1 \times X_2$ be c.νg.c., where $h(x) = (h_1(x), h_2(x))$. Then $h_i : X \rightarrow X_i$ is c.νg.c. for i = 1, 2.*
Proof: Let U_1 is regular open in X_1. Then Let $U_1 \times X_2$ is regular open in $X_1 \times X_2$, and $h^{-1}(U_1 \times X_2)$ is $\nu g-$closed in X. But $h_1^{-1}(U_1) = h^{-1}(U_1 \times X_2)$, therefore $h_1 : X \rightarrow X_1$ is c.νg.c. Similar argument gives $h_2 : X \rightarrow X_2$ is c.νg.c. and thus $h_i : X \rightarrow X_i$ is c.νg.c. for i = 1, 2.

In general we have the following extenstion of theorems 3.3 and 3.4:

Theorem 3.5: *(i) If $f : X \rightarrow \Pi Y_\lambda$ is c.νg.c, then $P_\lambda \circ f : X \rightarrow Y_\lambda$ is c.νg.c for each $\lambda \in \Lambda$, where P_λ is the projection of ΠY_λ onto Y_λ.*
(ii) $f : \Pi X_\lambda \rightarrow \Pi Y_\lambda$ is c.νg.c, iff $f_\lambda : X_\lambda \rightarrow Y_\lambda$ is c.νg.c for each $\lambda \in \Lambda$.

Note 3: Converse of Theorem 3.5 is not true in general, as shown by the following example.

Example 8: Let $X = X_1 = X_2 = [0, 1]$. Let $f_1 : X \rightarrow X_1$ be defined as follows: $f_1(x) = 1$ if $0 \leq x \leq \frac{1}{2}$ and $f_1(x) = 0$ if $\frac{1}{2} < x \leq 1$.
Let $f_2 : X \rightarrow X_2$ be defined as follows: $f_2(x) = 1$ if $0 \leq x < \frac{1}{2}$ and

$f_2(x) = 0$ if $\frac{1}{2} < x < 1$. Then $f_i : X \to X_i$ is clearly c.νg.c. for i = 1, 2., but $h(x) = (f_1(x_1), f_2(x_2)) : X \to X_1 \times X_2$ is not c.νg.c., for $S_{\frac{1}{2}}(1,0)$ is regular open in $X_1 \times X_2$, but $h^{-1}(S_{\frac{1}{2}}(1,0)) = \{\frac{1}{2}\}$ which is not $\nu g-$closed in X.

Remark 3: In general,
(i) The algebraic sum and product of two c.νg.c. functions is not c.νg.c. However the scalar multiple of a c.νg.c. function is c.νg.c.
(ii)The pointwise limit of a sequence of c.νg.c. functions is not c.νg.c. as shown by the following example.

Example 9: Let $X = X_1 = X_2 = [0,1]$. Let $f_1 : X \to X_1$ and $f_2 : X \to X_2$ are defined as follows: $f_1(x) = x$ if $0 < x < \frac{1}{2}$ and $f_1(x) = 0$ if $\frac{1}{2} < x < 1$; $f_2(x) = 0$ if $0 < x < \frac{1}{2}$ and $f_2(x) = 1$ if $\frac{1}{2} < x < 1$. Then their product is not c.νg.c.

Example 10: Let X = Y = [0, 1]. Let f_n is defined as follows: $f_n(x) = x_n$ for $n \geq 1$ then f is the limit of the sequence where $f(x) = 0$ if $0 \leq x < 1$ and $f(x) = 1$ if $x = 1$. Therefore f is not c.νg.c. For $(\frac{1}{2}, 1]$ is open in Y, $f^{-1}((\frac{1}{2}, 1])$ = (1) is not $\nu g-$closed in X.

However we can prove the following theorem.

Theorem 3.6: *Let $f_n : (X, d_X) \to (Y, d_Y)$, be c.$\nu g$.c., for n = 1, 2... and let $f : (X, d_X) \to (Y, d_Y)$ be the uniform limit of $\{f_n\}$, then $f : (X, d_X) \to (Y, d_Y)$ is c.νg.c.*

Problem: (i) Are sup{f, g} and inf{f, g} are c.νg.c. if f, g are c.νg.c.
(ii) Is $C_{c.\nu g.c}(X, R)$, the set of all c.νg.c. functions,
(1) a Group. (2) a Ring. (3) a Vector space. (4) a Lattice.
(iii) Suppose $f_i : X \to X_i (i = 1, 2)$ are c.νg.c. If $f : X \to X_1 \times X_2$ defined by $f(x) = (f_1(x), f_2(x))$, then f is c.νg.c. .
Solution: No.

Note 4: In general c.gpr.c; c.gp.c; c.pg.c and c.gα.c. are independent of c.νg.c. maps

Example 11: f as in Example 1 is c.νg.c, but not c.gpr.c.

Example 12: f as in Example 2 is c.gpr.c. but not c.νg.c.

Theorem 3.7: If f is $\nu g-$irresolute and g is c.νg.c.[c.g.c; c.rg.c], then $g \circ f$ is c.νg.c.

Theorem 3.8: If f is $\nu g-$irresolute, $\nu g-$open and $\nu GO(X) = \tau$ and g be any function, then $g \circ f$ is c.νg.c iff g is c.νg.c.
Proof: If part: Theorem 3.7
Only if part: Let A be closed in Z. Then $(g \circ f)^{-1}(A)$ is $\nu g-$open and hence open in X[by assumption]. Since f is $\nu g-$open $f(g \circ f)^{-1}(A) = g^{-1}(A)$ is $\nu g-$open in Y. Thus g is c.νg.c.

Corollary 3.1: If f is $\nu g-$irresolute, $\nu g-$open and bijective, g is a function. Then g is c.νg.c. iff $g \circ f$ is c.νg.c.

Theorem 3.9: If $g : X \to X \times Y$, defined by $g(x) = (x, f(x)) \forall x \in X$ be the graph function of $f : X \to Y$. Then g is c.νg.c iff f is c.νg.c.
Proof: Let $V \in C(Y)$, then $X \times V \in C(X \times Y)$. Since g is c.νg.c., then $f^{-1}(V) = g^{-1}(X \times V) \in \nu GO(X)$. Thus, f is c.νg.c.
Conversely, let $x \in X$ and F be closed in $X \times Y$ containing g(x). Then $F \cap (\{x\} \times Y)$ is closed in $\{x\} \times Y$ containing g(x). Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y : (x, y) \in F\}$ is closed in Y. Since f is c.νg.c. $\bigcup\{f^{-1}(y) : (x, y) \in F\}$ is $\nu g-$open in X. Further $x \in \bigcup\{f^{-1}(y) : (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is $\nu g-$open. Thus g is c.νg.c.

Theorem 3.10: *(i) If f is c.νg.c. and g is continuous then $g \circ f$ is c.νg.c.*
(ii) If f is c.νg.c. and g is nearly-continuous then $g \circ f$ is c.νg.c.
(iii)If f and g are c.rg.c. then $g \circ f$ is νg.c
(iv) If f is c.νg.c. and g is c.rg.c., then $g \circ f$ is semi-continuous and $\beta-$continuous.

Remark 4: In general, composition of two c.νg.c. functions is not c.νg.c. However we have the following example:

Example 13: Let X = Y = Z = {a, b, c} and $\tau = \wp(X)$; $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$, and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Let f and g be identity maps which are c.νg.c., then $g \circ f$ is c.νg.c.

Theorem 3.11: *Let X, Y, Z be spaces and every $\nu g-$closed set be open[r-open] in Y, then the composition of two c.νg.c. maps is c.νg.c.*

Theorem 3.12: *(i) If f is c.νg.c.[c.rg.c.] g is g-continuous[rg-continuous] and Y is $T_{\frac{1}{2}}$[$rT_{\frac{1}{2}}$] space, then $g \circ f$ is c.νg.c.*
(ii) If f is c.ν.c.[c.r.c.], g is continuous[r-continuous], then $g \circ f$ is c.νg.c.
(iii)If f is c.ν.c.[c.r.c.], g is g-continuous$\{$rg-continuous$\}$ and Y is $T_{\frac{1}{2}}\{rT_{\frac{1}{2}}\}$, then $g \circ f$ is c.νg.c.

Theorem 3.13: *(i) If $R\alpha C(X) = RC(X)$ then f is c.rα.c. iff f is c.rg.c.*
(ii) If $R\alpha C(X) = \nu gC(X)$ then f is c.rα.c. iff f is c.νg.c.
(iii)If $\nu gC(X) = RC(X)$ then f is c.rα.c. iff f is c.νg.c.
(iv) If $\nu gC(X) = \alpha C(X)$ then f is c.α.c. iff f is c.νg.c.
(v) If $\nu gC(X) = SC(X)$ then f is c.sg.c. iff f is c.νg.c.
(vi) If $\nu gC(X) = \beta C(X)$ then f is c.βg.c. iff f is c.νg.c.

Example 14: $X = Y = \{a, b, c\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f be identity function, then f is c.νg.c; c.sg.c. but not c.rg.c

Note 5: Pasting Lemma is not true with respect to c.νg.c. functions. However we have the following weaker versions.

Theorem 3.14: *Let X and Y be such that $X = A \cup B$. Let $f_{/A} : A \to Y$ and $g_{/B} : B \to Y$ are c.rg.c. such that $f(x) = g(x) \forall x \in A \cap B$. Suppose A and B are r-closed sets in X and RC(X) is closed under finite unions, then the combination $\alpha : X \to Y$ is c.νg.c.*

Theorem 3.15: Pasting Lemma *Let X and Y be such that $X = A \cup B$.Let $f_{/A} : A \to Y$ and $g_{/B} : B \to Y$ are c.νg.c. such that $f(x) = g(x)\ \forall x \in A \cap B$. Suppose A, B are r-closed sets in X and $\nu gC(X)$ is closed under finite unions, then the combination $\alpha : X \to Y$ is c.νg.c.*
Proof: Let F be open set in Y, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ where $f^{-1}(F)$ is $\nu g-$closed in A and $g^{-1}(F)$ is $\nu g-$closed in B $\Rightarrow f^{-1}(F)$ and $g^{-1}(F)$ are $\nu g-$closed in X $\Rightarrow f^{-1}(F) \cup g^{-1}(F)$ is $\nu g-$closed in X[by assumption] $\Rightarrow \alpha^{-1}(F)$ is $\nu g-$closed in X. Hence α is c.νg.c.

Theorem 3.16: *The following are equivalent:*
(i) f is c.νg.c.
(ii) $\forall x \in X$ and each $V \in C(Y, f(x))$, $\exists U \in \nu GO(X, x)$ and $f(U) \subset V$.
(iii)$f^{-1}(V)$ is $\nu g-$open in X whenever V is closed in Y.

Definition 3.2: A function f is said to be
(i) strongly $\nu g-$continuous if the inverse image of every set is $\nu g-$clopen.
(ii) perfectly $\nu g-$continuous if the inverse image of every open set is $\nu g-$clopen.
(iii)M-$\nu g-$open if the image of each $\nu g-$open set of X is $\nu g-$open in Y.

Theorem 3.17:
(i) Everly strongly νg.c function is c.νg.c. and νg.c.
(ii) Everly perfectly νg.c function is c.νg.c. and νg.c.
(iii) Everly strongly νg.c function is perfectly νg.c.

Theorem 3.18: *The following statements are equivalent for a function f:*
(1) f is c.νg.c.;
(2) $f^{-1}(F) \in \nu GO(X)$ for every $F \in C(Y)$;
(3) for each $x \in X$ and each $F \in C(Y, f(x))$, $\exists\, U \in \nu GO(X, x) \ni f(U) \subset F$;
(4) for each $x \in X$ and $V \in \sigma(Y)$ non-containing f(x), $\exists\, K \in \nu GC(X)$ non-containing $x \ni f^{-1}(V) \subset K$;
(5) $f^{-1}((\overline{G})^o) \in \nu GC(X)$ for every open subset G of Y;
(6) $f^{-1}(\overline{F^o}) \in \nu GO(X)$ for every closed subset F of Y.
Proof: (1)⇔ (2): Let $F \in C(Y)$. Then $Y - F \in RO(Y)$. By (1), $f^{-1}(Y - F) = X - f^{-1}(F) \in \nu GC(X)$. We have $f^{-1}(F) \in \nu GO(X)$. Reverse can be obtained similarly.

(2)⇒(3): Let $F \in C(Y, f(x))$. By (2), $f^{-1}(F) \in \nu GO(X)$ and $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then $f(U) \subset F$.

(3)⇒(2): Let $F \in C(Y)$ and $x \in f^{-1}(F)$. From (3), $\exists\, U_x \in \nu GO(X, x) \ni U_x \subset f^{-1}(F)$. We have $f^{-1}(F) = \bigcup_{x \in f^{-1}(F)} U_x$. Thus $f^{-1}(F)$ is $\nu g-$open.

(3)⇔(4): Let $V \in \sigma(Y)$ not containing f(x). Then, $Y - V \in C(Y, f(x))$. By (3), $\exists\, U \in \nu GO(X, x) \ni f(U) \subset Y - V$. Hence, $U \subset f^{-1}(Y - V) \subset X - f^{-1}(V)$ and then $f^{-1}(V) \subset X - U$. Take $H = X - U$, then $H \in \nu GC(X)$ non-containing x. The converse can be shown easily.

(1)⇔(5): Let $G \in \sigma(Y)$. Since $(\overline{G})^o \in \sigma(Y)$, by (1), $f^{-1}((\overline{G})^o) \in \nu GC(X)$. The converse can be shown easily.

(2)⇔(6): It can be obtained smilar as (1)⇔(5).

Example 15: Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Then the identity function $f\colon X \to X$ is c.νg.c. But it is not regular set-connected.

Theorem 3.19: *If f is c.νg.c. and $A \in RO(X)$[resp: RC(X)], then $f_{|A} : A \to Y$ is c.νg.c.*
Proof: Let $V \in \sigma(Y) \Rightarrow f^{-1}_{/A}(V) = f^{-1}(V) \cap A \in \nu GC(A)$. Hence $f_{/A}$ is c.νg.c.

Remark 5: Every restriction of an c.νg.c. function is not necessarily c.νg.c.

Theorem 3.20: *Let f be a function and $\Sigma = \{U_\alpha : \alpha \in I\}$ be a $\nu g-$cover of X. If for each $\alpha \in I$, $f_{|U_\alpha}$ is c.νg.c., then f is an c.νg.c. function.*

Proof: Let $F \in C(Y)$. $f_{|U_\alpha}$ is c.νg.c. for each $\alpha \in I$, $f^{-1}_{|U_\alpha}(F) \in \nu gO_{|U_\alpha}$. Since $U_\alpha \in \nu GO(X)$, $f^{-1}_{|U_\alpha}(F) \in \nu GO(X)$ for each $\alpha \in I$. Then $f^{-1}(F) = \bigcup_{\alpha\in I} f^{-1}_{|U_\alpha}(F) \in \nu GO(X)$. This gives f is an c.νg.c.

Theorem 3.21: *If f and g are functions. Then, the following properties hold:*
(1) If f is c.νg.c. and g is regular set-connected, then $g \circ f$ is c.νg.c. and νg.c.
(2) If f is c.νg.c. and g is perfectly continuous, then $g \circ f$ is νg.c. and c.νg.c.
Proof: (1) Let$V \in \eta(Z)$. Since g is regular set-connected, $g^{-1}(V)$ is clopen. Since f is c.νg.c., $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is $\nu g-$open and $\nu g-$closed. Therefore, $g \circ f$ is c.νg.c. and νg.c.

(2) can be obtained similarly.

Theorem 3.22: *If f is a surjective M-$\nu g-$open[resp: M-$\nu g-$closed] and g is a function such that $g \circ f$ is c.νg.c., then g is νg.c.*

Theorem 3.23: *If f is c.νg.c., then for each point $x \in X$ and each filter base Λ in X $\nu g-$converging to x, the filter base f(Λ) is rc-convergent to f(x).*

Theorem 3.24: *Let f be a function and $x \in X$. If there exists $U \in \nu GO(X, x)$ and $f_{|U}$ is c.νg.c. at x, then f is c.νg.c. at x.*
Proof: If $F \in C(Y, f(x))$. Since $f_{|U}$ is c.νg.c. at x, there exists $V \in \nu GO(U, x) \ni f(V) = (f_{|U})(V) \subset F$. Since $U \in \nu GO(X, x)$, $V \in \nu GO(X, x)$. Hence$f$ is c.νg.c. at x.

Lemma 3.1:
(i) If V is an open set, then $sCl_\theta(V) = sCl(V)$.
(ii)If V is an regular-open set, then sCl(V) = Int(Cl(V).

Lemma 3.2: *For $V \subset Y, \sigma)$, the following properties hold:*
(1) $\alpha\overline{V} = \overline{V}$ for every $V \in \beta O(Y)$,
(2) $\nu\overline{V} = \overline{V}$ for every $V \in SO(Y)$,
(3) $s\overline{V} = (\overline{V})^o$ for every $V \in RO(Y)$.

Theorem 3.25: *For a function f, the following properties are equivalent:*
(1) f is (νg, s)-continuous;
(2) f is c.νg.c.;
(3) $f^{-1}(V)$ is $\nu g-$open in X for each θ-semi-open set V of Y;
(4) $f^{-1}(F)$ is $\nu g-$closed in X for each θ-semi-closed set F of Y.
Proof: (1)⇒(2): Let $F \in RC(Y)$ and $x \in f^{-1}(F)$. Then $f(x) \in F$ and F is semi-open. Since f is (νg, s)-continuous, $\exists U \in \nu GO(X, x) \ni f(U) \subset \overline{F} = F$.

Hence $x \in U \subset f^{-1}(F)$ which implies that $x \in \nu g(f^{-1}(F))^0$. Therefore, $f^{-1}(F) \subset \nu g(f^{-1}(F))^0$ and hence $f^{-1}(F) = \nu g(f^{-1}(F))^0$. This shows that $f^{-1}(F) \in \nu GO(X)$. It follows that f is c.νg.c.

(2)⇒(3): Follows from the fact that every θ-semi-open set is the union of regular closed sets.

(3)⇔(4): This is obvious.

(4) ⇒ (1): Let $x \in X$ and $V \in SO(Y, f(x))$. Since $\overline{V}$ is closed, it is θ-semi-open. Now, put $U = f^{-1}(\overline{V})$. Then $U \in \nu GO(X, x)$ and $f(U) \subset \overline{V}$. Hence f is (νg, s)-continuous.

Theorem 3.26: *For a function f, the following properties are equivalent:*
(1) f is c.νg.c.;
(2) $f^{-1}(\overline{V})$ is $\nu g-$open in X for every $V \in \beta O(Y)$;
(3) $f^{-1}(\overline{V})$ is $\nu g-$open in X for every $V \in SO(Y)$;
(4) $f^{-1}((\overline{V})^o)$ is $\nu g-$closed in X for every $V \in RO(Y)$.
Proof: (1) ⇒ (2): Let $V \in \beta$O(Y). By Theorem 2.4 of [3] $\overline{V}$ is closed and by Theorem 3.18 $f^{-1}(\overline{V}) \in \nu GO(X)$.

(2) ⇒ (3): This is obvious since $SO(Y) \subset \beta O(Y)$.

(3) ⇒ (4): Let $V \in RO(Y) \Rightarrow Y - (\overline{V})^o$ is closed and hence it is semi-open. Then $X - f^{-1}((\overline{V})^o) = f^{-1}(Y - (\overline{V})^o) = f^{-1}(\overline{(Y - (\overline{V}))^o}) \in \nu GO(X)$. Hence $f^{-1}((\overline{V})^o) \in \nu GC(X)$.

(4) ⇒ (1): Let $V \in RO(Y)$. Then $f^{-1}(V) = f^{-1}((\overline{V})^o) \in \nu GC(X)$.

Corollary 3.2: *For a function f, the following properties are equivalent:*
(1) f is c.νg.c.;
(2) $f^{-1}(\alpha\overline{V})$ is $\nu g-$open in X for every $V \in \beta O(Y)$;
(3) $f^{-1}(\nu\overline{V})$ is $\nu g-$open in X for every $V \in SO(Y)$;
(4) $f^{-1}(s\overline{V})$ is $\nu g-$closed in X for every $V \in RO(Y)$.
Proof: This is an immediate consequence of Theorem 3.26 and Lemma 3.2.

The $\nu g-$frontier of $A \subset X$; is defined by $\nu gFr(A) = \nu g\overline{(A)} - \nu g\overline{(X - A)} = \nu g\overline{(A)} - \nu g(A)^0$.

Theorem 3.27: *$\{x \in X : f : X \to Y$ is not c.νg.c.$\}$ is identical with the union of the $\nu g-$frontier of the inverse images of closed sets of Y containing f(x).*

Proof: If f is not c.νg.c. at $x \in X$. By Theorem 3.18, $\exists$ a closed set $F \in C(Y, f(x) \ni f(U) \cap (\overline{Y-F}) \neq \phi$ for every $U \in \nu GO(X, x)$. Then $x \in \nu g(\overline{f^{-1}(Y-F)}) = \nu g(\overline{X - f^{-1}(F)})$. On the other hand, we get $x \in f^{-1}(F) \subset \nu g(\overline{f^{-1}(F)})$ and hence $x \in \nu gFr(f^{-1}(F))$.
Conversely, If f is c.νg.c. at x and let $F \in C(Y, f(x))$. By Theorem 3.18, there exists $U \in \nu GO(X, x) \ni x \in U \subset f^{-1}(F)$. Therefore, $x \in \nu g(f^{-1}(F))^o$. This contradicts that $x \in \nu gFr(f^{-1}(F))$. Thus f is not c.νg.c.

§4 Contra $\nu g-$Irresolute Maps

Definition 4.1: A function f is said to be contra $\nu g-$irresolute if the inverse image of every $\nu g-$open set is $\nu g-$closed.

Example 16:
(i) Let $X = Y = \{a, b, c\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} = \sigma$. Let f be identity map. Then f is contra $\nu g-$irresolute, contra rg-irresolute, contra gr-irresolute, contra sg-irresolute, contra gs-irresolute, contra g-irresolute, and contra r$\alpha-$irresolute but not contra-irresolute, contra r-irresolute, contra pre-irresolute, contra $\alpha-$irresolute and contra $\beta-$irresolute.

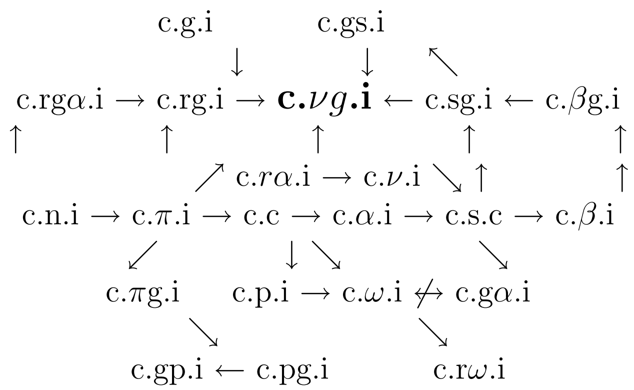
(ii) The identity map f in Example 7 is contra $\nu g-$irresolute, contra r-irresolute but not contra rg-irresolute, contra gr-irresolute, contra sg-irresolute, contra gs-irresolute, contra g-irresolute, contra continuous, contra-irresolute, contra pre-irresolute, contra $\alpha-$irresolute, contra $\beta-$irresolute, and contra r$\alpha-$irresolute.

Example 17: Let $X = Y = \{a, b, c, d\}; \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\} = \sigma$. Let f be defined as f(a) = f(b) = f(c) = d,f(d) = a. Then f is contra $\nu g-$irresolute and $\nu g-$irresolute.

Theorem 4.1: *(i) Let f be c.rg.c. and r-open, then f is contra $\nu g-$irresolute.*
(ii) f is contra $\nu g-$irresolute iff inverse image of every $\nu g-$closed set is $\nu g-$open.

Theorem 4.2: *If f; g are contra $\nu g-$irresolute, then $g \circ f$ is $\nu g-$irresolute.*

Remark 6: We have the following implication diagram for a function $f : X \to Y$



Example 18: The identity map f in Example 1 is contra $\nu g-$irresolute, contra-irresolute but not contra rgα-irresolute, contra rg-irresolute, contra gr-irresolute, contra sg-irresolute, contra gs-irresolute, contra g-irresolute, contra r-irresolute.

Theorem 4.3: *If f is contra $\nu g-$irresolute and*
(i) g is r-irresolute,then $g \circ f$ is contra $\nu g-$irresolute.
(ii)g is contra r-irresolute,then $g \circ f$ is $\nu g-$irresolute.

Note 6: contra $\nu g-$irresolute and c.νg.c.; contra gα-irresolute; contra pg-irresolute; contra gp-irresolute maps are independent to each other

Theorem 4.4: *(i) If $R\alpha C(X) = RC(X)$ and $R\alpha C(Y) = RC(Y)$, then f is contra r$\alpha-$irresolute iff f is contra r-irresolute.*
(ii) If $R\alpha C(X) = \nu gC(X)$ and $R\alpha C(Y) = \nu gC(Y)$, then f is contra r$\alpha-$irresolute iff f is contra $\nu g-$irresolute.
(iii) If $\nu gC(X) = RC(X)$ and $\nu gC(Y) = RC(Y)$, then f is contra r-irresolute iff f is contra $\nu g-$irresolute.
(iv) If $\nu gC(X) = \alpha C(X)$ and $\nu gC(Y) = \alpha C(Y)$, then f is contra $\alpha-$irresolute iff f is contra $\nu g-$irresolute.

Theorem 4.5: Pasting Lemma *Let X and Y be spaces such that $X = A \cup B$ and let $f_{/A} : A \to Y$ and $g_{/B} : B \to Y$ are contra $\nu g-$irresolute maps such that $f(x) = g(x)\ \forall x \in A \cap B$. Suppose A, B are r-open sets in X and $\nu gC(X)$ is closed under finite unions, then the combination $\alpha : X \to Y$ is contra $\nu g-$irresolute.*

Theorem 4.6: *(i) If f is contra $\nu g-$irresolute and g is νg.c.[rg.c.], then $g \circ f$ is c.νg.c.*
(ii)If f is contra $\nu g-$irresolute and g is c.νg.c.[c.rg.c.] then $g \circ f$ is νg.c.

Theorem 4.7: *If $\nu GO(Y,\sigma) = \sigma$ in Y, then f is contra $\nu g-$irresolute iff f is c.νg.c.*

Theorem 4.8: *If $\nu GO(X,\tau) = \tau$; $\nu GO(Y,\sigma) = \sigma$, then the following are equivalent:*
(i) f is c.g.c (ii) f is c.νg.c. (iii)f is contra $\nu g-$irresolute.

Theorem 4.9: *The set of all contra $\nu g-$irresolute mappings do not form a group under the operation usual composition of mappings.*

Theorem 4.10: *If f is contra $\nu g-$irresolute then for every subset A of X, $f(\nu g\overline{(A)}) \subset \nu g\overline{(f(A))}$.*
Proof: Let $A \subseteq X$ and consider $\nu g\overline{(f(A))}$ which is $\nu g-$closed in Y, then $f^{-1}(\nu g\overline{(f(A))})$ is $\nu g-$open in X, by theorem 4.1(ii). Furthermore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\nu g\overline{(f(A))})$ and $\nu g\overline{(A)} \subseteq f^{-1}(\nu g\overline{(f(A))})$, we have $f(\nu g\overline{(A)}) \subseteq f(f^{-1}(\nu g\overline{(f(A))}))$ $= (\nu g\overline{(f(A))} \cap f(Y)) \subseteq \nu g\overline{(f(A))})$. Hence $f(\nu g\overline{(A)} \subseteq \nu g\overline{(f(A)}$.

Theorem 4.11: *If f is contra $\nu g-$irresolute then for every subset A of Y, $\nu g\overline{(f^{-1}(\nu g\overline{(A)}))} \subset f^{-1}(\nu g\overline{(A)})$.*

§5 The Preservation Theorems and Some Other Properties

Theorem 5.1: *If f is c.νg.c.[resp: c.r.c] surjection and X is $\nu g-$compact, then Y is closed compact.*
Proof: Let $\{G_i : i \in I\}$ be any closed cover for Y. For G_i is closed in Y and f is c.νg.c., $f^{-1}(G_i)$ is $\nu g-$open in X. Thus $\{f^{-1}(G_i)\}$ forms a $\nu g-$open cover for X and hence have a finite subcover, since X is $\nu g-$compact. Since f is surjection, $Y = f(X) = \bigcup_{i=1}^{n} G_i$. Therefore Y is closed compact.

Theorem 5.2: *If f is a r-irresolute and continuous surjection and X is mildly compact (resp. mildly countably compact, mildly Lindelof), then Y is nearly compact (resp. nearly countably compact, nearly Lindelof) and S-closed (resp. countably S-closed, S-Lindelof).*
Proof: Let $V \in C(Y)$. Since f is r-irresolute and continuous, $f^{-1}(V)$ is regular-open and closed in X and hence $f^{-1}(V)$ is clopen. Let $\{V_\alpha : \alpha \in I\}$ be any closed (respectively open) cover of Y. Then $\{f^{-1}(V_\alpha : \alpha \in I\}$ is a clopen cover of X and since X is mildly compact, $\exists$ a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha : \alpha \in I_0\}$. Since f is surjective, we get $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$. Hence Y is S-closed (respectively nearly compact). The other proofs can be obtained similarly.

Theorem 5.3:*If f is c.νg.c.[c.rg.c.], surjection. Then the following statements hold:*
(i) If X is locally $\nu g-$compact, then Y is locally closed compact[locally nearly closed compact; locally mildly compact.]
(ii) If X is $\nu g-$Lindeloff[locally $\nu g-$lindeloff], then Y is closed Lindeloff[resp: locally closed Lindeloff; nearly closed Lindeloff; locally nearly closed Lindeloff; locally mildly lindeloff].
(iii)If X is $\nu g-$compact[countably $\nu g-$compact], then Y is S-closed[countably S-closed].
(iv) If X is $\nu g-$Lindelof, then Y is S-Lindelof[nearly Lindelof].
(v) If X is $\nu g-$closed[countably $\nu g-$closed], then Y is nearly compact[nearly countably compact].
(vi) X is $\nu g-$compact[$\nu g-$lindeloff], then Y is nearly closed compact; mildly closed compact[mildly closed lindeloff].

Theorem 5.4: *If f is c.νg.c.[contra $\nu g-$irreolute] surjection and X is $\nu g-$connected, then Y is connected[$\nu g-$connected]*
Proof: If Y is disconnected. Then Y = $V_1 \cup V_2$, where V_1 and V_2 are clopen in Y. Since f is c.νg.c., $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint $\nu g-$open sets in X and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$, which is a contradiction for $\nu g-$connectedness of X. Hence, Y is connected.

Corollary 5.1: *The inverse image of a disconnected[$\nu g-$disconnected] space under a c.νg.c.,[contra $\nu g-$irreolute] surjection is $\nu g-$disconnected.*

Theorem 5.5:*If f is c.νg.c., injection and*
(i) Y is UT_i, then X is $\nu g - T_i$ i = 0,1,2.
(ii) Y is UR_i, then X is $\nu g - R_i$ i = 0, 1.
(iii)Y is $UC_i[resp : UD_i]$ then X is $\nu g - T_i$[resp: $\nu g - D_i$], i = 0, 1, 2.
(iv)If f is closed and Y is UT_i, then X is $\nu g - T_i$, i = 3, 4.

Theorem 5.6: *If f is c.νg.c.[resp: c.rg.c] and Y is UT_2, then the graph G(f) of f is $\nu g-$closed in the product space $X \times Y$.*
Proof: Let $(x, y) \notin G(f) \Rightarrow y \neq f(x) \Rightarrow \exists$ disjoint clopen sets V and W $\ni f(x) \in V$ and y$\in W$. Since f is c.νg.c., $\exists U \in \nu GO(X) \ni x \in U$ and $f(U) \subset W$. Therefore $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is $\nu g-$closed in $X \times Y$.

Theorem 5.7: *If f is c.νg.c.[c.rg.c] and Y is UT_2, then $A = \{(x_1, x_2)|f(x_1) = f(x_2)\}$ is $\nu g-$closed in the product space $X \times X$.*
Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2) \Rightarrow \exists$ disjoint $V_j \in CO(\sigma) \ni$

$f(x_j) \in V_j$, and since f is c.νg.c., $f^{-1}(V_j) \in \nu GO(X, x_j)$ for each j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in \nu GO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is $\nu g-$closed.

Theorem 5.8: *If f is c.r.c.$\{$c.c.$\}$; $g : X \to Y$ is c.νg.c; and Y is UT_2, then $E = \{x \in X : f(x) = g(x)\}$ is $\nu g-$closed[and hence semi-closed and $\beta-$closed] in X.*

Theorem 5.9: *If f is c.νg.c. injection and Y is weakly Hausdorff, then X is $\nu g - T_1$.*

Proof: Suppose that Y is weakly Hausdorff. For any $x \neq y \in X, \exists V, W \in RC(Y) \ni f(x) \in V, f(y) \notin V, f(x) \notin W$ and $f(y) \in W$. Since f is c.νg.c., $f^{-1}(V)$ and $f^{-1}(W)$ are $\nu g-$open subsets of X such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is $\nu g - T_1$.

Theorem 5.10: *If X is $\nu g-$ultra-connected and f is c.νg.c., and surjective, then Y is hyperconnected.*

Proof: If Y is not hyperconnected, $\exists\ V \in \sigma(Y) \ni$ V is not dense in Y. Then $Y = B_1 \cup B_2; B_1 \cap B_2 = \phi$. Since f is c.νg.c. and onto, $A_1 = f^{-1}(B_1)$ and $A_2 = f^{-1}(B_2)$ are disjoint non-empty $\nu g-$closed subsets of X. By assumption, the $\nu g-$ultra-connectedness of X implies that A_1 and A_2 must intersect, which is a contradiction. Therefore Y is hyperconnected.

Theorem 5.11: *If for each $x_1 \neq x_2$ in a space X there exists a function f of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is c.νg.c., at x_1 and x_2, then X is $\nu g - T_2$.*

Proof: Let $x_1 \neq x_2$. By the hypothesis $\exists$ a function f which satisfies the condition of this theorem. Since Y is Urysohn and $f(x_1) \neq f(x_2)$, there exist open sets V_1 and V_2 containing $f(x_1)$ and $f(x_2)$, respectively, such that $\overline{V_1} \cap \overline{V_2} = \phi$. Since f is c.νg.c., at x_i, $\exists U_i \in \nu GO(X, x_i) \ni f(U_i) \subset \overline{V_i}$ for i = 1, 2. Hence $U_1 \cap U_2 = \phi$. Therefore, X is $\nu g - T_2$.

Corollary 5.2: *If f is an c.νg.c. injection and Y is Urysohn, then X is $\nu g - T_2$.*

§6 $\nu g-$Regular Graphs:

Recall that for a function f, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 6.1: A graph $G(f)$ of a function f is said to be $\nu g-$regular if for each $(x, y) \in (X \times Y) - G(f)$, $\exists U \in \nu GC(X, x)$ and $V \in RO(Y, y) \ni$

$(U \times V) \cap G(f) = \phi$.

Lemma 6.1: *The following properties are equivalent for a graph G(f) of a function:*
(1) G(f) is $\nu g-$*regular;*
(2) for each point $(x, y) \in (X \times Y) - G(f)$, $\exists U \in \nu GC(X, x)$ *and* $V \in RO(Y, y) \ni f(U) \cap V = \phi$.
Proof: It is an immediate consequence of definition of $\nu g-$regular graph and the fact that for any subsets $A \subset X$ and $B \subset Y, (A \times B) \cap G(f) = \phi$ iff $f(A) \cap B = \phi$.

Theorem 6.2: *If f is c.*νg*.c., and Y is* T_2 *, then G(f) is* $\nu g-$*regular graph in* $X \times Y$.
Proof: Assume Y is T_2. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is T_2, there exist disjoint open sets V and W containing f(x) and y, respectively. We have $((\overline{V})^o) \cap ((\overline{W})^o) = \phi$. Since f is c.νg.c., $f^{-1}((\overline{V})^o)$ is $\nu g-$closed in X containing x. Take $U = f^{-1}((\overline{V})^o)$. Then $f(U) \subset ((\overline{V})^o)$. Therefore, $f(U) \cap ((\overline{W})^o) = \phi$ and $G(f)$ is $\nu g-$regular in $X \times Y$.

Theorem 6.3: *Let f have a* $\nu g-$*regular graph G(f). If f is injective, then X is* $\nu g - T_1$.
Proof: Let $x \neq y \in X$. Then, we have $(x, f(y)) \in (X \times Y) - G(f)$. By definition 6.1, $\exists U \in \nu GC(X)$ and $V \in RO(Y) \ni (x, f(y)) \in U \times V$ and $f(U) \cap V = \phi$; hence $U \cap f^{-1}(V) = \phi$. Therefore, we have $y \notin U$. Thus, $y \in X - U$ and $x \notin X - U$. We obtain that $X - U \in \nu GO(X)$. This implies that X is $\nu g - T_1$.

Theorem 6.4: *Let f have a* $\nu g-$*regular graph G(f). If f is surjective, then Y is weakly* T_2.
Proof: Let $y_1 \neq y_2 \in Y$. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) - G(f)$. By definition 6.1, $\exists U \in \nu GC(X)$ and $F \in RO(Y) \ni (x, y_2) \in U \times F$ and $f(U) \cap F = \phi$; hence $y_1 \notin F$. Then $y_2 \notin Y - F \in RC(Y)$ and $y_1 \in Y - F$. This implies that Y is weakly T_2.

Example 19: Let $X = \{a, b, c\}, \tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$. Then, the identity function f is contra-$\nu g-$continuous but it is not weakly continuous.

Corollary 6.1:
(i) If f is M-$\nu g-$*open and c.*νg*.c., then f is al.*νg*.c.*
(ii)If f is c.νg*.c. and Y is almost regular, then f is al.*νg*.c.*

Definition 6.2:. A function f is said to be faintly $\nu g-$continuous if for each $x \in X$ and each θ-open set V of Y containing f(x), there exists $U \in \nu GO(X,x) \ni f(U) \subset V$.

Theorem 6.5: *Let Y be E.D. Then, f is c.νg.c. iff it is νg.c.*
Proof: Necessity. Let $x \in X$ and $V \in \sigma(Y, f(x))$. Since Y is E.D., V is clopen and hence V is closed. By Theorem 3.18, $\exists\, U \in \nu GO(X,x) \ni f(U) \subset V$. Therefore f is $\nu g-$continuous.
Sufficiency. Let F be any closed set in Y. Since Y is E.D., F is also open and $f^{-1}(F) \in \nu GO(X)$. Hence f is c.νg.c.

§7 Contra-$\nu g-$Closed Graphs

Definition 7.1: A function f is said to have a contra-$\nu g-$closed graph if for each $(x,y) \in (X \times Y) - G(f)$ there exists $U \in \nu GO(X,x)$ and a closed set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.

Lemma 7.1: *f has a contra-$\nu g-$closed graph iff for each $(x,y) \in (X \times Y) - G(f) \exists U \in \nu GO(X,x)$ and $V \in C(Y,y) \ni f(U) \cap V = \phi$.*

Theorem 7.1: *If f is c.νg.c., and Y is C_2, then G(f) is contra-$\nu g-$closed.*
Proof: Suppose that $(x,y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since Y is C_2, there exist open sets V and W in Y containing y and f(x), respectively, such that $\overline{V} \cap \overline{W} = \phi$. Since f is c.νg.c., there exists $U \in \nu GO(X,x) \ni f(U) \subset \overline{W}$. This shows that $f(U) \cap \overline{V} = \phi$ and hence $G(f)$ is ontra-$\nu g-$closed.

Corollary 7.1: *If f is c.νg.c. and Y is C_2, then $G(f)$ is contra-$\nu g-$closed.*

Theorem 7.2: *If f is an injective c.νg.c. function with the contra-$\nu g-$closed graph, then X is $\nu g - T_2$.*
Proof: Let $x \neq y \in X$. Since f is injective, $f(x) \neq f(y)$ and $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is contra-$\nu g-$closed, by Lemma 7.1 $\exists\, U \in \nu GO(X,x)$ and $V \in RC(Y, f(y)) \ni f(U) \cap V = \phi$. Since f is c.νg.c., by Theorem 3.18 $\exists\, G \in \nu GO(X,y) \ni f(G) \subset V$. Therefore, we have $f(U) \cap f(G) = \phi$; hence $U \cap G = \phi$. Hence X is $\nu g - T_2$.

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